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Book

THE

NATIONAL ARITHMETIC,

ON THE

INDUCTIVE SYSTEM,

COMBINING THE

ANALYTIC AND SYNTHETIC METHODS;

FORMING A COMPLETE

COURSE OF HIGHER ARITHMETIC.

By BENJAMIN GREENLEAF, A.M.

AUTHOR OF THE "COMMON SCHOOL ARITHMETIC," "ALGEBRA," ETC.

NEW ELECTROTYPE EDITION,

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PREFACE.

THE National Arithmetic was first presented to the American public in 1835. The generous favor with which it was received assured the author that he had not misunderstood the wants of the public in the department of arithmetical instruction, and that his labors had, to a considerable extent, supplied those wants.

During the ten years following, increased attention was given to the subject of popular education, and great improvements were made in methods of imparting knowledge. Accomplished teachers soon began to demand a work on Arithmetic, which should embody the numerous improvements which had enriched that science. In response to a demand so reasonable, the author was induced, in 1847, to prepare a revised and enlarged edition of the National Arithmetic. Aided by important suggestions from eminent teachers, and directly assisted by gentlemen intimately acquainted with arithmetical science, he was enabled to produce a work which, up to the present time, has been steadily increasing in public favor.

The last ten years have formed a period of unprecedented activity in all that relates to the interests of education. The numerous Arithmetics which, within this period, have become candidates for popular patronage, afford ample evidence that the department of knowledge to which they relate has meanwhile received its share of attention. Vigorous emulation among authors and publishers has produced thorough investigation, careful preparation, and valuable results.

The author of this work, wishing, if possible, to keep pace with the rapid march of improvement, has again thoroughly revised, rewritten, and considerably enlarged it. The results of a long experience as a mathematical instructor, and the suggestions of many distinguished teachers of the present day, are embodied in this volume.

In preparing this as well as the former editions of his National Arithmetic, the author has regarded the end to be sought in the study of Arithmetic as twofold, — *a practical knowledge of numbers, and the discipline of the mind.* With reference to the former, he has endeavored to present methods which are brief, accurate, and especially adapted to the wants of business life; with reference to the latter, he has aimed to give a clear and logical analysis of every operation, from the simplest to the most involved.

The author adheres to his opinion long since advanced, in relation

to the value of *rules* in an arithmetical treatise. It is not an easy thing for the experienced teacher to express in the most concise and accurate language the method of solving a problem. Much less can such an expression be given by the untrained scholar. Now, as precision in thought is essentially aided by precision in language, it is deemed expedient to furnish the scholar with rules which shall state in the fewest and clearest words the results of previous logical inductions. Moreover, when an intricate reasoning process may have been forgotten and cannot readily be recalled, the brief form of words impressed upon the memory in one's youth may oftentimes enable him in after life to perform an important mathematical operation in which he must otherwise have failed.

It will be observed, that, while the author has expressed in rules his modes of operating, he has in every case first given the analysis upon which each rule is based.

The author flatters himself that the present edition of the National Arithmetic embraces many improvements on former editions. He has endeavored to present clearer definitions, more rigid analyses, and briefer and more accurate rules. While almost every topic included in earlier editions has been treated in a more elaborate and comprehensive manner, this volume comprises a large amount of new matter, which it is believed will be found useful in business.

On comparing this with preceding editions, teachers will find extensive additions and improvements under the heads of Numeration, Addition, and the other fundamental rules, Properties of Numbers, Fractions, Ratio, Percentage, Notes and Banking, Roots, etc. Among the new material will be discovered methods of finding the greatest common divisor and the least common multiple of fractions, of reducing fractions to a common numerator, of contracting the operations in the multiplication and division of decimal fractions, of reducing continued fractions, of averaging accounts, of alligating, of extracting roots to any degree, and of reducing numbers from one system of notation to another.

Especial attention is invited to the section on averaging accounts, — a subject rarely taught in schools, though of great importance in the counting-room, — to the manner of treating the roots, and to the many new problems which will be found in all parts of the book.

In closing these remarks, the author desires to tender his hearty thanks to many teachers who have favored him with valuable suggestions; and to acknowledge in an especial manner his indebtedness to Mr. H. B. Maglathlin, who has been constantly associated with him in making this revision, and to whose accurate scholarship and sound judgment the value of the work is largely due.

Bradford, Mass., *May* 30, 1857.

CONTENTS.

INTRODUCTION.

HISTORY OF ARITHMETIC.

It is difficult to determine who was the inventor of Arithmetic, or in what age or among what people it originated. In ordinary history, we find the origin of the science attributed by some to the Greeks, by some to the Chaldeans, by some to the Phœnicians, by Josephus to Abraham, and by many to the Egyptians. The opinion, however, which modern investigations have rendered most probable, is, that Arithmetic, properly so called, is of Indian origin, — that is, that the science received its first definite form, and became the germ of modern Arithmetic, in the regions of the East.

It is evident, from the nature of the case, that some knowledge of numbers and of the art of calculation was necessary to men in the earliest periods of society, since without this they could not have performed the simplest business transactions, even such as are incidental to an almost savage state. The question, therefore, as to the invention of Arithmetic, deserves to be considered only as it respects the origin of the science as we now have it, and which, as all scholars admit, has reached a surprising degree of perfection. In this sense the honor of the invention must be awarded to the Hindoos.

The history of the various methods of Notation, or the different means by which numbers have been expressed by signs or characters, is one of much interest to the advanced and curious scholar; but the brevity of this sketch allows us barely to touch upon it here. Among the ancient nations which possessed the art of writing, it was a natural and common device to employ letters to denote what we express by our numeral figures. Accordingly we find, that, with the Hebrews and Greeks, the first letter of their respective alphabets was used for 1, the second for 2, and so on to the number 10, — the latter, however, inserting one new character to denote the number 6, and evidently in order that their notation might coincide with that of the Hebrews, the sixth letter of the Hebrew alphabet having no corresponding one in the Greek.

The Romans, as is well known, employed the letters of their alphabet as numerals. Thus I denotes 1; V, 5; X, 10; L, 50; C, 100; D, 500; and M, 1,000. The intermediate numbers were expressed by a repetition of these letters in various combinations; as, II for 2; VI for 6; XV for 15; IV for 4; IX for 9, &c. They frequently expressed any number of thousands by the letter or letters denoting so many units, with a line drawn above; thus, $\overline{\mathrm{V}}$, 5,000; $\overline{\mathrm{VI}}$, 6,000; $\overline{\mathrm{X}}$, 10,000; $\overline{\mathrm{L}}$, 50,000; $\overline{\mathrm{C}}$, 100,000; $\overline{\mathrm{M}}$, 1,000,000.

In the classification of numbers, as well as in the manner of expressing them, there has been a great diversity of practice. While we adopt the *decimal* scale and reckon by tens, the aborigines of Mexico, according to Humboldt, and some of the early nations of Europe, adopted the *vicenary*, reckoning by twenties; some of the Indian tribes, and several of the African tribes, use the *quinary*, reckoning by fives; and the Chinese for more than 4,000 years have used the *binary*, reckoning by twos. The adoption of one or another of these scales has been so general, that they have been regarded as natural, and accounted for by referring them to a common and natural cause. The reason for assuming the binary scale probably lay in the use of the two hands, which were employed as counters in computing; that for employing the quinary, in a similar use of the five fingers on either hand; while the decimal and vicenary scales had respect, the former to the ten fingers on the two hands, and the latter to the ten fingers combined with the ten toes on the naked feet, which were as familiar to the sight of a rude, uncivilized people as their fingers.—It is an interesting circumstance, that in the common name of our numeral figures, digits (*digiti*) or fingers, we preserve a memento of the reason why ten characters and our present decimal scale of numeration were originally adopted to express all numbers, even of the highest order.

It is now almost universally admitted, that our present numeral characters, and the method of estimating their value in a tenfold ratio from right to left, have decided advantages over all other systems, both of notation and numeration, that have ever been adopted. There have been those, as Leibnitz and De Lagni, who have advocated the binary scale; a few, with Claudius Ptolemy, have claimed advantages for the sexagenary scale, or that by sixty; and there are those who think that a *duodecimal* scale, and the use of twelve numeral figures instead of ten, would afford increased facility for rapid and extensive calculations; but most mathematicians are satisfied with the present number of numerals and the scale of numeration, which have attained an adoption all but universal.

It was long supposed, that for our modern Arithmetic the world is indebted to the Arabians. But this, as we have seen, is not the

case. The Hindoos at least communicated a knowledge of it to the Arabians, and, as we are not able to trace it beyond the former people, they must have the honor of its invention. They do not, however, claim this honor, but refer it to the Divinity, declaring that *the invention of nine figures, with device of place, is to be ascribed to the beneficent Creator of the universe.*

But though the invention of modern Arithmetic is to be ascribed to the Hindoos, the honor of introducing it into Europe belongs unquestionably to the Arabians. It was they who took the torch from the East and passed it along to the West. The precise period, however, at which this was done, it is not easy to determine. It is evident, that our numeral characters and our method of computing by them were in use among the Arabians about the beginning of the eighth century, when they invaded Spain, and it is probable that a knowledge of them was soon afterwards communicated to the inhabitants of Spain, and gradually to those of the other European countries.

It is said, that the celebrated Gerbert, afterward Pope Sylvester II., returning to France from Spain, where he had been to acquire a knowledge of the Arabic or Indian notation, about the year 970, introduced it among the French.

About the middle of the eleventh century it is supposed to have been introduced into England by John of Basingstoke, Archdeacon of Leicester.

The Arabic characters, having been first used by astronomers, became circulated over Europe in their almanacs; but do not seem to have secured general adoption in Europe earlier than the twelfth or thirteenth century.

The science of Arithmetic, like all other sciences, was very limited and imperfect at the beginning, and the successive steps by which it has reached its present extension and perfection have been taken at long intervals and among different nations. It has been developed by the necessities of business, by the strong love of certain minds for mathematical science and numerical calculation, and by the call for its higher offices by other sciences, especially that of Astronomy. In its progress, we find that the Arabians discovered the method of proof by casting out the 9's, and that the Italians early adopted the practice of separating numbers into periods of six figures, for the purpose of enumeration. To facilitate the process of multiplication, this latter people also introduced, probably from the writings of Boethius, the Multiplication Table of Pythagoras.

The invention of the Decimal Fraction was a great step in the advancement of arithmetical science, and the honor of it has generally been given to John Muller, commonly called Regiomontanus, about the year 1464. It appears, however, that Stevinus, in 1582, wrote

the first express treatise on the subject. The credit of first using the decimal point, by which the invention became permanently available, is given by Dr. Peacock to Napier, the inventor of Logarithms; but De Morgan says, that it was used by Richard Witt as early as 1613, while it is not shown that Napier used it before 1617. Circulating Decimals received but little attention till the time of Dr. Wallis, the author of the Arithmetic of Infinites. Dr. Wallis died at Oxford, in 1703.

The greatest improvement which the art of computation ever received was the invention of Logarithms, the honor of which is unquestionably due to Baron Napier, of Scotland, about the end of the sixteenth or the commencement of the seventeenth century.

The oldest treatises on Arithmetic now known are the 7th, 8th, 9th, and 10th books of Euclid's Elements, in which he treats of proportion and of prime and composite numbers. These books are not contained in the common editions of the great geometer, but are found in the edition by Dr. Barrow, the predecessor of Sir Isaac Newton in the mathematical chair at Cambridge. Euclid flourished about 300 B. C.

A century later, Eratosthenes invented a method, which is known as his "sieve," for separating prime numbers from others.

The next writer on Arithmetic mentioned in history is Nicomachus, the Pythagorean, who wrote a treatise relating chiefly to the distinctions and divisions of numbers into classes, as plain, solid, triangular, &c. He is supposed to have lived near the Christian era.

About the middle of the fourth century lived Diophantus, a celebrated mathematician, who, besides being the first known author on the subject of Algebra, composed thirteen books on Arithmetic, six of which are still extant.

The next writer of note is Boethius, the Roman, who, however, copied most of his work from Nicomachus. He lived at the beginning of the sixth century, and is the author of the well-known work on the Consolation of Philosophy.

The next writer of eminence on the subject is Jordanus, of Namur, who wrote a treatise about the year 1200, which was published by Joannes Faber Stapulensis in the fifteenth century, soon after the invention of printing.

The author of the first *printed* treatise on Arithmetic was Pacioli, or, as he is more frequently called, Lucas de Burgo, an Italian monk, who in 1484 published his great work entitled *Summa de Arithmetica*, &c., in which our present numerals appear under very nearly their modern form.

In 1522, Bishop Tonstall published a work on the Art of Computation, in the Dedication of which he says, that he was induced to study

Arithmetic to protect himself from the frauds of money-changers and stewards, who took advantage of the ignorance of their employers. In his preparation for this work, he professes to have read all the books which had been published on this subject, adding, also, that there was hardly any nation which did not possess such books.

About the year 1540, Robert Record, Doctor in Physic, printed the first edition of his famous Arithmetic, which was afterward augmented by John Dee, and subsequently by John Mellis, and which did much to advance the science and practice of Arithmetic in England in its early stages. This work, which is now quite a curiosity, effectually destroys the claim to originality in some things of which authors much more modern have obtained the credit. In it we find the celebrated case of a will, which we have in the Miscellaneous Questions of Webber and Kinne, and which, altered in language and the time of making the testament, is the 2nd Miscellaneous Question in the present work. This question is, by his own confession, older than Record, and is said to have been famous since the days of Lucas de Burgo. In Record it occurs under the "Rule of Fellowship." Record was the author of the first treatise on Algebra in the English language.

In 1556, a complete work on Practical Arithmetic was published by Nicolas Tartaglia, an Italian, and one of the most eminent mathematicians of his time.

From the time of Record and Tartaglia, works on Arithmetic have been too numerous to mention in an ordinary history of the science. De Morgan, in his recent work (*Arithmetical Books*), has given the names of a large number, with brief observations upon them, and to this the inquisitive student is referred for further information in regard both to writers and books on this subject since the invention of Printing. It is remarkable that De Morgan knew next to nothing of any American works on Arithmetic. He mentions the "American Accountant" by William Milns, New York, 1797, and gives the name of Pike (probably Nicholas Pike) among the names of which he had heard in connection with the subject. Of the compilation of Webber and the original work of Walsh, he seems to have been entirely ignorant.

The various signs or symbols, which are now so generally used to abridge arithmetical as well as algebraical operations, were introduced gradually, as necessity or convenience taught their importance. The earliest writer on Algebra after the invention of printing was Lucas de Burgo, above mentioned, and he uses *p* for plus and *m* for minus, and indicates the powers by the first two letters, in which he is followed by several of his successors. After this, Steifel, a German, who in 1544 published a work entitled *Arithmetica Integra*, added

considerably to the use of signs, and, according to Dr. Hutton, was the first who employed $+$ and $-$ to denote addition and subtraction. To denote the root of a quantity he also used our present sign $\surd$, originally *r*, the initial of the word *radix*, root. The sign $=$, to denote equality, was introduced by Record, the above-named English mathematician, and for this reason, as he says, that "noe 2 thynges can be moar equalle," namely, than two parallel lines. It is a curious circumstance that this same symbol was first used to denote subtraction. It was also employed in this sense by Albert Girarde, who lived a little later than Record. Girarde dispensed with the *vinculum* employed by Steifel, as in $\overline{3+4}$, and substituted the parenthesis $(3+4)$, now so generally adopted. The first use of the St. Andrew's cross, $\times$, to signify multiplication, is attributed to William Oughtred, an Englishman, who in 1631 published a work entitled *Clavis Mathematicæ*, or Key of Mathematics.

It was intended to notice several other works, ancient and modern, but the length to which this sketch has already extended forbids it.

We had thought of alluding to the ancient philosophic Arithmetic, and the elevated ideas which many of the early philosophers had of the science and properties of numbers. But a word must here suffice. Arithmetic, according to the followers of Plato, was not to be studied "with gross and vulgar views, but in such a manner as might enable men to attain to the contemplation of numbers; not for the purpose of dealing with merchants and tavern-keepers, but for the improvement of the mind, considering it as the path which leads to the knowledge of truth and reality." These transcendentalists considered perfect numbers, compared with those which are deficient or superabundant, as the images of the virtues, which, they allege, are equally remote from excess and defect, constituting a mean between them; as in the case of true courage, which, they say, lies midway between audacity and cowardice, and of liberality, which is a mean between profusion and avarice. In other respects, also, they regard this analogy as remarkable; perfect numbers, like the virtues, are "few in number and generated in a constant order; while superabundant and deficient numbers are, like vices, infinite in number, disposable in no regular series, and generated according to no certain and invariable law."

NOTE TO TEACHERS.

For the convenience of those who require a less extended course, several entire Articles, and some examples, have been marked (°), to be omitted at the option of the teacher.

ARITHMETIC.

DEFINITIONS.

ARTICLE **1.** QUANTITY is anything that can be increased, diminished, or measured; as time, weight, lines, surfaces, and solids.

2. A *unit* is a single thing or quantity regarded as a whole.

3. An *abstract unit* is one that has no reference to any particular thing or quantity.

4. A *concrete unit* is one that has reference to some particular thing or quantity.

5. A *number* is an expression of quantity, representing either a unit or a collection of units.

6. An *abstract number* is a number whose unit is abstract; as, one, six, nine.

7. A *concrete or denominate number* is a number whose unit is concrete; as, one dollar, six pounds, nine men.

8. A *simple number* is a unit, or a collection of units, either abstract, or concrete of a single kind or denomination; as, 1, 15, 1 book, 13 dollars.

9. The *unit of measure* of any quantity is one of the same kind with that by which the quantity is measured or compared; as, in the abstract number, six, the abstract unit is that of measure or comparison; and in six pounds, the concrete unit, one pound, is that of measure or comparison.

10. ARITHMETIC is the science of numbers and the art of computing by them. It treats of the properties and relations

of numbers, and teaches the methods of applying the principles of the science to practical purposes.

11. An *axiom* is a self-evident truth.

12. A *problem* is a question proposed for solution, or something to be done.

13. An *operation* is the process of finding, from given quantities, others that are required.

14. A *sign* is a symbol employed to indicate the relations of quantities, or operations to be performed upon them.

15. A *rule* is a direction for performing an operation.

16. An *example* is a particular application of a general principle or rule.

17. The *principal* or *fundamental* processes of arithmetic are Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

SIGNS.

18. The sign of *equality*, two short horizontal lines, $=$, is read *equal*, or *equal to*, and denotes that the quantities between which it is placed are equal to each other. Thus, 12 inches $=$ 1 foot, signifies that 12 inches are equal to 1 foot.

19. The sign of *addition*, an erect cross, $+$, is read *plus*, *and*, or *added to*, and denotes that the quantities between which it is placed are to be added together. Thus, $8 + 6$ signifies that 6 is to be added to 8.

20. The sign of *subtraction*, a short horizontal line, $-$, is read *minus*, or *less*, and denotes that the quantity on the right of it is to be subtracted from the quantity on the left. Thus, $8 - 6$ signifies that 6 is to be subtracted from 8.

21. The sign of *multiplication*, an inclined cross, $\times$, is read *times*, or *multiplied by*, and denotes that the quantities between which it is placed are to be multiplied together. Thus, 7×6 signifies that 7 is to be multiplied by 6.

22. The sign of *division*, a horizontal line between two dots, $\div$, is read *divided by*, and denotes that the quantity on the left of it is to be divided by that on the right. Thus, $42 \div 6$ signifies that 42 is to be divided by 6.

23. The sign of *aggregation*, a parenthesis, ( ), includ-

ing several numbers, or a vinculum, ——, drawn over them, indicates that the value of the expression is to be used as a single number. Thus, $(17 + 3) \times 5$, indicates that the sum of 17 and 3, or 20, is to be multiplied by 5; and $12 + \overline{(9 - 3) \div 2}$, indicates that the difference between 9 and 3 divided by 2, or 3, is to be added to 12.

AXIOMS.

24. Arithmetic, in common with other branches of the mathematics, is based upon axioms, few in number, and universally admitted to be so clearly true, that no process of reasoning can make them plainer; as,

1. If the same quantity, or equal quantities, be *added* to equal quantities, the *sums* will be equal.
2. If the same quantity, or equal quantities, be *subtracted* from equal quantities, the *remainders* will be equal.
3. If the same quantity, or equal quantities, be *added* to unequal quantities, the *sums* will be unequal.
4. If the same quantity, or equal quantities, be *subtracted* from unequal quantities, the *remainders* will be unequal.
5. If equal quantities be *multiplied* by the same quantity, or equal quantities, the products will be equal.
6. If equal quantities be *divided* by the same quantity, or equal quantities, the *quotients* will be equal.
7. If the same quantity be both *added* to and *subtracted* from another, the *value* of the latter will not be changed.
8. If a quantity be both *multiplied* and *divided* by the same quantity, its *value* will not be changed.
9. If two quantities be equally increased or diminished, their *difference* will not be changed.
10. Quantities which are equal to the *same* quantity are equal to each other.
11. Quantities which are *like parts* of equal quantities are equal to each other.
12. The *whole* of a quantity is greater than any of *its parts*.
13. The *whole* of a quantity is equal to the sum of *all its parts*.

NOTATION AND NUMERATION.

NOTATION.

25. NOTATION is the process of representing numbers by letters, figures, or other symbols.

The common methods of expressing numbers are three: by words, written or spoken; by letters, called the Roman method; and by figures, called the Arabic method.

26. In common language, we express numbers by the terms *one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty-one*, etc., giving a distinct name to each unit as far as *ten*, when we begin a second ten, and pass on to *twenty;* a third ten, and pass on to *thirty;* and so on to *forty, fifty, sixty, seventy, eighty*, and *ninety*. Proceeding thus we reach *ten tens*, which we call *one hundred*, when we begin a second hundred, and pass to *two hundred;* a third hundred, and pass to *three hundred;* and so on as far as *ten hundred*, which we call *one thousand. A thousand thousand* we call *one million; a thousand million, one billion; a thousand billion, one trillion;* and so on with numbers still higher.

NOTE 1.—The term *eleven* is a contraction of *one left* after ten; and *twelve*, of *two left* after ten. *Thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen*, are derived from *three* and *ten, four* and *ten, five* and *ten*, etc. *Twenty, thirty, forty, fifty, sixty, seventy, eighty*, and *ninety* are contractions of *two tens, three tens, four tens*, etc.

NOTE 2.—*Billion* is a contraction of the Latin *bis, twice*, and *million;* and *trillion*, of the Latin *tres, three*, and *million*. In like manner from the Latin numerals, *quatuor, four; quinque, five; sex, six; septem, seven; octo, eight; novem, nine; decem, ten; undecim, eleven; duodecim, twelve; tredecim, thirteen; quatuordecim, fourteen; quindecim, fifteen; sexdecim, sixteen; septendecim, seventeen; octodecim, eighteen; novemdecim, nineteen; viginti, twenty*,—are formed *quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions*, etc.

ROMAN NOTATION.

27. THE ROMAN NOTATION, so called from its having originated with the ancient Romans, employs in expressing numbers seven capital letters, viz.:

I,	V,	X,	L,	C,	D,	M.
one,	five,	ten,	fifty,	one hundred,	five hundred,	one thousand.

All intervening and succeeding numbers are expressed by use of these letters, either in repetitions or combinations. By a letter being written *after* another denoting equal or less value, the *sum* of their values is represented; as, II represents *two;* VI, *six.* By writing a letter denoting a less value *before* a letter denoting a greater, their *difference* of value is represented; as, IV represents *four;* XL, *forty.*

A dash (—) placed over a letter increases the value denoted by the letter a *thousand* times; as, $\overline{\text{V}}$ represents *five thousand;* $\overline{\text{IV}}$, *four thousand.*

TABLE.

I	denotes	one.	XXX	denotes	thirty.
II	"	two.	XL	"	forty.
III	"	three.	L	"	fifty.
IV	"	four.	LX	"	sixty.
V	"	five.	LXX	"	seventy.
VI	"	six.	LXXX	"	eighty.
VII	"	seven.	XC	"	ninety.
VIII	"	eight.	C	"	one hundred.
IX	"	nine.	CI	"	one hundred and one.
X	"	ten.	CC	"	two hundred.
XI	"	eleven.	CCC	"	three hundred.
XII	"	twelve.	CCCC	"	four hundred.
XIII	"	thirteen.	D	"	five hundred.
XIV	"	fourteen.	DC	"	six hundred.
XV	"	fifteen.	DCC	"	seven hundred.
XVI	"	sixteen.	DCCC	"	eight hundred.
XVII	"	seventeen.	DCCCC	"	nine hundred.
XVIII	"	eighteen.	M	"	one thousand.
XIX	"	nineteen.	MD	"	fifteen hundred.
XX	"	twenty.	MM	"	two thousand.
XXI	"	twenty-one.	$\overline{\text{XIX}}$	"	nineteen thousand.
XXII	"	twenty-two.	$\overline{\text{M}}$	"	one million.
XXIII	"	twenty-three.	$\overline{\text{MM}}$	"	two million.

NOTE 1.—The Roman method of Notation is now but little used, except in numbering sections, chapters, and other divisions of books; and for indicating the hours on the face of clocks, watches, or dials.

NOTE 2. — Formerly CIↃ was used to represent one thousand, and the prefixing of a C and the annexing of a Ↄ increased the number denoted *ten* times; thus, CCIↃↃ represented *ten* thousand, and CCCIↃↃↃ, one hundred thousand.

EXERCISES.

Represent the following numbers by letters: —

1. Forty-nine. Ans. XLIX.
2. Ninety-seven.
3. One hundred and eighty-eight.
4. Two hundred and nineteen.
5. Six hundred and sixty-three.
6. One thousand five hundred and six.
7. One thousand eight hundred and fifty-seven.
8. Four thousand four hundred and forty-four.
9. Eleven thousand nine hundred and eleven.
10. One hundred fifty thousand and fifty.
11. One million twenty thousand and twenty.
12. Three million one hundred thousand.

ARABIC NOTATION.

28. ARABIC NOTATION, so called from its having been made known through the Arabs, employs in expressing numbers *ten* characters or figures, viz.:

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	cipher.

The first *nine* are sometimes called *digits*, and the *cipher*, *naught* or *zero*.

29. The *place* of a figure is its particular position with regard to other figures; as in 61 (sixty-one) counting from the right, the 1 occupies the first place and the 6 the second place, and so on for any other like arrangement of figures.

30. The digits have been denominated *significant figures*, because each of itself expresses a positive value, always representing so many *units*, or *ones*, as its name indicates. But the *size* or *value* of the units represented by a figure differs with the place occupied by the figure. Thus in 235 (two hundred and thirty-five), each of the figures, without regard to its place, expresses units, or ones; but these units or ones differ in value. The 5 occupying the first place represents

5 single units; the 3 occupying the second place represents 3 tens, or 3 units each *ten times* the size or value of a unit of the first place; and the 2 occupying the third place represents 2 hundreds, or 2 units each *one hundred times* the size or value of a unit of the first place; *the value expressed by any figure being always made tenfold by each removal of it one place to the left.*

31. The *cipher* becomes significant when connected with other figures, by filling a place that otherwise would be vacant; as in 10 (ten) where it occupies the vacant place of units, and in 102 (one hundred and two) where it fills the vacant place of tens.

32. The *simple* value of a unit is the value expressed by a figure standing alone; or, in a collection, when standing in the right-hand place. Thus 2 alone, or in 32 (thirty-two), expresses a simple value of two single units or ones.

33. The *local* value of a unit is the value expressed by a figure when it is used in combination with another figure or figures, and depends upon the place the figure occupies. Thus, in 44 (forty-four), the 4 in the first place expresses the local value of 4 units, and the 4 in the second place, the local value of 4 tens, or forty.

34. The successive places occupied by figures are often called *orders.* Thus a figure in the first or units' place is called a figure of the *first* order, or of the order of *units;* a figure in the second place is a figure of the *second* order, or of the order of *tens;* in the third place, of the third order, or of the order of *hundreds;* and so on, each figure next to the left belonging to a distinct order, the unit of which is tenfold the size or value of a unit of the order at the right.

Exercises.

1. Write three units of the first order.

2. Write five units of the first order.

3. Write eight units of the second order, with seven of the first.

4. Write two units of the third order, with none of the second, and one of the first.

5. Write seven units of the fourth order, with two of the third, none of the second, and none of the first.

6. Write one unit of the fifth order, with none of the four lower orders.

7. Write six units of the sixth order, five of the fifth, four of the fourth, three of the third, one of the second, and two of the first.

8. Write one unit of the eighth order, with none of the seven lower orders.

9. Write nine units of the ninth order, with six of each of the eight lower orders.

10. Write two units of the twelfth order, with none of the eleventh, none of the tenth, one of the ninth, five of the eighth, nine of the seventh, none of the sixth, none of the fifth, none of the fourth, three of the third, none of the second, and three of the first.

11. Write three units of the fifteenth order, with none of the fourteenth, none of the thirteenth, none of the twelfth, one of the eleventh, seven of the tenth, five of the ninth, one of the eighth, none of the seventh, none of the sixth, five of the fifth, three of the fourth, two of the third, two of the second, and seven of the first.

12. Write four units of the twenty-fifth order, with three of the twenty-fourth, two of the twenty-third, none of the twenty-second, none of the twenty-first, none of the twentieth, none of the nineteenth, none of the eighteenth, none of the seventeenth, five of the sixteenth, and none of the fifteen lower orders.

NUMERATION.

35. Numeration is the process of reading numbers when expressed by figures.

36. There are two methods of numeration; the *French* and the *English*.

French Numeration.

37. The French method of numeration is that in general use on the continent of Europe and in the United States. Beginning at the right, figures occupying more than three places being separated into as many groups as possible of three figures each, called *periods*, it gives a distinct name to each period.

FRENCH NUMERATION TABLE.

Hundreds of Sextillions.	Tens of Sextillions.	Sextillions.	Hundreds of Quintillions.	Tens of Quintillions.	Quintillions.	Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
7	8	9,	1	2	3,	4	5	6,	7	8	9,	1	2	3,	4	5	6,	7	8	9,	1	2	3.
8th Period. Sextillions.			7th Period. Quintillions.			6th Period. Quadrillions.			5th Period. Trillions.			4th Period. Billions.			3d Period. Millions.			2d Period. Thousands.			1st Period. Units.		

The value of the number represented in the table is, seven hundred eighty-nine *sextillions*, one hundred twenty-three *quintillions*, four hundred fifty-six *quadrillions*, seven hundred eighty-nine *trillions*, one hundred twenty-three *billions*, four hundred fifty-six *millions*, seven hundred eighty-nine *thousands*, one hundred twenty-three.

38. The unit of the first period, or right-hand group, is 1; of the second, 1 thousand; of the third, 1 million; of the fourth, 1 billion; of the fifth, 1 trillion; of the sixth, 1 quadrillion; of the seventh, 1 quintillion; of the eighth, 1 sextillion, etc.

The periods above sextillions, in their order, are, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novemdecillions, Vigintillions, etc.

NOTE. — The idea of number is the latest and most difficult to form. Before the mind can arrive at such an abstract conception, it must be familiar with that process of classification by which we successively ascend from individuals to species, from species to genera, from genera to orders. The savage is lost in his attempts at numeration, and significantly expresses his inability to proceed by holding up his expanded fingers or pointing to the hair of his head. It is, indeed, difficult for any mind to form an adequate idea of the larger numbers. To count a million, at the rate of one in a second, would require upward of twenty-three days of twelve hours each. A billion is equal to a million a thousand times repeated, or a number so great, as to exceed all the seconds of time that would elapse in thirty-two years.

39. To read numbers represented by figures according to the French method; —

Begin at the right hand, and point off the figures into as many periods as possible of three places each.

Then, commencing at the left hand, read the figures of each period, giving the name of each period excepting that of units.

EXERCISES.

Read orally, or write in words, the numbers represented by the following figures, according to the French method: —

1.	31620	7.	21623417	13.	3256171894
2.	55216	8.	101315125	14.	5162251912
3.	29156	9.	876223399	15.	13141421211
4.	179213	10.	12345625	16.	457863497689
5.	918512	11.	21177792	17.	434378783434
6.	1219615	12.	6665551161	18.	7852463767445

19.	407000010703801	21.	4781278150166660607O7
20.	200070007801000	22.	800800800800800800800

23. 127081061071081010009007007

24. 4071441400700607000071101800808

40. To write numbers in figures according to the French method; —

Begin at the left hand, and write in each successive order the figure belonging to it.

If any intervening order would otherwise be vacant, fill the place by a cipher.

EXERCISES.

Represent by figures, and read, the following numbers, according to the French method: —

1. Twenty-nine.
2. Four hundred and seven.
3. Twenty-three thousand and seven.
4. Five millions and twenty-seven.
5. Seven millions, two hundred five thousand and five.
6. Two billions, two hundred seven millions, six hundred four thousand and nine.
7. One hundred five billions, nine hundred nine millions, three hundred eight thousand two hundred and one.

8. Nine quintillions, eight billions and forty-six.

9. Fifteen quintillions, thirty-one millions and seventeen.

10. Five hundred seven septillions, two hundred three trillions, fifty-seven millions and eighteen.

11. Nine nonillions, forty-seven trillions, seven billions, two millions, three hundred ninety-two.

12. Fifteen duodecillions, ten trillions, one hundred twenty-seven billions, twenty-six millions, three hundred twenty thousand four hundred twenty-six.

English Numeration.

41. The English method of numeration is that generally used in Great Britain, and in the British Provinces. It divides numbers into periods of *six* figures, and gives a distinct name to each.

ENGLISH NUMERATION TABLE.

Hund. of Thousands of Trillions.	Tens of Thousands of Trillions.	Thousands of Trillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hund. of Thousands of Billions.	Tens of Thousands of Billions.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hund. of Thousands of Millions.	Tens of Thousands of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
3	9	8	8	3	2,	5	6	3	8	7	1,	3	5	1	6	1	5,	1	2	3	5	6	1.
4th Period. Trillions.						3d Period. Billions.						2d Period. Millions.						1st Period. Units.					

The value of the figures in the above table, expressed in words according to the English method, is, Three hundred ninety-eight thousand, eight hundred thirty-two trillions; five hundred sixty-three thousand, eight hundred seventy-one billions; three hundred fifty-one thousand, six hundred fifteen millions; one hundred twenty-three thousand five hundred sixty-one.

42. To read numbers represented by figures according to the English method; —

Begin at the right hand, and point off the figures into periods of six places each.

Then, commencing at the left hand, read the figures of each period, giving the name of each period except that of units.

EXERCISES.

Read orally, or write in words, the numbers represented by the following figures, according to the English method: —

1.	23457896	4.	98765421910311
2.	325487691	5.	563241113232130001 2
3.	1678912161	6.	69617718891332014433455 67

43. To write numbers in figures according to the English method; —

Begin at the left hand, and write in each successive order the figure belonging to it.

If any intervening order would otherwise be vacant, fill the place by a cipher.

EXERCISES.

Represent by figures, and read, the following numbers, according to the English method: —

1. Thirty-two million three hundred.
2. Seven billion seventeen thousand.
3. Five hundred sixty thousand one hundred two million, nine hundred twenty-nine thousand four hundred eleven.
4. One trillion, seven hundred forty-eight thousand nine hundred fifty-five billion.

ADDITION.

44. ADDITION is the process of finding the sum of two or more numbers. The result obtained is called the *amount.*

Numbers can be added together only when their units are of the same kind. When the numbers added are simple, the process is termed *Addition of Simple Numbers.*

45. To add simple numbers.

Ex. 1. A man has three farms; the first contains 378 acres, the second 586 acres, and the third 168 acres. How many acres are there in the three farms. Ans. 1132.

OPERATION.
Acres.
3 7 8
5 8 6
1 6 8
Ans. 1 1 3 2

Having arranged the numbers so that all the units of the same order shall stand in the same column, we first add the column of *units;* thus, 8 and 6 are 14, and 8 are 22 units, = 2 tens and 2 units. We write the two units under the column of units, and *carry* or add the 2 tens to the column of tens; thus, 2 added to 6 make 8, and 8 are 16, and 7 are 23 *tens,* = 2 hundred and 3 tens. We write the 3 tens under the column of tens, and add the 2 hundred to the column of hundreds; thus, 2 added to one make 3, and 5 are 8, and 3 are 11 *hundred,* = 1 thousand and 1 hundred. We write the 1 hundred under the column of hundreds; and there being no other column to be added, we set down the 1 thousand in the thousands' place, and find the amount of the several numbers to be 1132.

In practice, it is better not to name each figure added, but only the results, thus, 8, 14, 22 units, = 2 tens and 2 units, etc.

RULE. — *Write the numbers so that all the figures of the same order shall stand in the same column.*

Add, upward, all the figures in the column of units, and, if the amount be less than ten, write it underneath. But if the amount be ten or more, write down the unit figure only, and add in the figure denoting the ten or tens with the next column.

Proceed in this way with each column, until all are added, observing to write under the last column its whole amount.

46. *First Method of Proof.* — Begin at the top and add the columns downward in the same manner as they were before added upward; and if the two sums agree, the work is presumed to be right.

The reason of this proof is, that, by adding downward, the order of the figures is inverted; and, therefore, any error made in the first addition would probably be detected in the second.

NOTE. — This method of proof is generally used in business.

47. *Second Method of Proof.* — Separate the numbers to be added into two parts, by drawing a horizontal line between them. Add the numbers below the line, and set down their sum. Then add this sum and the number or numbers above the line together; and if their sum is equal to the first amount, the work is presumed to be right.

The reason of this proof depends on the principle, *that the sum of all the parts into which any number is separated is equal to the whole.* (Art. 24, Ax. 13.)

EXAMPLES.

2.	2.	3.	3.
OPERATION.	OPERATION AND PROOF.	OPERATION.	OPERATION AND PROOF.
Dollars.	Dollars.	Tons.	Tons.
765	765	126	126
381	381	384	384
872	872	876	876
315	315	243	243
Ans. 2333	First am't 2333	Ans. 1629	First am't 1629
	1568		1503
	Ans. 2333		Ans. 1629

4.	5.	6.	7.	8.	9.	10.
Barrels.	Pounds.	Acres.	Cents.	Eagles.	Rods.	Poles.
123	678	456	789	456	781	889
456	901	789	987	781	175	776
789	278	127	123	197	564	432
341	633	815	321	715	337	876
1709	2490	2187	2220	2149	1857	2973

11.	12.	13.	14.	15.	16.
Ounces.	Inches.	Rods.	Furlongs.	Cords.	Feet.
7891	3256	6789	1234	4567	4561
3245	7890	1234	5678	8912	7890
6789	1234	5678	9012	3456	7658
1234	5678	6543	3456	7891	8888
5678	7801	1234	7891	4567	9199
24837	25859	21478	27271	29393	38196

17.	18.	19.	20.	21.	22.
Hogsheads.	Furlongs.	Miles.	Dollars.	Casks.	Pence.
1789	6781	7890	1785	7895	4371
6543	1371	1070	5678	5678	1699
2177	8715	4437	9137	7186	1098
8915	6371	6789	8171	5176	8816
6781	1234	5378	1888	4321	6171
4325	7171	1234	1919	4127	7185

23.	24.	25.	26.	27.
Shillings.	Tons.	Miles.	Trees.	Loads.
78956	12345	34567	76717	56789
32167	87655	78901	77776	12345
41328	34517	32199	67890	67819
45678	65483	17188	71444	34567
13853	79061	88888	47474	71888
71667	20939	12345	16175	33197

28.	29.	30.	31.
Acres.	Roods.	Poles.	Yards.
789516	451237	1234567	789123
377895	813715	8901234	456789
378567	679919	5678901	987654
832156	787651	3456789	357913
789567	637171	5432115	245678
813138	813785	7177444	999999

32. What is the sum of 15 + 26 + 18 + 91? Ans. 150.

33. What is the sum of 6789 + 5832 + 4671 + 8907? Ans. 26199.

34. Required the sum of 76 + 48 + 59 + 81. Ans. 264.

35. Required the sum of 123456 + 789012 + 345678 + 901234 + 567890 + 987654 + 321032 + 765437.

36. Required the sum of 876543 + 789112 + 345678 + 965887 + 445566 + 788743 + 399378 + 456789.

37. Required the sum of 789012 + 345678 + 901234 + 789037 + 891133 + 477666 + 557788 + 888878.

38. Required the sum of 987654 + 456112 + 222333 + 456789 + 987654 + 321178 + 123456 + 789561.

39. Required the sum of 678953 + 467631 + 117777 + 888888 + 444444 + 667679 + 998889 + 671236.

40. Required the sum of 783256 + 7128 + 39 + 432815 + 99 + 67851 + 125 + 641236 + 801 + 4328.

41. Required the sum of 12004 + 32 + 1 + 7836 + 100 + 46 + 3 + 6176 + 32 + 91876.

42. Add together 763, 4663, 37, 49763, 6178, and 671. Ans. 62075.

43. Add together 15, 7896, 1, 13, 106, 113, 156, 100, 2201.

44. A butcher sold to A 369 lbs. of beef, to B 169 lbs., to C 861 lbs., to D 901 lbs., to E 71 lbs., and to F 8716 lbs.; what did they all receive? Ans. 11087 lbs.

45. A owes to one creditor 596 dollars, to another 3961, to another 581, to another 6116, to another 469, to another 506, to another 69381, and to another 1261. What does he owe them all? Ans. $82871.

46. If a boy earn 17 cents a day, how much will he earn in 7 days? Ans. 119 cts.

47. If a man's wages be 19 dollars per month, what are they per year? Ans. $228.

48. If a boy receive a present every New Year's day of 1783 dollars, how much money will he possess when he is 21 years old? Ans. $37443.

48. Method of adding two or more columns at one operation.

Ex. 1. A merchant paid for cloth 219 dollars, for flour 416 dollars, for hardware 711 dollars, and for rent 93 dollars. How much did he pay in all? Ans. $1439.

OPERATION.
Dollars.

	2 1 9
	4 1 6
	7 1 1
	9 3
Ans.	1 4 3 9

Beginning with the number last written down, we add the units and tens; thus, 93 and 1 = 94, and 10 = 104, and 6 = 110, and 10 = 120, and 9 = 129, and 10 = 139. Of this amount we write the 9 units and 3 tens under the columns added; and add in the 1 hundred with the column of hundreds; thus, 1 (carried) and 7 = 8, and 4 = 12, and 2 = 14 hundred, of which we write the 4 hundred under the column of hundreds and the 1 thousand in the thousands' place; and find the whole amount to be 1439. In like manner may be added more than two columns at one operation.

NOTE. — The examples that follow can be performed as the above, or by the common method, or by both.

2. How many were the members of Congress in 1856, there being 2 Senators from each State, and Maine sending 6 Representatives, New Hampshire 3, Massachusetts 11, Rhode Island 2, Connecticut 4, Vermont 3, New York 33, New Jersey 5, Pennsylvania 25, Delaware 1, Maryland 6, Virginia 13, North Carolina 8, South Carolina 6, Georgia 8, Alabama 7, Mississippi 5, Louisiana 4, Tennessee 10, Kentucky 10, Ohio 21, Indiana 11, Illinois 9, Wisconsin 3, Iowa 2, Missouri 7, Arkansas 2, Michigan 4, Florida 1, Texas 2, California 2. Ans. 296.

3. Water-mills were invented in the year 555 after Christ; windmills 744 years after water-mills; pumps 126 years after windmills; printing 15 years after pumps; watches 37 years after printing; the spinning-wheel 53 years after watches; the steam-engine 119 years after the spinning-wheel; the fire-engine 14 years after the steam-engine; the spinning-frame 98 years after the fire-engine; and the electro-magnetic telegraph 71 years after the spinning-frame. What year was the electro-magnetic telegraph invented? Ans. 1832.

4. During the American Revolutionary war, the British lost at the battle of Lexington 273 men, at Bunker Hill 1054 men, at Long Island 400 men, at White Plains 300 men, at Fort Washington 1000 men, at Trenton 1020 men, at Princeton 400 men, at Hubbardton 200 men, at Bennington 800 men, at Brandywine 500 men, at Stillwater 600 men, at Germantown 500 men, at Saratoga 400 men, by Burgoyne's surrender 5791 men, at Fort Mercer 500 men, at Monmouth 400 men, at Rhode Island 260 men, at Brier Creek 23 men, at Stony Point 600 men, at Savannah 130 men, at Camden 325 men, at King's Mountain 1150 men, at Cowpens 800 men, at Guilford Court-House 600 men, at Hobkirk's Hill 250 men, at Eutaw Springs 700 men, at Yorktown 7000 men. What was the entire loss? Ans. 25976 men.

5. Required the number of square miles in the following States, there being in Maine 35,000, in New Hampshire 8,030, in Vermont 8,000, in Massachusetts 7,250, in Rhode Island 1,200, in Connecticut 4,750, in New York 46,000, in New Jersey 6,851, in Pennsylvania 47,000, in Ohio 39,964, in Michigan 56,243, in Indiana 33,809, in Illinois 55,405, in Wisconsin 53,924, in Iowa 50,914, and in California 188,982. Ans. 643,322.

6. Required the number of square miles in the following States, there being in Delaware 2,120, in Maryland 11,000, in Virginia 61,352, in North Carolina 45,500, in South Carolina 28,000, in Georgia 58,000, in Alabama 50,722, in Florida 59,268, in Mississippi 47,147, in Tennessee 44,000, in Kentucky 37,680, in Missouri 67,380, in Arkansas 52,198, in Louisiana 46,431, and in Texas 325,520. Ans. 936,318.

7. According to the census of 1850, Maine had 583,169

inhabitants, New Hampshire 317,976, Massachusetts 994,514, Rhode Island 147,545, Connecticut 370,792, Vermont 314,120, New York 3,097,394, New Jersey 489,555, Pennsylvania 2,311,786, Delaware 91,532, Maryland 583,034, District of Columbia 51,687, Virginia 1,421,661, North Carolina 869,039, South Carolina 668,507, Georgia 906,185, Florida 87,445, Alabama 771,623, Mississippi 606,526, Lousiana 517,762, Texas 212,592, Arkansas 209,897, Tennessee 1,002,717, Missouri 682,044, Kentucky 982,405, Ohio 1,980,329, Indiana 988,416, Illinois 851,470, Michigan 397,654, Wisconsin 305,391, Iowa 192,214, California 92,597, and the Territories 92,298. What was the whole number of inhabitants?

Ans. 23,191,876.

SUBTRACTION.

49. SUBTRACTION is the process of taking one number from another to find the difference.

When the two numbers are unequal, the larger is called the *Minuend* and the less number the *Subtrahend;* and when the numbers are equal, either is the *Minuend,* and the other is the *Subtrahend.* The result, or the number found by the subtraction, is called the *Difference,* or *Remainder.*

One number can be subtracted from another only when the units of both are of the same kind; and when the numbers are simple, the process is termed *Subtraction of Simple Numbers.*

50. To subtract simple numbers.

Ex. 1. From 935 take 673. Ans. 262.

OPERATION.

Minuend	9 3 5
Subtrahend	6 7 3
Remainder	2 6 2

We first take the 3 units from the 5 units, and find the difference to be 2 units, which we write under the figure subtracted. We then proceed to take the 7 tens from the 3 tens above it; but we here find a difficulty, since the 7 is greater than the 3, and cannot be subtracted from it. We therefore add 10 tens to the 3 tens, which makes 13 tens, and then subtract the 7 from 13, and 6 tens remain, which we write below. Then, to compensate for the 10 tens, equal to 1 hundred, added to the 3 tens in the minuend, we add 1 hundred to the 6 hundred of the subtrahend, which makes

7 hundreds, and subtract the 7 hundreds from 9 hundreds, and 2 hundreds remain. By adding the 10 tens to the minuend and the 1 hundred to the subtrahend, *the two numbers being equally increased, their difference is not changed.* (Art. 24, Ax. 9.) The remainder is 262.

NOTE. — The addition of 10 to the minuend is sometimes called *borrowing* 10, and the addition of 1 to the subtrahend is called *carrying* 1.

RULE. — *Place the less number under the greater, so that units of the same order shall stand in the same column.*

Commencing at the right hand, subtract each figure of the subtrahend from the figure above it.

If any figure of the subtrahend is larger than the figure above it in the minuend, add 10 *to that figure of the minuend before subtracting, and then add* 1 *to the next figure of the subtrahend.*

51. *First Method of Proof.* — Add the remainder and the subtrahend together, and their sum will be equal to the minuend, if the work is right.

This method of proof depends on the principle, *that the greater of any two numbers is equal to the less added to the difference between them.*

52. *Second Method of Proof.* — Subtract the remainder or difference from the minuend, and the result will be like the subtrahend, if the work is right.

This method of proof depends on the principle, *that the smaller of any two numbers is equal to the remainder obtained by subtracting their difference from the greater.*

EXAMPLES.

	2.	2.	3.	3.
	OPERATION.	OPERATION AND PROOF.	OPERATION.	OPERATION AND PROOF.
Minuend	469	469	788	788
Subtrahend	183	183	369	369
Remainder	286	286	419	419
		Min. 469		Sub. 369

	4.	5.	6.	7.
	Miles.	Gallons.	Minutes.	Pecks.
From	7654	7116	6178	4567
Take	1978	1997	1769	1978

	8. Barrels.	9. Degrees.	10. Furlongs.	11. Tons.
From	765116	56789	56781	71678
Take	716669	10091	39109	18819

	12. Hogsheads.	13. Bushels.	14. Yards.	15. Pounds.
From	611000	617853	7111111	999000
Take	199999	190909	909009	199919

	16. Roods.	17. Acres.	18. Poles.	19. Cords.
From	100200	511799	610000	789111
Take	98761	419109	166666	171670

	20. Dollars.	21. Eagles.	22. Guineas.
From	10000000	99999999	888888
Take	9099019	1000919	99999

	23. Seconds.	24. Hours.
From	1002003004005 00	600700800900
Take	90807060504039	191818917185

	25. Months.	26. Days.	27. Weeks.
From	61567101	1000000	10000000
Take	91678	1	9999999

28. What is the value of 6767851 — 81715?
29. What is the value of 761619161 — 916781?
30. What is the value of 31671675 — 361784?
31. What is the value of 16781321 — 100716?
32. What is the value of 1002007000 — 5971621?

33. Sir Isaac Newton was born in the year 1642, and he died in 1727; how old was he at the time of his decease? Ans. 85 years.

34. Gunpowder was invented in the year 1330; how long was this before the invention of printing, which was in 1440? Ans. 110 years.

35. The mariner's compass was invented in Europe in the year 1302; how long was this before the discovery of America by Columbus, which happened in 1492? Ans. 190 years.

36. What number is that, to which if 6956 be added, the sum will be one million? Ans. 993044.

37. A man bought an estate for seventeen thousand five hundred and sixty-five dollars, and sold it for twenty-nine thousand three hundred and seventy-five dollars. Did he gain or lose, and how much? Ans. Gained $11810.

38. Bonaparte was declared emperor in 1804; how many years since?

39. The union of the government of England and Scotland was in the year 1603; how long was it from this period to 1776, the time of the declaration of the independence of the United States? Ans. 173 years.

40. Jerusalem was taken and destroyed by Titus in the year 70; how long was it from this period to the time of the first Crusade, which was in the year 1096? Ans. 1026 years.

41. From the Creation to the Deluge was 1656 years; thence to the founding of Rome 1595 years; thence to the death of Charlemagne, which took place 814 years after Christ, 1567 years. In what year of the world was Christ born? Ans. 4004.

42. A gentleman 83 years old has two sons; the age of the older son added to his makes 128 years, and the age of the younger son is equal to the difference between the age of the father and that of the older son. How old is each of his sons? Ans. The older, 45 years; the younger, 38 years.

43. During the year 1810, there were manufactured in the United States one hundred and forty-six thousand nine hundred and seventy-four yards of cotton cloth; and during the year 1855, five hundred and twenty million yards. What was the increase? Ans. 519,853,026 yards.

53. Method of subtracting, when there are two or more subtrahends.

Ex. 1. From a pile of wheat containing 657 bushels, A is to have 141 bushels, B 244 bushels, C 134 bushels, and D the remainder. How many bushels is D to have? Ans. 138.

FIRST OPERATION.	Bushels.	SECOND OPERATION.	Bushels.
Minuend	657	Minuend	657
	141		141
	244		244
	134		134
Subtrahend	519	Remainder	138
Remainder	138		

In the first operation, *the several subtrahends are added for a single subtrahend* to be taken from the minuend. In the second, *the subtrahends are subtracted as they are added, at one operation*, thus: beginning with units, 4 and 4 and 1 = 9, which from 17 units leaves 8 units; passing to tens, 1 (carried) and 3 and 4 and 4 = 12 tens; reserving the left-hand figure to add in with the figures of the subtrahends in the next column, the right-hand figure, 2 tens, which we subtract from the 5 tens of the minuend, and have left 3 tens; and, passing to hundreds, we add in the left-hand figure 1, reserved from the 12 tens, which with the other figures 1 and 2 and 1 = 5 hundreds, which, taken from 6 hundreds, leaves 1 hundred; and 138 is the answer sought.

2. John Drew has a yearly income of 2,500 dollars; his family expenses are 1,300 dollars, his expenditures in improving his estate 450 dollars, and his contributions to several worthy objects 225 dollars. What remains to lay up or invest?

3. A speculator bought four village lots; for the first he paid 620 dollars; for the second, 416 dollars; for the third, 350 dollars; for the fourth, 225 dollars; and sold the whole for 2,000 dollars. What did he gain?

4. Daniel White, dying, left property to the amount of 27,563 dollars, of which his wife received 9,188 dollars, each of his two daughters, 4,594 dollars, and his only son the balance. What did his son receive? Ans. 9,187 dollars.

5. The United States contain 2,983,153 square miles, of which the Atlantic slope includes 967,576, the Pacific slope 778,266, and the Mississippi Valley the remainder. How many square miles does the Mississippi Valley contain?

Ans. 1,237,311.

6. The British North American Provinces contain 3,125,401 square miles; of which 147,832 square miles belong to Canada West; 201,989 to Canada East; 27,700 to New Brunswick; 18,746 to Nova Scotia; 2,134 to Prince Edward's Island; 57,000 to Newfoundland; 170,000 to Labrador; and the re-

mainder to the Hudson's Bay Territory. What number of square miles belong to the Hudson's Bay Territory?

Ans. 2,500,000.

7. James Howe has property to the amount of 63,450 dollars, and owes in all three debts; one of 1000 dollars, another of 350 dollars, and another of 12,468 dollars. How much has he after paying his debts?

8. The entire coinage of the mint of the United States, including the coinage of its branches, from 1792 to 1856, amounted in value to $498,197,382, of which $396,895,574 was gold, $100,729,602 silver, and the remainder of the amount copper. What was the value of the copper coinage?

Ans. $572,206.

MULTIPLICATION.

54. MULTIPLICATION is the process of taking one number as many times as there are units in another number.

In multiplication three terms are employed, called the *Multiplicand*, the *Multiplier*, and the *Product*.

The *multiplicand* is the number to be multiplied or taken.

The *multiplier* is the number by which we multiply, and denotes the number of times the multiplicand is to be taken.

The *product* is the result, or number produced by the multiplication.

The multiplicand and multiplier together are called FACTORS, from the product being *made* or produced by them.

When the multiplicand consists of a simple number, the process is termed *Multiplication of Simple Numbers.*

In the following table, the invention of Pythagoras, may be found all the elementary products necessary in performing any operation in multiplication, since the multiplication of numbers, however large, depends upon the product of one digit by another. The products, therefore, of each digit by any other, should be thoroughly committed to memory. Considerable more of the table, even, may be memorized with fully compensating results.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625

For example, suppose we wish to find the product of 7 by 5; we look for 7 at the top of the table, and for 5 at the left hand, and where the lines intersect is 35, the number sought; or, we may look for the 7 at the left hand, and the 5 at the top, and *find, where the lines* intersect, the same result.

55. The repeated addition of a number to itself is equivalent to a multiplication of that number. Thus, $7 + 7 + 7 + 7$ is equivalent to 7×4, the sum of the former and the product of the latter being the same. Hence multiplication has sometimes been called a *concise method of addition.*

56. The product must be of the same kind or denomination as the multiplicand, since the taking of a quantity any number of times does not alter its nature. Thus: 5, an *abstract number*, $\times 3 = 15$, an *abstract number;* and 9 yards $\times 7 = 63$ yards.

57. The multiplier must always be considered as an abstract number. Thus, in finding the cost of 4 *books* at 9 *dollars* each, we cannot multiply books and dollars together, which would be absurd, but we can, by regarding the 4 as an abstract number, take the 9 dollars, or cost of 1 book 4 times, and the product, 36 dollars, will be the result required.

58. The product of two factors will be the same, whichever is taken as the multiplier. Thus, $8 \times 6 = 6 \times 8 = 48$; and the cost of 5 hats at 2 dollars each gives the same product as 2 hats at 5 dollars each. Also, the product of any number of factors is the same, in whatever order they are multiplied. Thus, $2 \times 3 \times 5 = 3 \times 5 \times 2 = 5 \times 2 \times 3 = 30$.

59. A COMPOSITE number is a number produced by multiplying together two or more numbers greater than 1. Thus, 10 is a composite number, since it is the product of 2×5; and 18 is a composite number, since it is the product of $2 \times 3 \times 3$.

60. To multiply simple numbers.

Ex. 1. Let it be required to multiply 1538 by 9.

Ans. 13842.

	OPERATION.
Multiplicand	1 5 3 8
Multiplier	9
Product	1 3 8 4 2

Having written the multiplier, 9, under the unit figure of the multiplicand, we multiply the 8 *units* by the 9, obtaining 72 *units* = 7 tens and 2 units. We write down the 2 units in the units' place, and reserve the 7 tens to add to the product of the tens. We then multiply the 3 *tens* by 9, obtaining 27 *tens*, and, adding the 7 tens which were reserved, we have 34 tens = 3 hundreds and 4 tens. We write down the 4 tens in the tens' place, and reserve the 3 hundreds to add to the product of the hundreds. We next multiply the 5 *hundreds* by 9, obtaining 45 *hundreds*, and, adding the 3 hundreds which were reserved, we have 48 hun-

dreds = 4 thousands and 8 hundreds. We write down the 8 hundreds in the hundreds' place, and reserve the 4 thousands to add to the product of the thousands. By multiplying the 1 *thousand* by 9 we obtain 9 *thousands*, and, adding the 4 thousands reserved, we have 13 thousands, which we write down in full; — and the product is 13842.

2. Let it be required to multiply 2156 by 423.

Ans. 911988.

	OPERATION.
Multiplicand	2 1 5 6
Multiplier	4 2 3
Partial Products	6 4 6 8
	4 3 1 2
	8 6 2 4
Product	9 1 1 9 8 8

In this example the multiplicand is to be taken 423 times = 3 units times + 2 tens times + 4 hundreds times. 3 *units* times 2156 = 6468 *units;* 2 *tens* times 2156 = 4312 *tens;* and 4 *hundreds* times 2156 = 8624 *hundreds;* the sum of which partial products = 911988, or the total product required. In the operation the right-hand figure of each partial product is written directly under its multiplier, that units of the same order may stand in the same column, for convenience in adding.

Rule. — *Write the multiplier under the multiplicand, arranging units under units, tens under tens, &c.*

Multiply each figure of the multiplicand by each figure of the multiplier, beginning with the right-hand figure, writing the right-hand figure of each product underneath, and adding the left-hand figure or figures, if any, to the next succeeding product.

If the multiplier consists of more than one figure, the right-hand figure of each partial product must be placed directly under the figure of the multiplier that produces it. The sum of the partial products will be the whole product required.

Note. — When there are ciphers between the significant figures of the multiplier, pass over them in the operation, and multiply by the significant figures only, remembering to set the first figure of the product directly under the figure of the multiplier that produces it.

61. *First Method of Proof.* — Multiply the multiplier by the multiplicand, and, if the result is like the first product, the work is supposed to be right. (Art. 58.)

62. *Second Method of Proof.* — Divide the product by the multiplier, and, if the work is right, the quotient will be like the multiplicand.

Note. — This is the common mode of proof in business; but, as it anticipates the principles of division, it cannot be employed without a previous knowledge of that process.

63. *Third Method of Proof.* — Begin at the left hand of the

multiplicand, and add together its successive figures toward the right, till the sum obtained equals or exceeds the number *nine.* If it equals it, drop the nine, and begin to add again at this point, and proceed till you obtain a sum equal to, or greater than, nine. If it exceeds nine, drop the nine as before, and carry the excess to the next figure, and then continue the addition as before. Proceed in this way, till you have added all the figures in the multiplicand and rejected all the nines contained in it, and write the final excess at the right hand of the multiplicand.

Proceed in the same manner with the multiplier, and write the final excess under that of the multiplicand. Multiply these excesses together, and place the excess of nines in their product at the right.

Then proceed to find the excess of nines in the product obtained by the original operation; and, if the work is right, the excess thus found will be equal to the excess contained in the product of the above excesses of the multiplicand and multiplier.

NOTE. — This method of proof, though perhaps sufficiently sure for common purposes, is not always a test of the correctness of an operation. If two or more figures in the work should be transposed, or the value of one figure be just as much too great as another is too small, or if a nine be set down in the place of a cipher, or the contrary, the excess of nines will be the same, and still the work may not be correct. Such a balance of errors will not, however, be likely to occur.

EXAMPLES.

3. Multiply 7325 by 3612. Ans. 26457900.

	OPERATION.		PROOF BY MULTIPLICATION.
Multiplicand	7325	Multiplicand	3612
Multiplier	3612	Multiplier	7325
	14650		18060
	7325		7224
	43950		10836
	21975		25284
Product	26457900	Product	26457900

4. Required the product of 82967 by 652. Ans. 54094484.

	OPERATION.	PROOF BY THE NINES.
Multiplicand	82967	5 excess.
Multiplier	652	4 excess.
	165934	20 ⎫ 2 excess.
	414835	⎬
	497802	
Product	54094484	⎭ 2 excess.

5.	6.	7.	8.
789123	1234567	989898	3789588
4	5	2	8

9.	10.	11.	12.
678954	616783	789563	789567
24	36	57	98

13.	14.	15.	16.
892001	230442	425016	5061029
329	701	645	3408

17. What will 365 acres of land cost at 73 dollars per acre? Ans. $26645.

18. What will 97 tons of iron cost at 57 dollars a ton? Ans. $5529.

19. What will 397 yards of cloth cost at 7 dollars per yard? Ans. $2779.

20. What will 569 hogsheads of molasses cost at 37 dollars per hogshead? Ans. $21053.

21. If a man travel 37 miles in one day, how far will he travel in 365 days? Ans. 13505 miles.

22. If a vessel sails 169 miles in one day, how far will she sail in 144 days?

23. What will 698 barrels of flour cost at 7 dollars a barrel?

24. What will 376 lbs. of sugar cost at 13 cents a pound? Ans. 4888 cts.

25. What will 97 lbs. of tea cost at 93 cents a pound? Ans. 9021 cts.

26. If a regiment of soldiers consists of 1128 men, how many men are there in an army of 53 regiments?

Ans. 59784.

27. What is the product of 75432 × 47. Ans. 3545304.

28. What is the product of 76785316 × 7615.

Ans. 584720181340.

29. What is the product of 67853 × 8765.

Ans. 594731545.

30. What is the product of 3812345 × 31243.

Ans. 119109094835.

31. What is the product of 40670007 × 10002.

Ans. 406781410014.

32. What is the product of 31235678 × 10203.

Ans. 318697622634.

33. What is the product of 76786321 × 3007.

Ans. 230896467247.

34. What is the product of 6176777 × 22222.

Ans. 137260338494.

35. What is the product of 7060504 × 30204.

Ans. 213255462816.

36. Multiply 88888 by 4444. Ans. 395018272.

37. Multiply 7008005 by 10008. Ans. 70136114040.

38. Multiply 987648 by 481007. Ans. 475065601536.

39. Multiply 101010101 by 202020202.

Ans. 20406081008060402.

40. Multiply 304050607 by 3011101.

Ans. 915527086788307.

41. Multiply 908007004 by 500123.

Ans. 454115186861492.

42. Multiply 2003007001 by 6007023.

Ans. 12032109124168023.

43. Multiply 9000006 by 9000006.

Ans. 81000108000036.

44. A full-grown elm will, it is computed, yearly, on an average, produce three hundred twenty-nine thousand three hundred and seventy-five seeds. How many seeds will three such trees produce in fifty-three years. Ans. 52370625.

45. John Alden can plant 3 plats of corn, containing 11 rows of 67 hills each, in 1 day, and Loring Blanchard can

plant twice as much in the same time. How many hills can Blanchard plant in a month of 26 working days? Ans. 114972.

46. If the multiplicand be three hundred and seventy-five millions two hundred and ninety-six thousand three hundred and twenty-one, and the multiplier seventy-nine thousand and twenty-four, what will be the product?

Ans. 29657416470704.

64. When the multiplier is a composite number.

Ex. 1. What cost 35 acres of land at 316 dollars an acre?

Ans. 11060 dollars.

OPERATION.

3 1 6	dollars, cost of 1 acre.
7	
2 2 1 2	dollars, cost of 7 acres.
5	
1 1 0 6 0	dollars, cost of 35 acres.

The factors of 35 are 7 and 5. Now, if we multiply the price of one acre by 7, we get the cost of 7 acres; and, then, by multiplying the cost of 7 acres by the factor 5, it is evident, we obtain the cost of 5 times 7 acres, or 35 acres. Hence, when the multiplier is a composite number, we may

Multiply the multiplicand by one of the factors of the multiplier, and the product thus obtained multiply by another, and so on until each of the factors has been used as a multiplier; and the last product will be the one sought.

EXAMPLES.

2. Multiply 3121 by 81, using its factors.

3. What will 63 horses cost at 175 dollars each?

4. A certain house contains 87 windows, and each window has 32 panes of glass. How many panes in the whole house? Ans. 2784.

5. What is the product of 47134987 by 56?

Ans. 2639559272.

6. If a garrison consume 6231 pounds of bread in 1 day, how many pounds will the same consume in 144 days?

Ans. 897264 pounds.

65. When there are ciphers on the right in the multiplier or multiplicand, or both.

Ex. 1. In 1 yard there are 36 inches; how many inches in 10 yards? In 100 yards? Ans. 360, 3600.

OPERATION.

Multiplicand	3 6	3 6
Multiplier	1 0	1 0 0
Product	3 6 0	3 6 0 0
Or thus:	3 6 0,	3 6 0 0.

We annex one cipher to the multiplicand to multiply it by 10, and two ciphers to multiply it by 100; since annexing one cipher removes each figure of the multiplicand one place to the left, and thus increases it 10 times; annexing two ciphers removes each figure two places to the left, and increases it 100 times; and so on, each additional cipher having the effect to increase its value 10 times (Art. 30).

2. What will 700 bales of cotton cost at 40 dollars per bale? Ans. 28000 dollars.

OPERATION.

Multiplicand	7 0 0
Multiplier	4 0
Product	2 8 0 0 0

The multiplicand we resolve into the factors 7 and 100, and the multiplier into the factors 4 and 10. Now, it is evident (Art. 58), that, if these several factors be multiplied together, they will produce the same product as the original factors, 700 and 40. Thus $7 \times 4 = 28$, and $28 \times 100 = 2800$, and $2800 \times 10 = 28000$, the same result as in the operation.

Hence, when there are ciphers, one or more, on the right of the multiplier, or multiplicand, or both, we may, for the required product,

Multiply the significant figures together, and to their product annex as many ciphers as there are on the right in both multiplicand and multiplier.

Examples.

3. Multiply 1819 by 10.
4. Multiply 4106 by 100.
5. Multiply 10000 by 7000. Ans. 70000000.
6. Multiply 123000 by 78000. Ans. 9594000000.
7. Multiply 70000 by 10000. Ans. 700000000.
8. Multiply 900900 by 70070. Ans. 63126063000.
9. What must be the distance sailed by a steamship, whose average rate is 310 miles a day, in making a voyage from New York to Liverpool, in 12 days?
10. The annual salary of a member of Congress being 3,000 dollars, how much do 296 members receive?
Ans. 888,000 dollars.

11. The salary of the President of the United States is 25,000 dollars a year; how much will it amount to in 82 years? Ans. 2,050,000 dollars.

12. The earth is 95,000,000 of miles from the sun, and the planet Neptune is 30 times as far. How far is Neptune from the Sun? Ans. 2,850,000,000 miles.

DIVISION.

66. DIVISION is the process of finding how many times one number is contained in another; or the process of separating a number into a proposed number of equal parts.

In division there are three principal terms: the *Dividend*, the *Divisor*, and the *Quotient*.

The *dividend* is the number to be divided.

The *divisor* is the number by which we divide.

The *quotient* is the result, or number produced by the division, and denotes the number of times the divisor is contained in the dividend, or one of the equal parts into which the dividend is divided.

When the dividend does not contain the divisor an exact number of times, the *excess* is called a *remainder*, and may be regarded as a *fourth* term in the division.

When the dividend consists of a simple number, the process is termed *Division of Simple Numbers*.

67. Division is frequently indicated by writing the dividend above a short horizontal line and the divisor below; thus, $\frac{6}{2}$. The expression $\frac{6}{2} = 3$ is read, 6 divided by 2 is equal to 3.

Another method of indicating division, is by a curved line placed between the divisor and dividend. Thus, the expression 6) 12 shows that 12 is to be divided by 6.

68. When a number is divided into *two* equal parts, *one* of the parts is called *one half;* when divided into *three* equal parts, *one* of the parts is called *one third*, *two* of the parts *two thirds;* when divided into *four* equal parts, *one* of the

parts is called *one fourth*, *two* of the parts *two fourths*, *three* of the parts *three fourths;* etc.

Such equal parts are called FRACTIONS, since they are *fractured* or broken numbers. They are expressed by figures, in a form of division; thus, one half is written $\frac{1}{2}$; one third, $\frac{1}{3}$; two thirds, $\frac{2}{3}$; one fourth, $\frac{1}{4}$; two fourths, $\frac{2}{4}$; three fourths, $\frac{3}{4}$; and may also be read, one divided by two, one divided by three, and so on. In any fraction, expressed in the manner now explained, the number above the line is called the *numerator*, and that below the line its *denominator*. Thus, in $\frac{1}{2}$, 1 is the numerator, and 2 the denominator.

69. When the divisor and dividend are of the *same* kind or denomination, the quotient will denote the number of *times* the divisor is contained in the dividend, and will be an abstract number. Thus, to find how many pencils at 6 *cents* each can be bought for 24 *cents*, we inquire how many times 6 cents are contained in 24 cents, which are 4 times. Hence, 4 pencils, at 6 cents each, can be bought for 24 cents.

70. When the divisor and dividend are not of the same kind or denomination, the divisor must be regarded as an abstract number, and will denote the number of equal *parts* into which the dividend is divided, and the quotient will denote the number of units in each part, and will be of the same kind or denomination as the dividend. Thus, to find the cost of 1 *pencil*, when 4 *pencils* cost 24 *cents*, we separate or divide the 24 cents into 4 equal parts, of which one part is 6 cents. Hence, 1 pencil costs 6 cents, when 4 pencils cost 24 cents.

71. The remainder will always be of the same kind or denomination as the dividend, since it is a part of the dividend.

72. Division is the reverse of multiplication. The dividend answers to the product, and the divisor and quotient to the factors, of multiplication. In multiplication, two factors are given, to find their product; and in division, one of two factors and their product are given, to find the other factor.

73. To divide simple numbers.

Ex. 1. How many yards of cloth, at 4 dollars a yard, can be bought for 948 dollars? Ans. 237 yards.

OPERATION.

$$\begin{array}{l} \text{Divisor } 4\,)\,\underline{9\;4\;8} \text{ Dividend.} \\ \qquad\qquad\;\; 2\;3\;7 \text{ Quotient.} \end{array}$$

We first inquire how many times 4, the divisor, is contained in 9, the first left-hand figure of the dividend, which is *hundreds*, and find it contained 2 times, and 1 hundred remaining. We write the 2 directly under 9, its dividend, for the hundreds' figure of the quotient. To 4, the next figure of the dividend, which is *tens*, we regard as prefixed the 1 hundred that was remaining, which equals 10 tens, and thus form 14 tens, in which we find the divisor 4 to be contained 3 times, and 2 tens remaining. We write the 3 for the tens' figure in the quotient, and the 2 tens that were remaining, equal 20 units, we regard as prefixed to 8, the last figure of the dividend, which is *units*, in which the divisor 4 is contained 7 times. Writing the 7 for the units' figure of the quotient, we have 237 as the entire quotient, equal the number of yards of cloth at 4 dollars a yard that can be bought for 948 dollars.

2. How many times does 3979 contain 17?

Ans. $234\frac{1}{17}$ times.

OPERATION.

$$\begin{array}{l} \qquad\qquad\quad \text{Dividend.} \\ \text{Divisor } 17\,)\,3979\,(\,234\tfrac{1}{17} \text{ Quotient.} \\ \qquad\qquad\quad \underline{34} \\ \qquad\qquad\quad\;\; 57 \\ \qquad\qquad\quad\;\; \underline{51} \\ \qquad\qquad\quad\;\;\;\; 69 \\ \qquad\qquad\quad\;\;\;\; \underline{68} \\ \qquad\qquad\quad\;\;\;\;\;\; 1 \text{ Remainder.} \end{array}$$

We say, 17 in 39, 2 times. The 2 we write in the quotient. $17 \times 2 = 34$, which we write under the 39. $39 - 34 = 5$, to which bringing down the next figure of the dividend, we form 57. 17 in 57, 3 times. The 3 we write in the quotient. $17 \times 3 = 51$, which we write under the 57. $57 - 51 = 6$, to which bringing down the next figure of the dividend, we form 69. 17 in 69, 4 times. The 4 we write in the quotient. $17 \times 4 = 68$, which we write under the 69. $69 - 68 = 1$, a remainder, or a part of the dividend left undivided. 1 divided by $17 = \frac{1}{17}$ (Art. 68). The $\frac{1}{17}$ we write in the quotient, and obtain as the answer required $234\frac{1}{17}$.

In this illustration, to render the explanation the more concise, the naming of the denominations of the figures has been omitted.

When, as in the operation preceding the last, results only are written down, the method is called *short division;* and when, as in the last operation, the work is written out at length, it is called long division. The principle is the same in both cases. Hence the general

RULE.—*Beginning at the left, find how many times the divisor is contained in the fewest figures of the dividend that will contain it, for the first quotient figure.*

Multiply the divisor by this quotient figure, and subtract the product from the figures of the dividend used. With the remainder, if any, unite the next figure of the dividend.

Find how many times the divisor is contained in the number thus formed, and write the figure denoting the result at the right of the former quotient figure.

Thus proceed until all the figures of the dividend are divided.

NOTE 1. — The proper remainder is in all cases *less* than the divisor. If, in the course of the operation, it is at any time found to be as large as, or *larger* than, the divisor, it will show that there is an error in the work, and that the quotient figure should be increased.

NOTE 2. — If at any time the divisor, multiplied by the quotient figure, produces a product *larger* than the part of the dividend used, it shows that the quotient figure is too *large*, and must be diminished.

NOTE 3. — It will often happen that, when a figure of the dividend is taken, the number will not contain the divisor; and, in that case, a cipher must be placed in the quotient, and another figure of the dividend taken, and so on, until the number is large enough to contain the divisor.

NOTE 4. — If there be a remainder after dividing the last figure of the dividend, write it with the divisor underneath, with a line between them, at the right of the quotient.

74. *First Method of Proof.* — Multiply the divisor by the quotient, and to the product add the remainder, if any, and if the work be right, the sum thus obtained will be equal to the dividend.

NOTE. — This method follows from division being the reverse of multiplication. (Art. 72.)

75. *Second Method of Proof.* — Find the excess of nines in the divisor, quotient, and remainder. Multiply the excess of nines in the divisor and quotient together, and to the product add the excess of nines in the remainder. If the excess of nines in this sum equal the excess of nines in the dividend, the work may be supposed to be right.

76. *Third Method of Proof.* — Add together the remainder, if any, and all the products that have been produced by multiplying the divisor by the several quotient figures, and the result will be like the dividend, if the work be right.

77. *Fourth Method of Proof.* — Subtract the remainder, if any, from the dividend, and divide the difference by the quotient. The result will be like the original divisor, if the work be right.

NOTE. — The first method of proof (Art. 74) is usually most convenient, and is most commonly employed.

EXAMPLES.

3. Divide 9184 by 7. Ans. 1312.

OPERATION.

Divisor 7) 9 1 8 4 Dividend.
1 3 1 2 Quotient.

PROOF BY MULTIPLICATION.

1 3 1 2 Quotient.
7 Divisor.
9 1 8 4 Dividend.

4. Divide 18988 by 759. Ans. $25\frac{13}{759}$.

OPERATION.

Dividend.
Divisor 7 5 9) 1 8 9 8 8 (2 5 Quotient.
1 5 1 8
3 8 0 8
3 7 9 5
1 3 Remainder.

PROOF BY THE NINES.

Divisor = 3 excess.
Quotient = 7 excess.
Remainder = 4 excess.

$(3 \times 7) + 4 = 7$ excess.
Dividend = 7 excess.

5. Divide 147856 by 97. Ans. $1524\frac{28}{97}$.

OPERATION.

Dividend.
Divisor 9 7) 1 4 7 8 5 6 (1 5 2 4 Quotient.
+9 7
5 0 8
+4 8 5
2 3 5
+1 9 4
4 1 6
+3 8 8
+2 8 Remainder.

PROOF BY ADDITION.

9 7
4 8 5
1 9 4
3 8 8 } Products.
2 8 Remainder.
1 4 7 8 5 6 Dividend.

6. Divide 84645 by 285. Ans. 297.

OPERATION.

Dividend.
Divisor 2 8 5) 8 4 6 4 5 (2 9 7 Quotient.
5 7 0
2 7 6 4
2 5 6 5
1 9 9 5
1 9 9 5

PROOF BY DIVISION.

Dividend.
2 9 7) 8 4 6 4 5 (2 8 5 Divisor.
5 9 4
2 5 2 4
2 3 7 6
1 4 8 5
1 4 8 5

7.	8.	9.
3)67856336	7)178985	11)1667789
$22618778\frac{2}{3}$		

10.	11.	12.
5)334652	8)96723578	9)186731

13.	14.	15.
17)678916(	35)9106013(	91)6210011(

	Quotients.	Rem.
16. Divide 671678953 by 6.	111946492	1.
17. Divide 166336711 by 7.	23762387	2.
18. Divide 161331793 by 8.	20166474	1.
19. Divide 161677678 by 9.	17964186	4.
20. Divide 363895678 by 11.	33081425	3.
21. Divide 164378956 by 12.	13698246	4.
22. Divide 78950077 by 3.		1.
23. Divide 678956671 by 4.		3.
24. Divide 667788976 by 5.		1.
25. Divide 777777777 by 6.		3.
26. Divide 888888888 by 7.		6.
27. Divide 789636 by 46.		
28. Divide 7967848 by 52.	153227	44.
29. Divide 16785675 by 61.	275175	
30. Divide 675753 by 39.		
31. Divide 5678911 by 82.		1.
32. Divide 6716394 by 94.	71451	
33. Divide 1167861 by 135.	8650	111.
34. Divide 7861783 by 87.	90365	28.
35. Divide 1678567 by 365.	4598	297.
36. Divide 87635163 by 387.	226447	174.
37. Divide 34567890 by 6789.	5091	5091.
38. What is the value of 213255467083 ÷ 30204?		4267.
39. What is the value of 395020613 ÷ 4444?		2341.
40. What is the value of 7207276639 ÷ 9009?		4567.
41. What is the value of 4541151868702573 ÷ 500123?		8765.

42. How many barrels of flour, at 9 dollars a barrel, can be bought for 18621 dollars?

43. How much sugar at 15 dollars a hundred may be bought for 405 dollars?

44. A tailor has 938 yards of broadcloth; how many cloaks can be made of the cloth, if it require 7 yards to make one cloak?

45. What number multiplied by 1728 will produce 1705536?
Ans. 987.

46. A. Hartmann has sold his wagon to J. Herr for 85 dollars. He is to receive his pay in wood at 5 dollars a cord. How many cords will it require to pay for the wagon?
Ans. 17 cords.

47. The Bible contains 31,173 verses; how many must be read each day, that the book may be read through in a year of 365 days? Ans. $85\frac{148}{365}$ verses.

48. A train on the Liverpool Railroad runs at the rate of 65 miles an hour; how long would it take at that velocity to pass round the earth, the distance being about 25,000 miles?
Ans. $384\frac{8}{13}$ hours.

49. A gentleman possessing an estate of 66,144 dollars, bequeathed one fourth to his wife, and the remainder was divided among his 4 children; what was the share of each?
Ans. 12,402 dollars.

50. If the dividend is 6756785 and the quotient 193051, what is the divisor? Ans. 35.

51. A's age multiplied by 17, or B's age multiplied by 19, is equal to 1292 years, and the sum of their ages is equal to C's age multiplied by 3. What is the age of each?
Ans. A's 76 years; B's 68 years; C's 48 years.

78. When the divisor is a composite number.

Ex. 1. A farmer bought 21 horses for 2625 dollars; how many dollars did each cost? Ans. 125 dollars.

OPERATION.

$$\begin{array}{r l}
3\,)\,\underline{2\,6\,2\,5} & \text{dolls., cost of 21 horses.} \\
7\,)\,\underline{8\,7\,5} & \text{dolls., cost of 7 horses.} \\
1\,2\,5 & \text{dolls., cost of 1 horse.}
\end{array}$$

The factors of 21 are 3 and 7. Now, if we divide the 2625 dollars, the cost of 21 horses, by 3, we obtain 875 dollars, the cost of 7 horses, since there are 7 times 3 in 21. Then, dividing the 875 dollars, the cost of 7 horses, by 7, we obtain 125 dollars, the cost of 1 horse.

2. Divide 3515 by 42. Ans. $83\frac{29}{42}$.

OPERATION.	FINDING THE TRUE REMAINDER.
$2\,)\,\underline{3515}$	$4 \times 3 \times 2 = 24$, 1st Product.
$3\,)\,\underline{1757}$, 1, 1st Rem.	$2 \times 2 = \ \ 4$, 2d Product.
$7\,)\,\underline{585}$, 2, 2d Rem.	$\underline{\ \ 1}$, 1st Remainder.
83, 4, 3d Rem.	29, true Remainder.

Or, $(4 \times 3 \times 2) + (2 \times 2) + 1 = 29$, true Rem.

Using as divisors 2, 3, and 7, the factors of 42, we obtain for remainders, 1, 2, and 4.

The first remainder, 1, is a unit of the given dividend, since it is a part of it (Art. 71). The second remainder, 2, is units of the quotient 1757, whose units are 2 times as great as those of the given dividend. The third remainder, 4, is units of the quotient 585, whose units are 3 times as great as those of the quotient 1757, of which the units are 2 times as great as those of the given dividend. Now, these remainders must be all of the same units as the given dividend, to constitute the whole or true remainder. We therefore multiply the third remainder by 3 and 2, the divisors used in producing the quotient of which it is a part; and the second remainder by 2, the divisor used in producing the quotient of which it is a part; and the products with the first remainder added together give 29, the whole or true remainder sought. Hence, as shown by these illustrations, when the divisor is a composite number, we may

Divide the dividend by one of the factors, and the quotient thus found by another, and thus proceed till each of the factors has been made a divisor. The last quotient will be the quotient required.

If there be remainders, multiply each remainder, except the first, by all the divisors preceding the one which produced it; and the first remainder being added to the sum of the products, the amount will be the true remainder.

NOTE. — There will be but one product to add to the first remainder when there are only two divisors and two remainders.

EXAMPLES.

3. Divide 7704 by $24 = 4 \times 6$. Ans. 321.
4. Divide 8317 by $27 = 3 \times 9$.
5. Divide 3116 by $81 = 9 \times 9$.
6. Divide 61387 by $121 = 11 \times 11$. Ans. $507\frac{40}{121}$.
7. Divide 19917 by $144 = 12 \times 12$. Ans. $138\frac{45}{144}$.
8. Divide 91746 by $336 = 6 \times 7 \times 8$. Ans. $273\frac{18}{336}$.
9. At 45 dollars an acre, a farm of how many acres can be bought for 5464 dollars? Ans. $121\frac{19}{45}$ acres.

79. When the divisor contains one or more ciphers at the right hand.

Ex. 1. If 10 men receive 792 dollars for a job of work, what will be each man's share of it? Ans. $79\frac{2}{10}$ dollars.

OPERATION.

$1|0\,)\,\underline{7\,9\,|\,2}$

Quotient 7 9, 2 Rem.

Or thus: 7 9 | 2.

To multiply by 10, we annex one cipher, which removes the figures one place to the left, and thus makes the value denoted tenfold (Art. 65). Now, it is obvious, that, if we *reverse* the process, and cut off the right-hand figure by a line, we remove the remaining figures *one* place to the *right*, and consequently *diminish* the value denoted by each the same as dividing by 10. The figures on the left of the line are the quotient, and the one on the right is the remainder, which may be written over the divisor and annexed to the quotient. Hence each man's share is $79\frac{2}{10}$.

2. How many years will it take a man at a yearly salary of 700 dollars to earn 3664 dollars? Ans. $5\frac{164}{700}$ years.

OPERATION.

$1|0\,0\,)\,\underline{3\,6\,|\,6\,4}$

$7\,)\,\underline{3\,6},\ 6\,4$, 1st Rem.

$5,\ \ 1$, 2d Rem.

Or thus: $7|0\,0\,)\,\underline{3\,6\,|\,6\,4}$

$5,\ 1\,6\,4.$

The divisor, 700, may be resolved into the factors 7 and 100. We first divide by the factor 100, by cutting off two figures at the right, and get 36 for the quotient, and 64 for a remainder. We then divide the quotient, 36, by the other factor, 7, and obtain 5 for a quotient and 1 for a remainder. The last remainder, 1, being multiplied by the divisor, 100, and 64, the first remainder, added, we obtain 164 for the true remainder (Art. 77); and for the answer required, $5\frac{164}{700}$ years. Hence, when the divisor contains one or more ciphers at the right, we may, to perform the division,

Cut off the ciphers from the right of the divisor, and the same number of figures from the right of the dividend; and then divide the remaining figures of the dividend by the remaining figures of the divisor.

NOTE. — When, by the operation, there is a remainder, to it must be annexed the figures cut off from the dividend to form the true remainder. Should there be no last remainder, then the significant figures, if any, cut off from the dividend, will form the true remainder.

EXAMPLES.

	Quotient.	Rem.
3. Divide 123456789 by 10.	12345678	9.
4. Divide 987654300 by 100.		

		Quotients	Rem.
5.	Divide 32100 by 6000.	5	2100.
6.	Divide 3678953 by 326100.	11	91853.
7.	Divide 1637851 by 500000.	3	137851.
8.	Divide 41111111 by 1100000.	37	411111.
9.	Divide 89765432156 by 1000000.		

10. The entire annual loss to the United States in consequence of intemperance has been estimated to be about 98,400,000 dollars. How many schools at a yearly expense of 600 dollars would that sum support?

Ans. 164,000 schools.

11. The late war with Russia was carried on at a cost of 600,000,000 dollars to Great Britain. Allowing that country to have a population of 28,000,000, what was the cost to each individual?

12. If light moves at the rate of 192,000 miles in a second, how long is it in passing from the sun to the earth, a distance of 95,000,000 of miles. Ans. $494\frac{152}{192}$ seconds.

GENERAL PRINCIPLES AND APPLICATIONS.

80. In division, the value of the quotient depends upon the relative values of the divisor and dividend.

81. *If the dividend be multiplied, or the divisor divided, by any number, the quotient is multiplied by the same number.* Thus, if the dividend be 20 and the divisor 4, the quotient will be 5; but if the dividend be multiplied by any number, as 2, and the divisor remain unchanged, the quotient will be 2 times as large as before, or 10; as $(20 \times 2) \div 4 = 10$; and if the divisor be divided by the 2, and the dividend remain unchanged, the quotient will be, likewise, 2 times as large, or 10; as $20 \div (4 \div 2) = 10$.

82. *If the dividend be divided, or the divisor multiplied, by any number, the quotient is divided by the same number.* Thus, if the dividend be 32 and the divisor 8, the quotient will be 4. But if the dividend be divided by any number, as 2, and the

5*

divisor remain unchanged, the quotient will be only half as large as before, or 2; as $(32 \div 2) \div 8 = 2$; and if the divisor be multiplied by 2, and the dividend remain unchanged, the quotient will be, likewise, only half as large, or 2; as $32 \div (8 \times 2) = 2$.

83. *If the dividend and divisor be both multiplied, or both divided, by the same number, the quotient will not be changed.* Thus, if the dividend be 16 and the divisor 4, the quotient will be 4. Now, if we multiply the dividend and divisor by some number, as 2, their relative values are not changed, and we obtain 32 and 8 respectively, and $32 \div 8 = 4$, the same as the original quotient. Also, if we divide the dividend and quotient by some number, as 2, their relative values are not changed, and we obtain 16 and 2 respectively, and $16 \div 2 = 8$, the same quotient as before.

84. *If a factor in any number is rejected or cancelled, the number is divided by that factor.* Thus, if 24 is the dividend and 6 the divisor, the quotient will be 4. Now, since the divisor and quotient are the two factors which, being multiplied together, produce the dividend (Art. 72), it follows, if we *reject* or *cancel* the factor 6, the remaining 4 is the quotient; and, by the operation, the dividend 24 has been divided by 4.

CANCELLATION.

85. CANCELLATION is the method of abbreviating arithmetical operations by rejecting any factor or factors common to the divisor and dividend.

Ex. 1. Sold 19 thousand shingles at 4 dollars a thousand, and received pay in wood at 4 dollars a cord; how many cords of wood was received? Ans. 19 cords.

OPERATION.

$$\begin{array}{l} \text{Dividend} \\ \text{Divisor} \end{array} \frac{\cancel{4} \times 19}{\cancel{4}} = 19 \text{ Quotient.}$$

Having indicated by signs the multiplication and division required by the question, then, since dividing both dividend and divisor by the same number will not change the quotient (Art. 83), we divide them by the common factor 4, by cancelling it in both, and obtain 25 for the quotient.

2. Divide the product of 15, 3, 28, and 13, by the product of 7, 30, and 4.

OPERATION.

$$\begin{array}{ll} \text{Dividend} & \cancel{15} \times 3 \times \cancel{28} \times 13 \\ \text{Divisor} & \cancel{7} \times \cancel{30} \times \cancel{4} \\ & \quad\quad 2 \end{array} = \frac{39}{2} = 19\tfrac{1}{2} \text{ Quotient.}$$

The product of the 7 and 4 in the divisor equals the 28 in the dividend; we therefore cancel all these numbers. Finding 15 in the dividend to be a factor of 30 in the divisor, we cancel both of the numbers, and use the remaining factor 2 in place of the 30. There now being no factor common to both dividend and divisor uncancelled, we multiply together the remaining factors in the dividend, and divide the product by the remaining factor in the divisor, and obtain the quotient $19\frac{1}{2}$.

RULE. — *Cancel the factor or factors common to the dividend and divisor, and then divide the product of the factors remaining in the dividend by the product of those remaining in the divisor.*

NOTE. — 1. In arranging the numbers for cancellation, the dividend may be written above the divisor, with a horizontal line between them, as in division (Art. 67); or, as some prefer, the dividend may be written on the right of the divisor, with a vertical line between them.

NOTE. — 2. Cancelling a factor does not leave 0, but the quotient 1, to take its place, since rejecting a factor is the same as dividing by that factor (Art. 84). Therefore, for every factor cancelled, either in the dividend or divisor, the factor 1 remains.

EXAMPLES.

✓ 3. Multiply 24 by 16, and divide the product by 12.
Ans. 32.

✓ 4. Divide 48 by 16, and multiply the quotient by 8.
Ans. 24.

5. Divide the product of 7, 10, 12, and 5, by the product of 14, 18, and 6.

6. If 15 be multiplied by 7, 27, and 40, and the product divided by 54 multiplied by 14, 10, and 2, what will be the result? Ans. $7\frac{1}{2}$.

7. Divide the product of 13, 15, 20, and 5, by the product of 26, 10, 2, and 3. Ans. $12\frac{1}{2}$.

8. Divide the product of 28, 27, 21, 15, and 18, by the product of 7, 54, 7, 3, and 9.

9. How many pounds of butter at 28 cents a pound will be required to pay for 56 pounds of sugar at 11 cents a pound? Ans. 22 pounds.

10. A. Holmes sold 14 boxes of soap, each containing 24 pounds, at 9 cents a pound, and received for pay 63 barrels of ashes, each containing 3 bushels. What was allowed a bushel for the ashes? Ans. 16 cents.

11. M. Gardner sold 5 piles of brick, each containing 12 thousand, at 7 dollars a thousand, and was paid in wood, 3 ranges, at 4 dollars a cord. How many cords in each range? Ans. 35 cords.

12. A merchant exchanged 8 cases of shoes, each containing 60 pairs, at 75 cents a pair, for a certain number of casks of molasses, each containing 90 gallons, at 40 cents a gallon. How many casks did he get? Ans. 10 casks.

CONTRACTIONS IN MULTIPLICATION.

86. A CONTRACTION is the process of shortening any operation.

87. When the multiplier is 13, 14, etc., or 1 with a significant figure annexed.

Ex. 1. Multiply 3126 by 14. Ans. 43764.

FIRST OPERATION.	SECOND OPERATION.
3126 × 14	3126
12504	14
43764 Ans.	43764 Ans.

In the first operation, we multiply the multiplicand by the 4 *units* of the multiplier, and write the product under the multiplicand one place to the right. To this partial product we add the multiplicand, since, as it stands, it represents the product of the multiplicand by the 1 *ten* of the multiplier; and obtain 43764, the answer required. In the second operation, we add in the multiplicand taken as the product by the 1 *ten* of the multiplier, as we multiply by the 4 *units;* thus, $6 \times 4 = 24$, of which we write down the 4 and carry the 2. $2 \times 4 = 8, + 2$ (carried) $+ 6$ (from the multiplicand) $= 16$, of which we write down the 6 and carry the 1. $1 \times 4 = 4, + 1 + 2 = 7$, which we write down. $3 \times 4 = 12, + 1 = 13$ of which we write down the 3 and carry the 1. $3 + 1 = 4$, which we write down; and have as the entire result 43764 as before.

RULE. — *Write the product by the units' figure of the multiplier under the multiplicand, one place to the right, and add them together.* Or,

Multiply each figure of the multiplicand by the units' figure of the multiplier, and, after the units' place, add in the preceding figure of the multiplicand.

EXAMPLES.

2. Multiply 63013 by 17. Ans. 1071221.
3. Multiply 79245 by 19.
4. Multiply 32067812 by 16. Ans. 513084992.

88. When the multiplier is 101, 102, etc., or 1 with one or more ciphers and a significant figure annexed.

Ex. 1. Multiply 8107 by 103. Ans. 835021.

OPERATION.

```
8107 × 103
  24321
-------
835021 Ans.
```

We multiply by 3, the *units'* figure of the multiplicand, and write the product under the multiplicand, two places to the right, so that the multiplicand, as it stands over this partial product, will represent the product of the multiplicand by the 1 *hundred* of the multiplier; and, adding these, we obtain 835021, the result required. For the reason given, if the multiplier had been such as to have contained one more intervening cipher, we should have written the product by the units' figure three places to the right, and so on, one place farther to the right for every additional intervening cipher.

RULE. — *Write the product by the units' figure of the multiplier under the multiplicand, as many places to the right as there are in the multiplier intervening ciphers plus* 1; *and add them together.*

EXAMPLES.

2. Multiply 6651 by 108. Ans. 718308.
3. Multiply 111223 by 104. Ans. 11567192.
4. Multiply 2042 by 1009.

89. When the multiplier is 21, 31, etc., or 1 with a significant figure prefixed.

Ex. 1. Multiply 3113 by 41. Ans. 127633.

OPERATION.

```
  3113 × 41
12452
------
127633 Ans.
```

We multiply by 4, the *tens'* figure of the multiplier, and write the product under the multiplicand, one place to the left, so that the multiplicand, as it stands over this partial product, will represent the product of the multiplicand by the 1 *unit* of the multiplier; and, adding these, obtain the result required.

RULE. — *Write the product by the tens' figure of the multiplier under the multiplicand, one place to the left, and add them together.*

EXAMPLES.

2. Multiply 13317 by 51. Ans. 679167.
3. Multiply 71389 by 21.
4. Multiply 12062 by 91. Ans. 1097642.

90. When the multiplier is 201, 301, etc., or 1 with one or more ciphers and a significant figure prefixed.

Ex. 1. Multiply 14118 by 601. Ans. 8484918.

OPERATION.

```
  14118 × 601.
84708
-------
8484918 Ans.
```

We multiply by 6, the hundreds' figure of the multiplier, and write the product under the multiplicand, two places to the left, so that the multiplicand, as it stands over this partial product, will represent the product of the multiplicand by the 1 *unit* of the multiplier; and adding these we have the answer required.

RULE. — *Write the product by the hundreds' figure of the multiplier under the multiplicand, as many places to the left as there are in the multiplier intervening ciphers plus* 1; *and add them together.*

EXAMPLES.

2. Multiply 8360 by 7001. Ans. 58528360.
3. Multiply 10613 by 801. Ans. 8501013.
4. Multiply 91603 by 2001.

91. When the multiplier or multiplicand has a fraction annexed.

Ex. 1. Multiply 426 by $7\frac{1}{3}$. By $8\frac{2}{3}$. Ans. 3124; 3692.

OPERATION.

426
$7\frac{1}{3}$
$2982 =$ Product by 7.
$142 =$ Product by $\frac{1}{3}$.
$3124 =$ Product by $7\frac{1}{3}$.

OPERATION.

426
$8\frac{2}{3}$
$3408 =$ Product by 8.
$284 =$ Product by $\frac{2}{3}$.
$3692 =$ Product by $8\frac{2}{3}$.

In multiplying 426 by $7\frac{1}{3}$, we first obtain the product of 426 by 7, and then the product of 426 by $\frac{1}{3}$, and, adding these two partial products together, have 3124, the product by $7\frac{1}{3}$. In multiplying by $\frac{1}{3}$, we take one third of the multiplicand, by dividing it by 3; thus, $426 \times \frac{1}{3} = 426 \div 3 = 142$. In multiplying 426 by $8\frac{2}{3}$, we proceed as in the other case, except in obtaining the product by the fraction;

we take two thirds of the multiplicand, by taking one third of it 2 times; thus, $426 \times \frac{2}{3} = (426 \div 3) \times 2 = 284$.

If the fraction had been annexed to the multiplicand instead of the multiplier, we then would have found the product of the fraction and multiplier, for a partial product, in like manner as above.

RULE. — *Multiply the fractional part and the whole number separately, and add the products.*

EXAMPLES.

2. Multiply 915 by $22\frac{2}{5}$. Ans. 20496.
3. Multiply $1224\frac{1}{7}$ by 18. Ans. $22034\frac{4}{7}$.
4. If $69\frac{1}{6}$ miles make 1 degree, how many miles are 180 degrees? Ans. 12450 miles.

92. When the multiplier is a convenient part of a number of tens, hundreds, or thousands.

NOTE. — The following are some of the convenient parts often occurring as multipliers; $2\frac{1}{2} = \frac{1}{4}$ of 10; $3\frac{1}{3} = \frac{1}{3}$ of 10; $12\frac{1}{2} = \frac{1}{8}$ of 100; $16\frac{2}{3} = \frac{1}{6}$ of 100; $25 = \frac{1}{4}$ of 100; $33\frac{1}{3} = \frac{1}{3}$ of 100; $125 = \frac{1}{8}$ of 1000; $166\frac{2}{3} = \frac{1}{6}$ of 1000; $250 = \frac{1}{4}$ of 1000; $333\frac{1}{3} = \frac{1}{3}$ of 1000.

Ex. 1. Multiply 785643 by 25. Ans. 19641075.

OPERATION.

```
4)78564300
  19641075 Product.
```

We multiply by 100, by annexing two ciphers to the multiplicand (Art. 65), and obtain a product 4 times as large as it should be, since 25, the multiplier, is only *one fourth part* of 100; we therefore take *one fourth part* of that product, by dividing by 4, for the true product.

Upon the same principle, if the multiplier had been $3\frac{1}{3}$, we should have annexed *one* cipher and divided by 3, or, if the multiplier had been 125, we should have annexed *three* ciphers and divided by 8. Hence the following

RULE. — *Multiply by the number of tens, hundreds, or thousands, of which the multiplier is a part, and, of the product thus found, take the same part.*

EXAMPLES.

2. Multiply 68056 by $12\frac{1}{2}$. Ans. 850700.
3. Multiply 17924 by $2\frac{1}{2}$.
4. Multiply 192378 by $16\frac{2}{3}$. Ans. 3206300.
5. Multiply 12345678 by 125. Ans. 1543209750.
6. How much can be earned in one year of 313 working days, at $3\frac{1}{3}$ dollars a day?

7. What will a farm containing 534 acres cost, at $33\frac{1}{3}$ dollars an acre?

8. In a certain large field there are 771 rows of corn, each containing 250 hills; how many hills in all?

Ans. 192750 hills.

9. From a port in Louisiana there were exported, in a given time, 9168 boxes of sugar, averaging $166\frac{2}{3}$ pounds to a box; required the whole number of pounds.

Ans. 1528000 pounds.

10. An agent has bought for the army, at different times, in the aggregate, 1993 horses, at an average price of 125 dollars each. How much did he pay for them in all?

Ans. 249125 dollars.

11. How many are $333\frac{1}{3}$ times 28044? Ans. 9348000.

93.° When a part of the multiplier is a factor of another part.

Ex. 1. Multiply 3263 by 568. Ans. 1853384.

OPERATION.

3 2 6 3	Multiplicand.
5 6 8	Multiplier.
2 6 1 0 4	= Product by 8 units.
1 8 2 7 2 8	= Product by 56 tens.
1 8 5 3 3 8 4	= Product by 568.

We regard the multiplier as separated into two parts, 56 *tens* and 8 *units*, or 560 + 8; of which the smaller part is evidently a factor of the larger, since the 56 tens, or 560, is equal to 7 *tens* × 8. We next multiply by the 8 *units*, obtaining the product for that part of the multiplier. Now, as this product is the same as that by the factor 8 of the other part of the multiplier, we multiply it by 7 *tens*, obtaining the product of the multiplicand by 8 × 7 *tens* or 56 *tens*. These products of the parts, 560 and 8, added together, give the true product by 568.

RULE. — *Multiply first by the smaller part of the multiplier; and then that partial product by a factor, or factors, of a larger part; and so on with all the parts. The sum of the several partial products will be the product required.*

NOTE. — Care must be taken in writing down the partial products to have the units of the different orders stand in their proper places for adding.

EXAMPLES.

2. Multiply 112345678 by 288144486.

Ans. 32371787641631508.

OPERATION.

```
             112345678 Multiplicand.
             288144486 Multiplier.
             ---------
             674074068 = Product by 6 units.
          5392592544   = 1st product × 8 tens for product by 48 tens:
     1617777632        = 2d product × 3 thousands for product by 144 thousands.
323555552 64           = 3d product × 2 millions for product by 288 millions.
-----------------------
32371787641631508 = Product by 288144486.
```

3. Multiply 61370913 by 96488. Ans. 5921556653544.

4. Multiply 8649347864 by 1325769612.
Ans. 11467042561708308768.

94. When the multiplier is any number of nines.

Ex. 1. Multiply 87654 by 999. Ans. 87566346.

OPERATION.

```
87654000 = 87654×1000
   87654 = 87654×   1
--------          ----
87566346 = 87654× 999
```

By annexing three ciphers to the multiplicand we take it 1000 times, or 1 time more than is required by the given multiplier. We therefore from this result subtract the multiplicand taken once, and thus obtain the product by 999. In like manner we may multiply by any number of nines. Hence the

RULE. — *Annex as many ciphers to the multiplicand as there are nines in the multiplier, and from the number thus produced subtract the given multiplicand.*

NOTE. — To multiply by any number of *threes*, find the product for the same number of *nines*, by the rule, and take *one third* of it by dividing by 3; and to multiply by any number of *sixes*, take *twice* the product of the same number of *threes*, by multiplying it by 2.

2. Multiply 7777777 by 9999. Ans. 77769992223.
3. Multiply 416231 by 99999.
4. Multiply 987654 by 333333. Ans. 329217670782.
5. Multiply 876543 by 66666. Ans. 58435615638.
6. Multiply 999999 by 9999. Ans. 9998990001.
7. Multiply 32567895 by 3333.
8. Multiply 66666 by 66666. Ans. 4444355556.
9. Multiply 912345678 by 99. Ans. 90322222122.
10. Multiply 1234567 by 9999. Ans. 12344435433.
11. Multiply 98123452 by 999999. Ans. 98123353876548.

CONTRACTIONS IN DIVISION.

95. When the divisor is a convenient part of a number of tens, hundreds, or thousands.

Ex. 1. Divide 19641075 by 25. Ans. 785643.

OPERATION.

$$\begin{array}{r} 19641075 \\ 4 \\ \hline 785643|00 \end{array} \text{ Quotient.}$$

By multiplying both divisor and dividend by 4, which does not change the relation of the one to the other (Art. 83), the divisor becomes 100, which enables us to perform the division by simply cutting off two figures at the right of the dividend (Art. 79). In like manner, if the divisor had been $3\frac{1}{3}$, by taking both it and the dividend 3 times as large, we could have performed the division by simply cutting off one figure at the right of the dividend; or, if the divisor had been 125, by taking both it and the dividend 8 times as large, we could have performed the division by simply cutting off three figures at the right of the dividend. Hence the

RULE. — *Multiply both divisor and dividend by that number which will change the divisor to a number of tens, hundreds, or thousands, and then divide.*

EXAMPLES.

2. Divide 89630 by $3\frac{1}{3}$. Ans. 26889.
3. Divide 123450 by $16\frac{2}{3}$.
4. Divide 18621 by $12\frac{1}{2}$. Ans. $1489\frac{68}{100}$.
5. Divide 317121 by $2\frac{1}{2}$. Ans. $126848\frac{4}{10}$.
6. Divide 876735 by $33\frac{1}{3}$. Ans. $26302\frac{5}{100}$.
7. Divide 123456 by 125.
8. Divide 61678500 by 250. Ans. 246714.
9. J. Cushing bought a number of horses, at $166\frac{2}{3}$ dollars each, for $ 9500; required the number bought. Ans. 57 horses.
10. A company has received 12000 dollars from the sale of piano-fortes, at an average price of $333\frac{1}{3}$ dollars each; required the number sold.
11. How many shares of the Illinois Central Railroad, at 125 dollars each, can be bought for 150000 dollars? Ans. 1200 shares.
12. How many cows, at $33\frac{1}{3}$ dollars each, may be purchased for $333\frac{1}{3}$ dollars.

13. How many books, at $2\frac{1}{2}$ dollars each, may be purchased for 120 dollars? Ans. 48 books.

14. A certain magazine contains 616350 pounds of powder, in kegs of 25 pounds each; required the number of kegs.
Ans. 24654 kegs.

96.° When the divisor is any number of nines.

Ex. 1. Divide 316234 by 99. Ans. $3194\frac{28}{99}$.

OPERATION.

$$\begin{array}{r|l} 3\,1\,6\,2 & 3\,4 \\ 3\,1 & 9\,6 \\ 1 & 2\,7 \\ & 2\,8 \\ \hline \end{array}$$

$3\,1\,9\,4\,\frac{28}{99}$ Ans.

We first divide by 100, or $99 + 1$, by cutting off two figures at the right of the dividend, obtaining for the first partial quotient 3162 and a remainder 34. Since the divisor used was 1 larger than the given divisor, 99, the quotient obtained denotes that an excess of 3162, or a number equal to the quotient itself, must be added to the 34 for the true remainder, $34 + 3162 = 3196$, which exceeding the divisor 99, we write it for a second dividend; and dividing by 100, or $99 + 1$, as before, we obtain for the second partial quotient 31, and a remainder 96. To the remainder 96 we add 31, the excess denoted by the last quotient, and obtain 127 for a third dividend, which being divided by 100, or $99 + 1$ gives the third partial quotient 1, and a remainder 27. To the remainder 27 adding 1, the excess denoted by the last quotient, we have for the true remainder 28, which is $\frac{28}{99}$. The sum of the partial quotients with the final remainder annexed gives $3194\frac{28}{99}$, the quotient required.

The above process is based upon the principle that $10 = 9 + 1$; $100 = 99 + 1$; $1000 = 999 + 1$, etc.; consequently $20 = (2 \times 9) + 2$, and $37 = (3 \times 9) + 3 + 7$; $200 = (2 \times 99) + 2$; $3859 = (38 \times 99) + 38 + 59$; $15987 = (15 \times 999) + 15 + 987$, etc. Hence, $316234 = 316200 + 34 = (3162 \times 99) + 3162 + 34$; $3162 + 34 = 3196 = 3100 + 96 = (31 \times 99) + 31 + 96$; $31 + 96 = 127 = 100 + 27 = (1 \times 99) + 1 + 27$; $1 + 27 = 28 = \frac{28}{99}$; and $3162 + 31 + 1 + \frac{28}{99} = 3194\frac{28}{99}$, the answer, as before obtained. By like process, and upon the same principle, may the quotient be found for any number of nines.

RULE. — *Add 1 to the given divisor, and, by it thus increased, divide by cutting off figures at the right of the dividend. To the figures cut off on the right add those on the left for a true remainder, of which, if it equal or exceed the given divisor, make a second dividend, and divide as before. Proceed thus till there shall be no remainder as large as the given divisor; and the sum of the several quotients, with the last remainder, if any, will be the answer required.*

NOTE. — When the last remainder is the same as the given divisor, it must be cancelled, and 1 written as a partial quotient.

EXAMPLES.

2. Divide 341 by 9.

OPERATION.

3 4	1
3	5
	8
3 7 $\frac{8}{9}$ Ans.	

3. Divide 123332544 by 999.

OPERATION.

1 2 3 3 3 2	5 4 4
1 2 3	8 7 6
1	$\cancel{9}\cancel{9}\cancel{9}$
1 2 3 4 5 6 Ans.	

4. Divide 12332655 by 999. Ans. 12345.

5. Divide 987551235 by 9999.

6. Divide 91234567798765432l1 by 999999999.
Ans. 9123456789.

97.° Abbreviated method of long division.

Ex. 1. Divide 34634 by 134. Ans. $258\frac{62}{134}$.

OPERATION.

$$
\begin{array}{l}
134\,)\,34634\,(\,258\tfrac{62}{134}\text{ Ans.} \\
\quad\quad\ \ 783 \\
\quad\quad\ \ 1134 \\
\quad\quad\quad\ \ 62
\end{array}
$$

The operation we perform thus: finding the first quotient figure to be 2, we say 4 × 2 = 8; 16 — 8 = 8; 3 × 2 = 6, and 1 carried = 7; 14 — 7 = 7; 1 × 2 = 2, and 1 carried = 3; 3 — 3 = 0. We now bring down 3, and find the next quotient figure to be 5, then, 4 × 5 = 20; 3 — 0 = 3; 3 × 5 = 15, and 2 carried = 17; 8 — 7 = 1; 1 × 5 = 5, and 1 carried = 6; 7 — 6 = 1. We next bring down 4, and find the next quotient figure to be 8; then, 4 × 8 = 32; 4 — 2 = 2; 3 × 8 = 24, and 3 carried = 27; 13 — 7 = 6; 1 × 8 = 8, and 2 + 1 carried = 11; 1 — 1 = 0; 1 — 1 = 0. Hence, this method is that of ordinary long division (Art. 73), abridged by *subtracting each figure of the product of the divisor and a quotient figure, as it is obtained, and writing down only the remainders.*

EXAMPLES.

2. Divide 39006 by 44. Ans. $886\frac{22}{44}$.

3. A certain orchard contains 1088 trees in 34 rows; how many trees in each row? Ans. 32 trees.

4. A speculator sold 191 mules at a gain of 5157 dollars; what was the gain on each?

5. The population of Massachusetts, in 1855, was 1,133,123; how many would that be to each of its 7750 square miles of surface? Ans. $146\frac{1623}{7750}$.

98. When the divisor or dividend has a fraction annexed.

Ex. 1. Divide 3692 by $8\frac{2}{3}$. Ans. 426.

OPERATION.

```
8 2/3 | 3692
3     |    3
----------------
26 ) 11076 ( 426 Ans.
     104
     ---
      67
      52
      ---
      156
      156
      ---
```

We bring the divisor and dividend to the same fractional parts as those denoted by the given fraction, which are *thirds*, by multiplying them both by 3, the number of thirds in 1. We thus change the divisor to 26 thirds, and the dividend to 3692 thirds, without changing the quotient (Art. 83); and dividing obtain 426, the answer required.

RULE. — *Reduce the divisor and dividend to the same fractional parts as are denoted by the given fraction, and then divide as in whole numbers.*

NOTE. — In case there should be a remainder after the division, its true value may be found as in Art. 76.

EXAMPLES.

2. Divide 13120 by $9\frac{1}{4}$. Ans. $1418\frac{14}{37}$.

3. Divide $76672\frac{6}{7}$ by 36. Ans. $2129\frac{202}{252}$.

4. Divide $2090\frac{1}{3}$ by 205. Ans. $10\frac{121}{615}$.

5. If $16\frac{1}{2}$ feet make a rod, how many rods are there in 10626 feet?

6. If $69\frac{1}{6}$ miles make a degree, how many degrees in 12450 miles? Ans. 180 degrees.

7. How many barrels of pork, at $17\frac{3}{4}$ dollars a barrel, may be bought for $5591\frac{1}{4}$ dollars?

8. A merchant has sold 2667 yards of silk in dress patterns of $10\frac{1}{2}$ yards each; required the number of patterns.

Ans. 254 patterns.

9. How many plats, each of $272\frac{1}{4}$ square feet, are equal in extent to 136125 square feet? Ans. 500 plats.

10. How many days will 119 pounds of bread last a man, if he consume $2\frac{3}{7}$ pounds per day? Ans. 49 days.

11. How many casks of $31\frac{1}{4}$ gallons each will be required to hold $12968\frac{3}{4}$ gallons of cider? Ans. 415 casks.

PROBLEMS,

FOUNDED UPON THE FUNDAMENTAL RULES.

99. THE following problems are founded upon the general principles of *addition*, *subtraction*, *multiplication*, and *division*, the fundamental operations of arithmetic, which have already been explained.

1. The *parts* of a number being given, to find the *number*.—*Add the parts together* (Art. 47).

2. The *sum* of two numbers and *one* of the numbers being given, to find the *other* number.—*From the sum subtract the given number* (Art. 50).

3. The *difference* between two numbers and the *larger* number being given, to find the *smaller*.—*From the larger number subtract the difference* (Art. 52).

4. The *difference* between two numbers and the *smaller* number being given, to find the *larger*.—*Add the smaller number and the difference together* (Art. 51).

5. The *sum* and the *difference* of two numbers being given, to find the *numbers*.—*From the sum subtract the difference and divide the remainder by 2, for the smaller number; add the difference to the smaller number, for the larger* (Art. 52).

NOTE.—In like manner, when the sum and differences are given, may be found any number of required numbers. After subtracting the differences from the sum, if there are 3 required numbers, divide by 3 for the smaller number; if 4, divide by 4, and so on.

6. The *product* of two numbers, and *one* of the numbers being given, to find the *other* numbers.—*Divide the product by the given number* (Art. 62).

7. The *product* of three numbers, and *two* of the numbers being given, to find the *other* number.—*Divide the given product by the product of the two given numbers* (Art. 72).

8. The *dividend* and *quotient* being given, to find the *divisor*.—*Divide the dividend by the quotient* (Art. 77).

9. The *divisor* and *quotient* being given, to find the *dividend*.—*Multiply the divisor and quotient together* (Art. 74).

EXAMPLES.

1. A carpenter has contracted to build one house for 2763 dollars, another for 4650 dollars, and a third for 8950 dollars. How much is he to receive for them all? Ans. 16363 dollars.

2. N. Chandler has invested in railroad stock and a small farm 929 dollars. If the amount invested in the stock was 279 dollars, how much did the farm cost him? Ans. 650 dollars.

3. Mount Black, the highest peak of the Blue Ridge, is 6476 feet high, which is 242 feet higher than Mount Washington, the highest peak of the White Mountains. What is the height of Mount Washington?

4. The city of Mexico, in 1519, was taken by Cortes, and, 328 years after, by General Scott. In what year did it yield to Scott? Ans. 1847.

5. Two travellers, A and B, meeting on a journey, found they had both travelled 1963 miles, and that A had travelled 199 miles more than B. What distance had each travelled?
Ans. A, 1081 miles; B, 882 miles.

6. A father gave his three sons 4698 dollars, of which James received 250 dollars more than George, and Edwin 410 dollars more than George. What sum did each receive?
Ans. George $1346; James $1596; Edwin $1756.

7. There was paid for 217 chests of tea 8463 dollars. How much was that a chest?

8. How many weeks will 684 bushels of oats last 19 horses, each horse consuming 3 bushels a week?

9. On dividing 3808 dollars among a certain number of men, it was found that the share of each was 224 dollars. Required the number of men. Ans. 17 men.

10. A certain missionary society divides its income among 99 missions, giving to each an average of 575 dollars. What is its income? Ans. 56925 dollars.

11. The product of 96, 22, and one other number is 63360. What is the other number? Ans. 30.

12. The divisor being 13 and the quotient 1101, what is the dividend? Ans. 14313.

MISCELLANEOUS EXAMPLES.

1. What is the distance by railroad from Boston to Galena, it being from Boston to Albany 200 miles, from Albany to Niagara Falls 305, from Niagara Falls to Detroit 230, from Detroit to Chicago 282, from Chicago to Galena 171?

2. Sold J. Weimer my best horse for 175 dollars, my second-best chaise for 87 dollars, and a good harness for 31 dollars. He has paid me in cash 38 dollars, and has given me an order on S. Lantz for 12 dollars. How many dollars remain due?

3. Bought 97 barrels of molasses at $ 5 a barrel. Gave 17 barrels to support the poor, and the remainder was sold at $ 8 a barrel. Did I gain or lose, and how much? Ans. $ 155 gain.

4. It requires 1728 cubic inches to make one cubic foot; required the number of cubic inches in 3787 cubic feet.

5. If a garrison of 987 men are supplied with 175686 pounds of beef, how much will there be for each man? Ans. 178 lbs.

6. Albert Peyton sold off from his farm 120 acres, gave his son 80 acres, and had remaining 160 acres; what number of acres did his farm contain before he disposed of any portion of it? Ans. 360 acres.

7. The annual revenue of a gentleman being $ 8395, how much per day is that equivalent to, there being 365 days in a year? Ans. $ 23.

8. What is the difference between half a dozen dozen, and six dozen dozen? Ans. 792.

9. Bought of F. Johnson 8 barrels of flour at $ 7 per barrel, and 3 hundred-weight of sugar at $ 8 per hundred. What was the amount of his bill?

10. George Adams bought an equal number of cows and oxen for 3952 dollars. For the cows he paid 31 dollars each, and for the oxen 45 dollars each. How many of each kind did he buy? Ans. 52.

11. If a certain quantity of provisions will sustain 13 men 4 days, how long would it sustain 1 man? Ans. 52 days.

12. The Globe Manufacturing Company has a capital of 250,000 dollars, divided into 500 shares. How much is each share? Ans. 500 dollars.

13. Purchased a farm of 500 acres for \$17,876. I sold 127 acres of it at \$47 an acre, 212 acres at \$96 an acre, and the remainder at \$37 an acre. What did I gain by my bargain? Ans. \$14,402.

14. There is a certain island 18 miles in circuit, which A and B undertake to travel round, both starting from the same point and going round in the same direction. When A has travelled 17 miles and B 7 miles, how far apart are they? Ans. 8 miles.

15. If 15 men can reap a certain field in 5 days, how long will it take 1 man to reap the same? Ans. 75 days.

16. What number is that, which being multiplied by 24, the product divided by 10, the quotient multiplied by 2, 32 subtracted from the product, the remainder divided by 4, and 8 subtracted from the quotient, the remainder shall be 2? Ans. 15.

17. From $126 + \overline{(16 + 4) \times 2}$ take $(48 \div 2) + \overline{(34 \times 6) \div (17 - 5)}$. Ans. 125.

18. There are in the library of a certain school 683 books, which number will give 23 books to each pupil, and 16 books over; what is the number of pupils?

19. R. Howland in making a journey, after walking 12 miles and travelling 40 miles by stage, went by steamboat 5 times the distance he had travelled by stage, and by cars 6 times as far as he had walked, and 7 miles besides. What was the length of his journey? Ans. 331 miles.

20. At an election A and B were candidates for the same office, and the whole number of votes cast for them was 9891, of which B received 1211 majority. What number of votes did each receive? Ans. A, 4340; B, 5551.

21. The product of 3 numbers is 4080; one of the numbers is 15, and another 16. What is the third number?

22. How many tons of coal at 6 dollars a ton will be required to pay for 17 yards of cloth at 4 dollars a yard and 32 bushels of wheat at 2 dollars a bushel? Ans. 22 tons.

23. James Cooper has manufactured in 4 years 5608 pairs of shoes, making each successive year 100 pairs more than the year before; how many pairs did he manufacture each year? Ans. 1252; 1352; 1452; 1552.

24. How many years will it take a young man earning 45 dollars, and spending 35 dollars of it, every month, if he have already 620 dollars, to lay up enough more to pay for a house costing 1100 dollars? Ans. 4 years.

25. If the product of the divisor by the quotient be 19782, and the remainder 31, what is the dividend? Ans. 19813.

26. John Franklin purchased railroad stock to the amount of 4473 dollars, and sold a part of it for 1885 dollars, obtaining 65 dollars a share, which was at a loss of 6 dollars on every share sold; but the stock advancing much in value, he was able to dispose of the balance so as to gain by the whole operation 812 dollars. What did he get a share for the balance of the stock? Ans. 100 dollars.

UNITED STATES MONEY.

ART. **100.** UNITED STATES MONEY is the legal currency of the United States. It was established by Congress in 1786. Its denominations and their relative values are shown in the following

TABLE.

10 Mills	make	1 Cent,	marked	ct.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	$.
10 Dollars	"	1 Eagle,	"	E.

Mills.		Cents.		Dimes.		Dollars.		Eagle.
10	=	1						
100	=	10	=	1				
1000	=	100	=	10	=	1		
10000	=	1000	=	100	=	10	=	1

In accounts and ordinary business transactions, the only denominations mentioned are dollars and cents, eagles being expressed as dollars, dimes as cents, and mills as a fraction of a cent.

The dollar is the integer, or unit of measure of United States money; and therefore dimes are *tenths* of a dollar; *cents, hundredths*; and mills, *thousandths*.

101. Parts of a dollar, or any quantity thus expressed, in tenths, hundredths, etc., are termed DECIMALS, or fractions, whose denominator (Art. 68) is 1, with one or more ciphers annexed. They are usually expressed by simply writing the numerator (Art. 68) with a point (.) before it, called the *decimal point*, or *separatrix;* the first place at the right of the point being tenths; the second place, hundredths; the third place, thousandths; the fourth place, ten-thousandths; and so on. Thus, $\frac{1}{10}$ is written .1; $\frac{1}{100}$ is written .01; $\frac{1}{1000}$ is written .001; $\frac{1}{10000}$ is written .0001, etc.

102. In writing dollars and cents together, the decimal point or separatrix is placed between the dollars and the cents or decimal part; and since cents occupy two places, the place of dimes and of cents, when the number of cents is less than 10, a cipher must be written before them, in the place of dimes. Thus, $30.375 is read, thirty dollars thirty-seven cents five mills, or thirty dollars and three hundred seventy-five thousandths of a dollar; $12.05 is read, twelve dollars five cents, or twelve dollars and five hundredths of a dollar, etc.

103. The denominations of United States Money increasing from right to left, and decreasing from left to right, in the same manner as do the units of the several orders in simple whole numbers, they may therefore be added, subtracted, multiplied, and divided according to the same rules.

104. The COINS of the United States are of gold, silver, and copper.

The *gold* coins are the double eagle, eagle, half-eagle, quarter-eagle, three-dollars, and dollar.

The *silver* coins are the dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent-piece.

The *copper* coins are the cent and two-cent-piece.

NOTE. — All the gold and silver coins of the United States are now made of one purity, 9 parts of pure metal and 1 part alloy. The alloy for the silver is copper, and that for the gold 1 part copper and 1 part silver. The cent and two-cent-piece, by the law of 1864, are composed of 95 parts of pure copper and 5 parts of tin and zinc.

The standard weight of the eagle, as fixed by present laws, is 258 grains Troy, and the gold coins in proportion according to their values. The weight of the silver dollar is 412½ grains; half-dollar, 192 grains; quarter-dollar, 96

grains; dime $38\frac{2}{5}$ grains; half-dime, $19\frac{1}{5}$ grains; three-cent piece, $11\frac{52}{100}$ grains; the cent, old coinage, 168 grains; the cent, new coinage, 48 grains.

The weight of the silver dollar, it will be seen, is greater in proportion to its value than the other silver coins. This is owing to their standard weight having been reduced, while that of the dollar remained unchanged; but since this reduction of weight of the smaller silver coins, no more of the silver dollar appear to have been coined. In circulation, the gold dollar, which of late years has been extensively coined, has almost entirely taken the place of the silver dollar.

The symbol $, or dollar sign, represents, probably, the letter U written upon an S, denoting U. S. (United States).

REDUCTION OF UNITED STATES MONEY.

105. Reduction of United States Money is changing the units of one of its denominations to the units of another, either of a higher or lower denomination, without altering their value.

106. To reduce units from a higher denomination to a lower.

Ex. 1. Reduce 58 dollars to cents and mills.

Ans. 5800 cents; 58000 mills.

OPERATION.

$$\begin{array}{rl} 58 & \text{dollars.} \\ 100 & \\ \hline 5800 & \text{cents.} \\ 10 & \\ \hline 58000 & \text{mills.} \end{array}$$

Or thus: 58000 mills.

We multiply the 58 by 100, because 100 cents make 1 dollar; and multiply the 5800 by 10, because 10 mills make 1 cent. Hence,

To reduce dollars to cents, annex TWO *ciphers; to reduce dollars to mills, annex* THREE *ciphers; and to reduce cents to mills, annex* ONE *cipher.*

Note. — Dollars, cents, and mills, expressed by a single number, are reduced to mills by merely removing the separating point; and dollars and cents, by annexing one cipher and removing the separatrix.

107. To reduce units from a lower denomination to a higher.

Ex. 1. Reduce 58000 mills to cents and to dollars.

Ans. 5800 cents; 58 dollars.

OPERATION.

$$\begin{array}{r|l} 10) & 58000 \text{ mills.} \\ 100) & 5800 \text{ cents.} \\ & 58 \text{ dollars.} \end{array}$$

We divide the 58000 by 10, because 10 mills make 1 cent; and divide the 5800 by 100, because 100 cents make 1 dollar. Hence,

To reduce mills to cents, cut off ONE *figure on the right; to reduce cents to dollars, point off* TWO *figures; and to reduce mills to dollars, point off* THREE *figures.*

EXAMPLES.

2. Reduce $765 to cents.
3. Change 726 mills to cents. Ans. $72\frac{6}{10}$ cents.
4. How many dollars are 329 cents? Ans. $3.29.
5. Change 12345 mills to dollars. Ans. $12.345.
6. Reduce $123.56 to mills. Ans. 123560 mills.
7. Reduce 2 eagles, 2 dollars, and 2 dimes to cents. Ans. 2220 cts.

ADDITION OF UNITED STATES MONEY.

108. Ex. 1. Add together 17 dollars 13 cents 5 mills; 8 dollars 4 cents 6 mills; 63 dollars 20 cents 3 mills; and 29 dollars 87 cents 5 mills. Ans. $118.259.

OPERATION.

$ cts. m.
17.135
8.046
63.203
29.875
Ans. 118.259

We write units of the same denomination in the same column, and add as in addition of simple numbers (Art. 45), and separate the dollars from the cents in the answer by the decimal point.

RULE. — *Write dollars, cents, and mills, so that units of the same denomination shall stand in the same column.*

Add as in addition of simple numbers, and place the decimal point directly under that above.

Proof. — The proof is the same as in addition of simple numbers.

EXAMPLES.

2.	3.	4.	5.
$ cts. m.	$ cts. m.	$ cts. m.	$ cts. m.
375.875	78.193	171.013	861.073
671.127	18.014	382.094	516.716
387.143	91.038	999.900	344.673
184.189	16.817	155.068	617.814
147.758	81.476	48.153	169.973
63.072	43.184	49.619	810.426
1829.164			

N 7

6. Add the following sums, $ 18.165, $ 701.63, $ 151.161, $ 375.089, and $ 471.017. Ans. $ 1717.062.

7. Bought a horse for eighty-seven dollars nine cents, a pair of oxen for sixty-five dollars twenty cents, and six gallons of molasses for two dollars six cents five mills; what was the amount of my bill? Ans. $ 154.355.

8. Sold a calf for three dollars eight cents, a bushel of corn for ninety-seven cents five mills, and three bushels of rye for three dollars five cents; what was the amount received? Ans. $ 7.105.

SUBTRACTION OF UNITED STATES MONEY.

109. Ex. 1. From 106 dollars 7 cents 7 mills, take 92 dollars 83 cents 8 mills. Ans. $ 13.239.

OPERATION.

	$ cts. m.
	1 0 6.0 7 7
	9 2.8 3 8
Ans.	1 3.2 3 9

We write the less number under the greater, place mills under mills, cents under cents, and dollars under dollars, and subtract as in subtraction of simple numbers (Art. 50), and separate the dollars from the cents in the answer by the decimal point.

RULE. — *Write the several denominations of the subtrahend under the corresponding ones of the minuend.*

Subtract as in subtraction of simple numbers, and place the decimal point directly under that above.

Proof. — The proof is the same as in subtraction of simple numbers.

EXAMPLES.

2.	3.	4.	5.
$ cts. m.	$ cts. m.	$ cts. m.	$ cts. m.
8 7 1.1 6 1	4 7 8.4 7 7	1 6 7.1 6 3	1 6 3.1 6 7
8 9.9 1 8	1 9 9.9 9 1	9 8.0 9 7	9.0 9 8
7 8 1.2 4 3			

6. Bought a farm for $ 1728.90, and sold it for $ 3786.98; what did I gain by my bargain?

7. Gave $ 79.25 for a horse, and $ 106.875 for a chaise, and sold them both for $ 200; what did I gain?

8. Bought a farm for $ 8967, and sold it for nine thousand eight hundred seventy-six dollars seventy-five cents; what did I gain? Ans. $ 909.75.

9. Bought a barrel of flour for $ 7.50, three bushels of rye for $ 2.75, and three cords of wood at $ 5.25 a cord; I sold the flour for $ 6.18, the rye for $ 3.00, and the wood for $ 6.75 a cord; what was gained by the bargain? Ans. $ 3.43.

10. A young lady went a "shopping." Her father gave her a twenty-dollar bill. She purchased a dress for $ 8.16, a muff for $ 3.19, a pair of gloves for $ 1.12, a pair of shoes for $ 1.90, a fan for $ 0.19, and a bonnet for $ 3.08; how much money did she return to her father? Ans. $ 2.36.

MULTIPLICATION OF UNITED STATES MONEY.

110. Ex. 1. What will 365 barrels of flour cost at $ 5.75 a barrel? Ans. $ 2098.75.

OPERATION.

```
   $5.75
     365
   -----
    2875
   3450
  1725
 --------
 $2098.75 Ans.
```

We multiply as in multiplication of simple numbers, and, obtaining the product in cents, the lowest denomination of the multiplicand, we reduce them to dollars by pointing off two places at the right for cents (Art. 107).

RULE. — *Multiply as in multiplication of simple numbers.*
The product will be in the lowest denomination in the multiplicand, which must be pointed off as in reduction of United States money.

Proof. — The proof is the same as in multiplication of simple numbers.

EXAMPLES.

2. What will 126 pounds of butter cost at 13 cents a pound?

3. What will 63 pounds of tea cost at 93 cents a pound?

4. What will 43 tons of hay cost at 13 dollars 75 cents a ton? Ans. $ 591.25.

5. If 1 pound of pork is worth 7 cents 3 mills, what are 46 pounds worth? Ans. $ 3.358.

6. If 1 hundred of beef cost 3 dollars 28 cents, what are 76 hundred worth?

7. What will 96 thousand feet of boards cost at 11 dollars 67 cents a thousand? Ans. $ 1120.32.

8. If a barrel of cider be sold for 2 dollars 12 cents, what will be the value of 169 barrels? Ans. $ 358.28.

9. What will be the value of a hogshead of wine, containing 63 gallons, at 1 dollar 63 cents a gallon? Ans. $ 102.69.

10. Sold a sack of hops, weighing 396 pounds, at 11 cents 3 mills a pound; to what did it amount? Ans. $ 44.748.

11. Sold 19 cords of wood, at $ 5.75 a cord; to what did it amount?

12. Sold 169 tons of timber, at $ 4.68 a ton; what did I receive?

13. Sold a hogshead of sugar, weighing 465 pounds; to what did it amount, at 14 cents a pound? Ans. $ 65.10.

14. What will be the amount of 789 pounds of leather at 18 cents a pound? Ans. $ 142.02.

15. What will be the expense of 846 pounds of sheet-lead at 5 cents 7 mills a pound? Ans. $ 48.222.

16. When potash is sold for $ 132.55 a ton, what will be the price of 369 tons? Ans. $ 48910.95.

17. What will 365 pounds of beeswax cost at 18 cents 4 mills a pound? Ans. $ 67.16.

18. If 1 pound of tallow cost 7 cents 3 mills, what are 968 pounds worth? Ans. $ 70.664.

DIVISION OF UNITED STATES MONEY.

111. Ex. 1. If 78 barrels of fish cost $ 303.42, what will 1 barrel cost? Ans. $ 3.89.

OPERATION.

```
78 ) $303.42 ( $3.89
      234
      ---
       694
       624
       ---
        702
        702
        ---
```

We divide as in division of simple numbers, and point off as many places for cents in the quotient as there are for cents in the dividend.

RULE. — *Divide as in division of simple numbers.*

The quotient will be in the lowest denomination of the dividend, which must be pointed off as in reduction of United States money.

NOTE. — When the dividend consists of dollars only, and is either smaller than the divisor or not divisible by it without a remainder, reduce it to a lower denomination by annexing two or three ciphers, as the case may require, and the quotient will be cents or mills accordingly.

When it is required to find the number of times one sum of money is contained in another, both sums, if of different denominations, must be reduced to the same, before dividing.

Proof.—The proof is the same as in division of simple numbers.

EXAMPLES.

2. Bought 11 chests of tea for $ 234.30. What did I give a chest? Ans. $ 21.30.

OPERATION.

```
11 ) $ 2 3 4.3 0
Ans.   $ 2 1.3 0
```

3. How many yards of cloth at 9 cents a yard can be bought for $ 28.89? Ans. 321 yards.

OPERATION.

```
9 ) 2 8 8 9
Ans.  3 2 1 yards.
```

4. When 19 bushels of fine salt can be bought for $ 30.87½, what costs one bushel?

5. At 12 cents a pound, how many pounds of sugar can be bought for $ 51?

6. If 78 barrels of fish cost $ 303.42, what will one barrel cost?

7. Bought 42 bushels of pears for $ 73.50; what cost one bushel? Ans. $ 1.75.

8. How many yards of broadcloth at $ 2.75 a yard can be bought for $ 904.75? Ans. 329 yards.

9. When rye is sold at the rate of 628 bushels for $ 471.00, what is that a bushel? Ans. $ 0.75.

10. If it should cost $ 1460 to support a family 365 days, what would be the expense a day? Ans. $ 4.

11. How many gallons of oil at $ 1.62½ a gallon can be bought for $ 234?

12. If 1624 pounds of pork cost $ 97.44, what cost one pound? Ans. $ 0.06.

13. If 47 thousand shingles cost $ 176.25, what is the cost per thousand? Ans. $ 3.75.

14. Bought 148 tons of plaster of Paris for $ 337.44; what was it per ton? Ans. $ 2.28.

15. At $ 37.75 an acre, how many acres of land may be bought for $ 1774.25? Ans. 47 acres.

16. At 67 cents a pound, how many chests of tea, each weighing 59 pounds, can be bought for $ 672.01? Ans. 17 chests.

17. How many bushels of wheat at $ 1.25 a bushel can be bought for $ 863.75? Ans. 691 bushels.

18. Sold 169 tons of timber for $ 790.92; what cost one ton?

19. What cost one pound of leather, if 789 pounds cost $ 142.02?

20. If 369 tons of potash cost $ 48910.95, what will be the price of one ton? Ans. $ 132.55.

21. Bought 47 hogsheads of salt, each hogshead containing 7 bushels, for $ 368.48; what cost one bushel? Ans. $ 1.12.

GENERAL PRINCIPLES AND APPLICATIONS.

112. ANALYSIS is an examination of a question by resolving it into its parts, in order to consider them separately, and thus, by reasoning from the nature of the question, to render each step in the solution plain and intelligible.

113. An *aliquot part* of any number or quantity is such a part as will exactly divide that number or quantity. Thus, 3 is an aliquot part of 6; and 4, an aliquot part of 16.

114. The method of performing operations by means of aliquot parts is called PRACTICE, or ANALYSIS by ALIQUOT PARTS.

115. In business calculations frequent use is made of the following

ALIQUOT PARTS OF A DOLLAR.

50 cents $= \frac{1}{2}$ of $ 1.	$12\frac{1}{2}$ cents $= \frac{1}{8}$ of $ 1.
$33\frac{1}{3}$ cents $= \frac{1}{3}$ of $ 1.	10 cents $= \frac{1}{10}$ of $ 1.
25 cents $= \frac{1}{4}$ of $ 1.	$6\frac{1}{4}$ cents $= \frac{1}{16}$ of $ 1.
20 cents $= \frac{1}{5}$ of $ 1.	5 cents $= \frac{1}{20}$ of $ 1.

PRACTICAL QUESTIONS BY ANALYSIS.

116. The price of a unit of any quantity being given, to find the cost of that quantity.

Ex. 1. If 1 barrel of flour cost $ 7, what will 125 barrels cost?

ANALYSIS. Since 1 barrel costs $ 7, 125 barrels will cost 125 times as much: $ 7 $\times$ 125 $=$ $ 875.

2. If 1 pound of beef cost 13 cents, what will 914 pounds cost? Ans. $ 118.82.

3. If 1 yard of calico cost 23 cents, what will $31\frac{1}{2}$ yards cost?

4. When 44 cents are paid for 1 pound of tea, how much must be paid for 15 chests of tea, each containing 47 pounds?

5. At $39\frac{1}{2}$ cents a pound, how much must be paid for 9 bales of wool, each containing 317 pounds? Ans. $ $1126.93\frac{1}{2}$.

6. If the cost of building 1 mile of railroad be $ 41315, what will be that of 113 miles? Ans. $ 4668595.

117. The price of a unit of any quantity being a given aliquot part of a dollar, to find the cost of that quantity.

7. At $12\frac{1}{2}$ cents a yard, what will 208 yards of calico cost? Ans. $ 26.

ANALYSIS. Since, at $ 1 a yard, the cost would be as many dollars as there are yards, the cost at $12\frac{1}{2}$ cents, or $\frac{1}{8}$ of $ 1, will be one eighth as many dollars as there are yards, or as many dollars as 8 is contained times in 208; $208 \div 8 = 26$. Therefore 208 yards cost $ 26. That is, we *take such a part of the number denoting the quantity as the price is of* 1 *dollar.*

8. What will be the cost of 362 pounds of feathers at $33\frac{1}{3}$ cents a pound? Ans. $ $120.66\frac{2}{3}$.

9. At 50 cents a bushel, what will 7 loads of potatoes cost, each containing 30 bushels? Ans. $ 105.

10. At $6\frac{1}{4}$ cents a pound, how much must be paid for 1163 pounds of shingle nails?

11. What will be the cost of 96 yards of broadcloth at $ $4.33\frac{1}{3}$ a yard? Ans. $ 416.

ANALYSIS. Since, at $ 1 a yard, the cost would be as many dollars as there are yards, the cost at $ $4.33\frac{1}{3}$, or $ $4\frac{1}{3}$, will be $4\frac{1}{3}$ times as much: $ $4\frac{1}{3} \times 96 =$ $ 416.

12. If the cost of 1 pair of boots be $ $3.16\frac{2}{3}$, what will be the cost of 20 cases, each case containing 60 pairs?

13. At $ 2.25 each, what will 150 hats cost?

Ans. $ 337.50.

14. What will 98 dozen knives cost at $ $5.12\frac{1}{2}$ a dozen?

Ans. $ 502.25.

15. If James Spooner spends uselessly every day for cigars $6\frac{1}{4}$ cents, how much does he thus spend in 360 days?

16. At $ 2.20 a bushel, what will 15 car-loads of wheat, each containing 212 bushels, cost? Ans. $ 6996.

118. The price of any article sold by the 100, or 1000, with the quantity, being given, to find the cost of that quantity.

17. What will 732 fishes cost at $ 5 a hundred?

Ans. $ 36.60.

ANALYSIS. Since 100 fishes cost $ 5, 1 fish will cost $\frac{1}{100}$ of $ 5, or $ $\frac{5}{100}$; and if 1 fish cost $ $\frac{5}{100}$, 732 fishes will cost 732 times as much: $ 5 × 732 = $ 3660; $ 3660 ÷ 100 = $ 36.60. That is, we *multiply the quantity and price together, and cut off two figures at the right* (Art. 79).

18. How much must be paid for 950 apple-trees at 20 dollars a hundred?

19. What will 7235 arithmetics cost at $ 45 a hundred?

Ans. $ 3255.75.

20. What will it cost to excavate 19875 solid feet of earth, at the rate of 52 cents per hundred solid feet? Ans. $ 103.35.

21. What is the value of 1765 feet of timber at $ 3 a hundred? Ans. $ 52.95.

22. What is the value of 2355 cedar rails at $ 5.50 per hundred?

23. What will 3100 bricks cost at $ 6 a thousand?

Ans. $ 18.60.

ANALYSIS. Since 1000 bricks cost $ 6, 1 brick will cost $\frac{1}{1000}$ of $ 6, or $ $\frac{6}{1000}$; and, if 1 brick cost $ $\frac{6}{1000}$, 3100 will cost 3100 times as much: $ 6 × 3100 = $ 18600; $ 18600 ÷ 1000 = $ 18.60. That is, we *multiply the quantity and price together, and cut off three figures at the right* (Art. 79).

24. At $ 142.50 a thousand, what will 6150 chestnut posts cost? Ans. $ 876.375.

25. How much must be paid for 15750 feet of boards at $ 30 a thousand?

26. At $ 2.25 per thousand, how much must be paid for 3550 laths? Ans. $ $7.98\frac{3}{4}$.

27. I have bought of L. T. Robbins 363 feet of plank at

\$ 2.50 per hundred; 3150 pickets at \$ 14 per thousand; and 350 feet of boards at \$ 20 per thousand. What is the amount of the whole? Ans. \$ 60.17$\frac{1}{2}$.

28. What must be paid for 92350 railroad ties, at \$ 135 per thousand?

119. The price of any article sold by the ton of 2000 pounds, with the quantity, being given, to find the cost of that quantity.

29. At \$ 8 a ton, what will 2550 pounds of coal cost? Ans. \$ 10.20.

ANALYSIS. Since 2000 pounds cost \$ 8, 1000 pounds will cost one half of \$ 8, or \$ 4; and, if 1000 pounds cost \$ 4, 1 pound will cost $\frac{1}{1000}$ of \$ 4, or \$ $\frac{4}{1000}$, and 2550 pounds will cost 2550 times as much: $\$4 \times 2550 = \$10200$; $\$10200 \div 1000 = \$10.20$. That is, we *multiply the number denoting the quantity and half the price together, and cut off three figures at the right* (Art. 79).

30. How much must be paid for 3760 pounds of hay, when the price is \$ 15 a ton? Ans. \$ 28.20.

31. What is the value of 19 casks of plaster of Paris, each weighing 500 pounds, at \$ 9 a ton?

32. How much must be paid for transporting 5163 pounds of freight, at \$ 3.50 per ton?

33. At \$ 46 a ton, what will 33550 pounds of railroad iron cost? Ans. \$ 771.65.

34. G. Reed has bought 30 bags of superphosphate of lime, each bag containing 125 pounds, at \$ 45 per ton; and 7500 pounds of Peruvian guano, at \$ 53 per ton. What is the cost of the whole? Ans. \$ 283.125.

35. I have bought 6350 pounds of anthracite coal at \$ 7.12$\frac{1}{2}$ per ton, and 3560 pounds of Pictou coal at \$ 7.25 per ton. What is the cost of the whole? Ans. \$ 35.526$\frac{7}{8}$.

120. The price of any quantity, and the quantity, being given, to find the price of a unit of that quantity.

36. If 125 barrels of flour cost \$ 875, what will 1 barrel cost? Ans. \$ 7.

ANALYSIS. Since 125 barrels cost \$ 875, 1 barrel will cost

as many dollars as 125 is contained times in 875; $\$875 \div 125 = \$7$.

37. If 914 pounds of beef cost \$118.82, what will 1 pound cost? Ans. \$0.13.

38. If 96 yards of broadcloth can be bought for \$400, for what can 1 yard be bought? Ans. \$4.16$\frac{2}{3}$.

39. If \$510 are paid for 120 pairs of boots, what is the cost of 1 pair? What is the cost of 17 pairs?

40. A merchant paid \$270 for 300 casks of lime; what did he pay a cask? What did he pay for 33 casks?
Ans. \$29.70.

41. When boards are \$25 per thousand, what are they per foot? What per hundred? Ans. \$2.50.

42. If 3 pieces of calico, each containing 30 yards, cost \$19.80, what does 1 yard cost? Ans. \$0.22.

43. When 10 firkins of butter, each containing 56 pounds, can be bought for \$112, what is the price per pound?
Ans. \$0.20.

121. The price of any quantity, and the price of a unit of that quantity, being given, to find the quantity.

44. At \$7 per barrel, how many barrels of flour can be purchased for \$875? Ans. 125 barrels.

Analysis. Since 1 barrel will cost \$7, as many barrels can be purchased with \$875 as 7 is contained times in 875; $875 \div 7 = 125$; therefore, there can be purchased 125 barrels.

45. At \$3 a yard, how many yards of broadcloth can I purchase with \$456? Ans. 152 yards.

46. How many cords of wood, at \$4.25 each, can be bought for \$357?

47. At 33$\frac{1}{3}$ cents a pound, how many pounds of feathers can be bought for \$120.66$\frac{2}{3}$? Ans. 362 pounds.

48. At 50 cents a bushel, how many barrels of apples, each containing 3 bushels, can be purchased for \$40.50?
Ans. 27 barrels.

49. Expended for hemp \$11600, at \$232 per ton. How many tons did I buy? Ans. 50 tons.

50. How many boxes of raisins, at \$3.50 per box, can be bought for \$756? Ans. 216 boxes.

BILLS.

122. A BILL is a paper, given by merchants, containing a statement of goods sold, and their prices.

An *invoice* is a bill of merchandise shipped or forwarded to a purchaser, or selling agent.

The *date* of a bill is the time of the transaction.

The bill is *against* the party owing, and in *favor* of the party who is to receive the amount due.

A bill is receipted, when the receiving of the amount due is acknowledged by the party in whose favor it is, as in bill 1. A clerk, or any other authorized person, may, in his stead, receipt for him, as in bill 2.

When the items of a bill have been rendered at different dates, the several times may be given at the left hand, as in bill 4.

When the bill is in the form of an account, containing items of debt and credit in its settlement, it is required to find the difference due, or balance, as in bill 4.

NOTE. — A *due-bill* is a written acknowledgment that a debt is due, and is often given in making settlements to remove cause of subsequent disputes as to a claim. A bill is sometimes settled or receipted by a due-bill, as in bill 4.

What is the cost of each article in, and the amount due of, each of the following bills?

(1.) *Boston, July* 4, 1857.

Mr. JAMES DOW,

Bought of DENNIS SHARP,

17	*yds.*	*Flannel,*	*at*	$0.45	
19	"	*Shalloon,*	"	.37	
16	"	*Blue Camlet,*	"	.46	
13	"	*Silk Vesting,*	"	.87	
9	"	*Cambric Muslin,*	"	.63	
25	"	*Bombazine,*	"	.56	
17	"	*Ticking,*	"	.31	
19	"	*Striped Jean,*	"	.16	
					$ 61.33

Received payment,

DENNIS SHARP.

(2.) *Chicago, Jan.* 1, 1857.

Mr. SAMUEL SMITH,

Bought of DAVID JOHNSON,

13 *lbs.*	*Tea,*	*at*	$0.98
16 "	*Coffee,*	"	.15
36 "	*Sugar,*	"	.13
47 "	*Cheese,*	"	.09
12 "	*Pepper,*	"	.19
7 "	*Ginger,*	"	.17
13 "	*Chocolate,*	"	.61
			——— $

Received payment,

DAVID JOHNSON,
by J. KEEN.

(3.) *Philadelphia, Sept.* 5, 1856.

Messrs. WILSON, NILES, & CO.,

Bought of PECK & BLISS,

2 *doz.*	*National Arithmetics,*	*at* $6.00	
3 "	*Com. Sch. Arithmetics,*	" 5.00	
5 "	*Primary Arithmetics,*	" 1.80	
17	*Parker's Exercises in Composition,*	" .25	
13	*Maglathlin's National Speaker,*	" .60	
19	*Leverett's Cæsar,*	" .50	
3	*Greenleaf's Algebra,*	" .60	
7	*Silliman's Chemistry,*	" 1.12½	
15	*Olney's U. S. History,*	" .22	
5	*Imp. Quarto Bibles,*	" 15.00	
3	*Webster's Quarto Dictionaries,*	" 4.50	
5	*Felton's Greek Readers,*	" 1.50	
1	*Kane's Arctic Explorations,*	" 4.50	
		———	$171.02½

Received payment,

PECK & BLISS.

(4.) *New Orleans, Jan.* 1, 1856.

Mr. ALBERT CRAWFORD,

To BALDWIN, SHERWOOD, & RICE, *Dr.*

1855.								
Jan.	2.	*For*	17	*yds.*	*Broadcloth,*	*at*	$ 5.25	
"	15.	"	29	"	*Cassimere,*	"	1.62	
Feb.	3.	"	60	"	*Bleached Shirting,*	"	.17	
"	7.	"	49	"	*Ticking,*	"	.27	
"	15.	"	18	"	*Blue Cloth,*	"	3.19	
June	17.	"	27	"	*Habit do.*	"	2.75	
Aug.	3.	"	75	"	*Flannel,*	"	.61	
Sept.	19.	"	36	"	*Plaid Prints,*	"	.75	
Dec.	2.	"	49	"	*Brown Sheeting,*	"	.18	
								$ 372.90.

Cr.

Jan.	28.	*By cash,*			$ 83.00
Feb.	19.	" 3 *M. Boards,*	*at*	$ 30.00	
May	9.	" 7 *M. Shingles,*	"	4.00	
June	17.	" 4 *days' labor,*	"	2.00	
July	3.	" 5 " "	"	1.75	
Oct.	7.	" 7 *C. Timber,*	"	2.25	
Nov.	4.	" *Cash,*			60.00
Dec.	29.	" *Draft,*			45.00
					$ 338.50
		Bal. due B., S., & R.,			$ 34.40

Jan. 5. *Settled by due-bill,*

BALDWIN, SHERWOOD, & RICE.

(5.) *St. Louis, May* 1, 1856.

Mr. BENJAMIN TREAT,

Bought of JOHN TRUE,

37 *chests*	*Green Tea,*	*at*	$ 25.50	
41 "	*Black do.*	"	16.17	
40 "	*Imperial Tea,*	"	97.75	
13 *crates*	*Liverpool Ware,*	"	169.37	
				$

Received payment,

JOHN TRUE.

(6.) *Portland, May* 16, 1857.

Mr. J. C. PORTER,

Bought of WILLARD & HALE,

17 *bbl.*	*Canada Flour,*	*at*	$ 8.25
50 *lbs.*	*Dupont's Eagle Gunpowder,*	"	.50
140 "	*Sheet Zinc,*	"	.08½
120 "	*Prussian Blue,*	"	.63
			——— $

Received payment,

WILLARD & HALE.

(7.) *New York, July* 11, 1856.

Mr. JOHN CUMMINGS,

Bought of LORD & SECOMB,

97 *bbl.*	*Genesee Flour,*	*at*	$ 6.25
167 "	*Philadelphia do.*	"	5.95
87 "	*Baltimore do.*	"	6.07
196 "	*Richmond do.*	"	5.75
275 "	*Howard St. do.*	"	7.25
69 *bu.*	*Rye,*	"	1.16
136 "	*Virginia Corn,*	"	.67
68 "	*North River do.*	"	.76
169 "	*Wheat,*	"	1.37
76 *tons*	*Lehigh Coal,*	"	9.67
89 "	*Iron,*	"	69.70
49	*Grindstones,*	"	3.47
39	*Pitchforks,*	"	1.61
197	*Rakes,*	"	.17
86	*Hoes,*	"	.69
78	*Shovels,*	"	1.17
187	*Spades,*	"	.85
91	*Ploughs,*	"	11.61
83	*Harrows,*	"	17.15
47	*Handsaws,*	"	3.16
35	*Mill-saws,*	"	18.15
47 *cwt.*	*Steel,*	"	9.47
57 "	*Lead,*	"	6.83
			——— $ 17,315.32.

Received payment,

E. T. LOWE,

for LORD & SECOMB.

LEDGER ACCOUNTS.

123. The principal book of accounts among merchants is called a ledger. In it are brought together scattered items of accounts, often making long columns. As a rapid way of finding the amounts, accountants generally add more than one column at a single operation (Art. 48). The examples below may be added both by the usual method and by that which is more rapid.

1.	2.	3.	4.
$ cts.	$ cts.	$ cts.	$ cts.
3.33	91.16	134.62	120.03
9.10	40.44	511.42	63.17
16.37	60.66	27.34	100.00
41.30	91.31	33.35	781.14
31.70	4.00	60.40	151.61
57.71	6.34	149.31	31.21
3.60	7.19	921.31	105.52
5.00	28.37	39.16	306.06
.50	81.00	110.05	17.75
.13	6.00	104.03	85.25
1.55	15.00	363.40	13.00
7.91	43.05	9.00	10.00
1.19	10.03	11.12	135.75
9.00	95.05	0.10	111.15
13.40	61.18	17.91	163.05
37.65	112.15	11.13	712.15
45.75	100.61	45.92	31.15
5.25	160.00	17.75	95.25
7.75	39.41	13.85	25.17
5.31	22.39	11.01	77.25
8.13	92.77	62.19	123.10
4.27	33.01	95.17	116.51
2.64	64.32	19.10	223.06
11.12	59.11	111.00	966.75
41.21	17.89	195.25	102.15
33.79	761.15	613.20	4106.31
47.31	823.54	108.09	6102.19
10.00	238.47	334.05	7119.81

COMPOUND NUMBERS.

124. A COMPOUND NUMBER is a collection of concrete units of different denominations; as, 5 pounds and 6 ounces; 4 feet and 5 inches.

125. A *scale* expresses the law of relation between the different units of a number.

The different units of simple numbers have a uniform *tenfold* increase from lower to higher orders, and a like decrease from higher to lower orders. They, therefore, are said to have a *uniform scale.*

In compound numbers, the names of different measuring units (Art. 9) are included in the expression of a single quantity, so that the relation of the units of one order to those of another is that of a *varying scale;* as in the expression of pounds, shillings, pence, and farthings, it is 4, 12, and 20.

REDUCTION OF COMPOUND NUMBERS.

126. REDUCTION is the process of changing numbers from one denomination to another, without altering their values.

It is of two kinds, Reduction Descending and Reduction Ascending.

Reduction Descending is changing numbers of a higher denomination to a lower denomination; as pounds to shillings, &c. It is performed by multiplication.

Reduction Ascending is changing numbers of a lower denomination to a higher denomination; as farthings to pence, &c. It is the reverse of Reduction Descending, and is performed by division.

ENGLISH MONEY.

127. English or Sterling Money is the currency of England.

TABLE.

4 Farthings (qr. or far.)	make	1 Penny,	d.
12 Pence	"	1 Shilling,	s.
20 Shillings	"	1 Pound,	£.

far.		d.		s.		£.
4	=	1				
48	=	12	=	1		
960	=	240	=	20	=	1

NOTE 1. — The symbol £. stands for the Latin word *libra*, signifying a pound; *s.* for *solidus*, a shilling; *d.* for *denarius*, a penny; *qr.* for *quadrans*, a quarter.

NOTE 2. — Farthings are sometimes expressed in a fraction of a penny; thus, 1 *far.* = $\frac{1}{4}$ d.; 2 far. = $\frac{1}{2}$ d.; 3 far. = $\frac{3}{4}$ d.

NOTE 3. — The term *sterling* is probably from Easterling, the popular name of certain early German traders in England, whose money was noted for the purity of its quality.

NOTE 4. — The English coins consist of the five-sovereign piece, the double-sovereign, the sovereign, and the half-sovereign, made of *gold;* the crown, the half-crown, the shilling, the six-pence, the four-pence, the three-pence, the two-pence, the one-and-a-half-pence, and the penny, made of *silver;* the penny, the half-penny, the farthing, and the half-farthing, made of *copper*.

The sovereign represents the pound sterling, whose legal value in United States money is $4.84. The value of the English guinea is 21 shillings sterling. The guinea, the five-guinea, the half-guinea, the quarter-guinea, and the seven-shilling piece, are no longer coined.

The guinea is so called because the gold of which the first guineas were made was brought from Guinea, in Africa.

The English gold coins are now made of 11 parts of pure gold, and 1 part of copper, or some other alloy; and the silver coin, of 37 parts of pure silver, and 3 parts of copper.

The present standard weight of the sovereign is $123\frac{171}{623}$ grains Troy; the crown, $436\frac{4}{11}$ grains; the copper penny, $291\frac{2}{3}$ grains.

128. To change numbers expressed in one or more denominations to their equivalents in one or more other denominations.

Ex. 1. In 48£. 12s. 7d. 2far. how many farthings?

OPERATION.

$$\begin{array}{rl}
48\text{£.}\ 12\text{s.}\ 7\text{d.}\ 2\text{far.} & \\
20 & \\
\hline
972 & \text{shillings.} \\
12 & \\
\hline
11671 & \text{pence.} \\
4 & \\
\hline
\text{Ans. } 46686 & \text{farthings.}
\end{array}$$

We multiply the 48 by 20, because 20 shillings make 1 pound, and to this product we add the 12 shillings in the question, and obtain 972 shillings. We then multiply by 12, because 12 pence make 1 shilling, and to the product we add the 7 pence, and obtain 11671 pence. Again, we multiply by 4, because 4 farthings make 1 penny, and to this product we add the 2 farthings, and obtain 46686 farthings, the answer sought.

Ex. 2. In 46686 farthings how many pounds?

OPERATION.

4) 4 6 6 8 6 far.
12) 1 1 6 7 1 d. 2 far.
20) 9 7 2 s. 7 d.
4 8 £. 1 2 s.

Ans. 48£. 12s. 7d. 2 far.

We divide by 4, because 4 farthings make 1 penny, and the result is 11671 pence, and 2 farthings remaining. We then divide by 12 because 12 pence make 1 shilling, and the result is 972 shillings, and 7 pence remaining. Lastly, we divide by 20, because 20 shillings make 1 pound, and the result is 48 pounds, and 12 shillings remaining. By annexing to the last quotient the several remainders, we obtain 48£. 12s. 7d. 2far. as the required result.

From these illustrations, for the two kinds of reduction, we deduce the following

RULE. — FOR REDUCTION DESCENDING. *Multiply the highest denomination given by the number of units required of the next lower denomination to make one in the denomination multiplied. To this product add the corresponding denomination of the multiplicand, if there be any. Proceed in this way, till the reduction is brought to the denomination required.*

FOR REDUCTION ASCENDING. *Divide the lower denomination given by the number of units required of that denomination to make one of the next higher. The quotient thus obtained divide in like manner, and so proceed until it is brought to the denomination required. The last quotient, with the several remainders, if there be any, annexed, will be the answer.*

EXAMPLES.

3. In 127£. 15s. 8d. how many farthings?
4. In 122672 farthings how many pounds?
5. How many farthings in 28£. 19s. 11d. 3 far.?
6. How many pounds in 27839 farthings?
7. In 378£. how many pence?
8. In 90720 pence how many pounds?
9. Reduce 967 guineas to pounds.
10. Reduce 1015£. 7s. to guineas.

AVOIRDUPOIS WEIGHT.

129. Avoirdupois or Commercial Weight is used in weighing almost every kind of goods, and all metals except gold and silver.

Table.

16 Drams (dr.)	make	1 Ounce,	oz.
16 Ounces	"	1 Pound,	lb.
25 Pounds	"	1 Quarter,	qr.
4 Quarters	"	1 Hundred Weight,	cwt.
20 Hundred Weight	"	1 Ton,	T.

dr.		oz.		lb.		qr.		cwt.		T.
16	=	1								
256	=	16	=	1						
6400	=	400	=	25	=	1				
25600	=	1600	=	100	=	4	=	1		
51200	=	32000	=	2000	=	80	=	20	=	1

Note 1. — The *oz.* stands for *onza*, the Spanish for ounce, and in *cwt.* the *c* stands for *centum*, the Latin for *one hundred*, and *wt* for *weight*.

Note 2. — The laws of most of the States, and common practice at the present time, make 25 pounds a quarter, as given in the table. But formerly, 28 pounds were allowed to make a quarter, 112 pounds a hundred, and 2240 pounds a ton, as is still the standard of the United States government in collecting duties at the custom-houses.

Note 3. — The term *avoirdupois* is from the French *avoir du poid*, signifying to have weight.

Note 4. — The standard avoirdupois pound of the United States is the weight, taken in the air, of $27\frac{7015}{10000}$ cubic inches of distilled water, at its maximum density, or when at a temperature of $39\frac{83}{100}$ degrees Fahrenheit, the barometer being at 30 inches. It is the same as the Imperial pound avoirdupois of Great Britain, which is the weight of $27\frac{7274}{10000}$ cubic inches of distilled water at the temperature of 62 degrees.

Examples.

1. In 165T. 13cwt. 3qr. 19lb. 14oz. how many ounces?
2. In 5302318 ounces how many tons?
3. If a load of hay weigh 3T. 16cwt. 2qr. 18lb., required the weight in ounces.
4. In 122688 ounces how many tons?
5. Required the number of drams in 2T. 17cwt. 3qr. 16lb. 15oz. 13dr.
6. In 1482749 drams how many tons?
7. What is the value of 7T. 17cwt. at 7 cents per pound? Ans. $1099.00.
8. What will 19cwt. 3qr. 20lb. of sugar cost at 9 cents per pound? Ans. $179.55.

TROY OR MINT WEIGHT.

130. Troy or Mint Weight is the weight used in weighing gold, silver, jewels, and liquors; and in philosophical experiments.

Table.

24 Grains (gr.)	make	1 Pennyweight,	pwt.
20 Pennyweights	"	1 Ounce,	oz.
12 Ounces	"	1 Pound,	lb.

gr.		pwt.		oz.		lb.
24	=	1				
480	=	20	=	1		
5760	=	240	=	12	=	1

Note 1. — Troy weight was introduced into Europe from Cairo in Egypt, in the 12th century, and was first adopted in Troyes, a city in France, where great fairs were held, whence it may have had its name.

Note 2. — A grain or corn of wheat, gathered out of the middle of the ear, was the origin of all the weights used in England. Of these grains, 32, well dried, were to make one pennyweight. But in later times it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, being the least weight now in use, from which the rest are computed.

Note 3. — Diamonds and other precious stones are weighed by what is called *Diamond Weight*, of which 16 *parts* make 1 *grain;* 4 grains, 1 *carat.* 1 grain Diamond Weight is equal to $\frac{4}{5}$ grains Troy, and 1 carat to $3\frac{1}{5}$ grains Troy. In weighing pearls, the pennyweight is divided into 30 grains instead of 24, so that 1 pearl grain is equal to $\frac{4}{5}$ grains Troy. The carat as a weight must not be confounded with the *assay carat*, a term whose use is to indicate a proportional part of a weight, as in expressing the fineness of gold, each carat means a twenty-fourth part of the entire mass used. Thus, pure gold is termed 24 carat gold, and gold that is not pure is termed 18 carat gold, 20 carat gold, &c., as its mass may be 18 twenty-fourths, 20 twenty-fourths, &c. pure gold. Each assay carat is subdivided into 4 assay grains, and each assay grain into 4 assay quarters.

Note 4. — The Troy pound, the *standard unit of weight* adopted by the United States Mint, is the same as the Imperial Troy pound of Great Britain, and is equal to the weight, taken in the air, of $22\frac{7944}{10000}$ cubic inches of distilled water, at its maximum density, the barometer being at 30 inches.

Examples.

1. How many grains in 28lb. 11oz. 12pwt. 15gr. Troy?
2. In 166863 grains Troy how many pounds?
3. If a silver pitcher weigh 3lb. 10oz., what is its weight in grains?
4. How many pounds Troy in 22080 grains?

5. What is the value of 73lb. 11oz. of standard silver at \$0.062 per pennyweight?

6. How many pounds of standard silver can be purchased for \$1099.88, at the rate of \$0.062 per pennyweight?

7. A Californian has 57lb. 7oz. of pure gold. What is its value at \$20.593$\frac{1}{5}$ per ounce? Ans. \$14229.901$\frac{1}{5}$.

8. What is the value of a mass of standard gold weighing 19lb. 6oz. 16pwt. at 93 cents per pennyweight?

Ans. \$4367.28.

9. I have a lump of pure silver weighing 13lb. 9oz. What is its value at \$1.385$\frac{7}{10}$ per ounce? Ans. \$228.640$\frac{1}{2}$.

APOTHECARIES' WEIGHT.

131. Apothecaries' Weight is used in mixing medical prescriptions.

TABLE.

20 Grains (gr.)	make	1 Scruple,	sc. or ℈.
3 Scruples	"	1 Dram,	dr. or ʒ.
8 Drams	"	1 Ounce,	oz. or ℥.
12 Ounces	"	1 Pound,	lb. or ℔.

gr.		sc.		dr.		oz.		lb.
20	=	1						
60	=	3	=	1				
480	=	24	=	8	=	1		
5760	=	288	=	96	=	12	=	1

NOTE 1.—In this weight the pound, ounce, and grain are the same as in Troy Weight.

NOTE 2.—Medicines are usually bought and sold by Avoirdupois Weight.

NOTE 3.—In estimating the weight of fluids, 45 drops, or a common tea-spoonful, make about 1 fluid dram; 2 common table-spoonfuls, about 1 fluid ounce; a wineglassful, about 1$\frac{1}{2}$ fluid ounces; and a common teacupful, about 4 fluid ounces.

EXAMPLES.

1. In 23℔ 9℥ 0ʒ 2℈ 13gr. how many grains?
2. In 136853 grains how many pounds?
3. How many scruples in 23lb.?
4. How many pounds in 6624 scruples?
5. In 47℔ 0℥ 0ʒ 1℈ 19gr. how many grains?
6. In 270759 grains how many pounds?

7. A physician bought 1 pound of ipecacuanha for $ 1.80, and retailed it out in doses of 5 grains, at $12\frac{1}{2}$ cents each. How much did he get for it over the cost? Ans. $ 142.20.

Avoirdupois, Troy, and Apothecaries' Weight Compared.

132. The relative value of the pound, and its subdivisions, of the several weights, in Troy grains, and in denominations of each other, is shown in the following

Table.

1 lb. Av.	=	7000 gr. Tr.	=	1lb. 2oz. 11pwt. 16gr. Tr.
1 lb. Tr. or Ap.	=	5760 "	=	13oz. $2\frac{114}{175}$dr. Av.
1 oz. Tr. or Ap.	=	480 "	=	1oz. $1\frac{97}{175}$dr. Av.
1 oz. Av.	=	$437\frac{1}{2}$ "	=	18pwt. $5\frac{1}{2}$gr. Tr.
1 dr. Ap.	=	60 "	=	$2\frac{34}{175}$dr. Av.
1 dr. Av.	=	$27\frac{11}{32}$ "	=	1pwt. $3\frac{11}{32}$gr. Tr.
1 pwt. Tr.	=	24 "	=	$\frac{768}{875}$dr. Av.
1 sc. Ap.	=	20 "	=	$\frac{128}{175}$dr. Av.
1 gr. Tr. or Ap.	=	1 "	=	$\frac{32}{875}$dr. Av.

Note. — To change a quantity from one weight to its equivalent in another weight, *reduce the given quantity to Troy grains, and then find their value in denominations of the weight required.*

Examples.

1. Change 13lb. 6oz. Avoirdupois weight to Troy weight.

2. Change 16lb. 3oz. 1pwt. 1gr. Troy weight to Avoirdupois weight.

3. Change 3lb. 8oz. 10pwt. to drams of Apothecaries' weight. Ans. 356dr.

4. Change 356 drams Apothecaries' weight to Troy weight. Ans. 3lb. 8oz. 10pwt.

5. An apothecary bought by Avoirdupois weight 2lb. 8oz. of quinine at $ 2.40 per ounce, which he retailed at 20 cents a scruple. What was his gain on the whole?

6. If I should buy by Avoirdupois weight 12lb. of opium at $37\frac{1}{2}$ cents per ounce, and sell it by Troy weight at 40 cents per ounce, should I gain or lose by so doing? Ans. Lose $ 2.

LINEAR OR LONG MEASURE.

133. Linear or Long Measure is used in measuring distances in any direction.

TABLE.

12	Inches (in.)	make	1	Foot,	ft.
3	Feet	"	1	Yard,	yd.
$5\frac{1}{2}$	Yards, or $16\frac{1}{2}$ Feet,	"	1	Rod, or Pole,	rd.
40	Rods	"	1	Furlong,	fur.
8	Furlongs, or 320 Rods,	"	1	Mile,	m.
3	Miles	"	1	League,	lea.
$69\frac{1}{6}$	Miles (nearly)	"	1	Degree on the equator,	deg. or °.
360	Degrees.	"	1	Great Circle of the Earth.	

in.		ft.		yd.		rd.		fur.		m.
12	=	1								
36	=	3	=	1						
198	=	$16\frac{1}{2}$	=	$5\frac{1}{2}$	=	1				
7920	=	660	=	220	=	40	=	1		
63360	=	5280	=	1760	=	320	=	8	=	1

NOTE 1.—12 lines make 1 inch; 4 inches, 1 hand; 6 feet, 1 fathom; 120 fathoms, 1 cable-length; $7\frac{1}{3}$ cable-lengths, 1 mile; $\frac{1}{60}$ of a degree of the circumference of the earth, 1 knot, or geographical mile, equal to $1\frac{11}{72}$ statute miles.

NOTE 2.—The *yard* adopted by the United States government as the *standard unit* of linear measure is the same as the imperial yard of Great Britain, which, as compared with a pendulum vibrating seconds in the latitude of London, the pendulum moving in a vacuum, at the level of the sea, and at the temperature of 62° Fahrenheit, should bear the proportion of 36 to $39\frac{1393}{10000}$ inches. A *metre*, the unit of linear measure, as established by the French government, is equal to about $39\frac{37}{100}$ English inches.

NOTE 3.—The English statute mile is the same as that of the United States, but that of other countries differs in value from it; as the German short mile is equal to 6857 yards, or about $3\frac{9}{10}$ English miles; the German long mile, to 10125 yards, or about $5\frac{3}{4}$ English miles; the Prussian mile, to 8237 yards, or about $4\frac{7}{10}$ English miles; the Spanish common league, to 7416 yards, or about $4\frac{1}{5}$ English miles; the Spanish judicial league, to 4635 yards, or about $2\frac{5}{8}$ English miles.

NOTE 4.—A degree of longitude is $\frac{1}{360}$ of any circle of latitude. As the circles of latitude diminish in length, the degrees of longitude vary in length under different parallels of latitude. Thus, under the equator, the length of a degree of longitude is about $69\frac{1}{6}$ statute miles; at 25° of latitude $62\frac{7}{10}$ miles; at 40° of latitude, 53 miles; at 42° of latitude, $51\frac{1}{3}$ miles; at 49° of latitude, $45\frac{1}{2}$ miles; at 60°, $34\frac{7}{12}$ miles.

EXAMPLES.

1. In 96deg. 56m. 7fur. 32rd. 12ft. 6in. how many inches?
2. In 424320486 inches how many degrees?
3. How many feet in 79 miles?
4. Required the miles in 417120 feet.
5. How many inches in 396 furlongs?
6. Required the furlongs in 3136320 inches?
7. How many inches from Haverhill to Boston, the distance being 30 miles?
8. Required the miles in 1900800 inches.

CLOTH MEASURE.

134. Cloth Measure is used in measuring cloth, ribbons, lace, and other articles sold by the yard or ell.

TABLE.

$2\frac{1}{4}$ Inches (in.)	make	1 Nail,	na.
4 Nails	"	1 Quarter of a yard,	qr.
4 Quarters	"	1 Yard,	yd.
3 Quarters	"	1 Ell Flemish,	E. F.
5 Quarters	"	1 Ell English,	E. E.

in.		na.		qr.		E. F.		yd.		E. E.
$2\frac{1}{4}$	=	1								
9	=	4	=	1						
27	=	12	=	3	=	1				
36	=	16	=	4	=	$1\frac{1}{3}$	=	1		
45	=	20	=	5	=	$1\frac{2}{3}$	=	$1\frac{1}{4}$	=	1

NOTE 1. — The Ell French is 6 quarters; the Ell Scotch, 4qr. $1\frac{1}{5}$in.

NOTE 2. — Cloth measure is a species of linear measure, and the yard and inch are the same in both.

EXAMPLES.

1. In 17yd. 3qr. 2na. how many nails?
2. In 286 nails how many yards?
3. In 365yd. 1qr. 3na. how many nails?
4. In 5847 nails how many yards?
5. In 71E. E. 4qr. how many nails?
6. In 1436 nails how many ells English?
7. What cost 47yd. 3qr. of silk velvet at $ 1.25 per quarter?
8. A merchant bought a roll of cloth containing $31\frac{1}{5}$E. E. and paid for it at the rate of $ 3 per yard. What did it cost him? Ans. $ 117.

SURFACE OR SQUARE MEASURE.

135. Square Measure is used in measuring surfaces of all kinds.

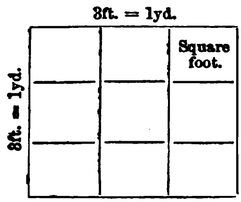
TABLE.

144	Square inches (sq. in.)	make	1	Square foot,	ft.
9	Square feet	"	1	Square yard,	yd.
30¼	Square yards	"	1	Square rod or pole,	p.
40	Square rods	"	1	Rood,	R.
4	Roods	"	1	Acre,	A.
640	Acres	"	1	Square mile,	S. M.

in.		ft.		yd.		p.		R.		A.		S.M.
144	=	1										
1596	=	9	=	1								
39204	=	272¼	=	30¼	=	1						
1568160	=	10890	=	1210	=	40	=	1				
6272640	=	43560	=	4840	=	160	=	4	=	1		
4014489600	=	27878400	=	3097600	=	102400	=	2560	=	640	=	1

NOTE. — A *square* is a figure bounded by four equal lines, perpendicular to each other.

When the four lines are each 1 foot in length, the space enclosed is 1 *square foot;* when 1 yard in length, 1 *square yard;* when 1 rod in length, 1 *square rod;* and so for any other dimension.



In this diagram the *large* square represents a square *yard*, and each of the *smaller* squares within it represents one square *foot*. Now, since there are *three* rows of small squares, and *three* square feet in each row, there will be 3 sq. ft. × 3 = 9 sq. ft. in the large square. But the large square is 3ft. in length and 3ft. in breadth; hence,

To find the contents of a square, multiply the numbers denoting its length and breadth together.

EXAMPLES.

1. In 57A. 3R. 27p. 21yd. 8ft. 57in. how many square inches?
2. In 363331893 square inches how many acres?
3. How many square feet in 25 acres?
4. How many acres in 1089000 square feet?
5. How many square rods in 365 square miles?
6. How many square miles in 37376000 square rods?
7. How many acres in 12345678 square inches?

Ans. 1A. 3R. 34p. 27yd. 4ft. 54in.

8. Bought 39A. 2R. 16p. of land for $3.75 per square rod, and sold the same for $0.25 per square foot. What did I gain by my bargain? Ans. $407,484.00.

SURVEYORS' MEASURE.

136. This measure is used by surveyors in measuring land, roads, &c.

Table.

$7\frac{92}{100}$	Inches (in.)	make	1 Link	l.
25	Links	"	1 Pole,	p.
100	Links, 4 Poles, or 66 Feet,	"	1 Chain,	ch.
10	Chains	"	1 Furlong,	fur.
8	Furlongs, or 80 Chains,	"	1 Mile,	m.

Inches.		Link.		Pole.		Chain.		Furlong.		Mile.
$7\frac{92}{100}$	=	1								
198	=	25	=	1						
792	=	100	=	4	=	1				
7920	=	1000	=	40	=	10	=	1		
63360	=	8000	=	320	=	80	=	8	=	1

Note 1. — Gunter's chain, in length 4 poles, or 66 feet, and divided into 100 links, is that mostly used in ordinary land surveys; but in locating roads, and like public works, an engineer's chain is usually 100 feet in length, containing 120 links, each 10 inches long.

Note 2. — A section of government lands is 1 square mile, or 640 acres. An acre, as a square piece of land, will measure on each side about 209 feet or 70 paces. 625 square links make 1 square rod or pole; 16 square rods make 1 square chain, and 10 square chains make 1 acre.

Note 3. — A rod or pole is sometimes called a perch, and each of the names given to this measure is expressive of the instrument by which it was formerly measured.

Examples.

1. How many links in 46m. 3fur. 5ch. 25l.?

2. In 371525 links how many miles?

3. In 97m. 0fur. how many links?

4. In 776000 links how many miles?

5. The extent of a certain farm is found, by survey, to be 1377 square chains (Note 2). How many acres does it contain? Ans. 137A. 2R. 32p.

6. What will be the cost of a field measuring 2,126,250 square links, at $80 per acre? Ans. $1701.00.

CUBIC OR SOLID MEASURE.

137. Cubic or Solid Measure is used in measuring such bodies or things as have length, breadth, and thickness; as timber, stone, &c.

TABLE.

1728	Cubic inches (cu. in.)	make 1	Cubic foot,	cu. ft.
27	" feet	" 1	" yard,	cu. yd.
40	" feet	" 1	Ton,	T.
16	" feet	" 1	Cord foot,	c. ft.
8	Cord feet, or 128 Cubic feet,	" 1	Cord of wood,	C.

in.		ft.		yd.		T.		C.
1728	=	1						
46656	=	27	=	1				
69120	=	40	=	$1\frac{13}{27}$	=	1		
221184	=	128	=	$4\frac{20}{27}$	=	$3\frac{1}{5}$	=	1

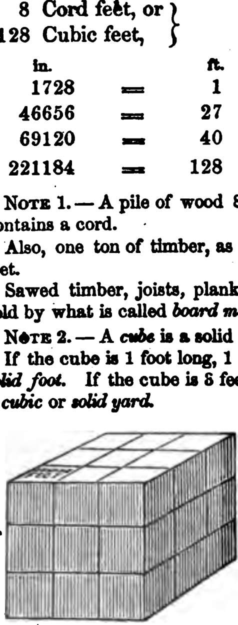
NOTE 1.— A pile of wood 8ft. in length, 4ft. in breadth, and 4ft. in height, contains a cord.

Also, one ton of timber, as usually surveyed, contains $50\frac{93}{100}$ cubic or solid feet.

Sawed timber, joists, plank, and scantlings are now generally bought and sold by what is called *board measure.*

NOTE 2.— A *cube* is a solid bounded by six square and equal sides.

If the cube is 1 foot long, 1 foot wide, and 1 foot high, it is called a *cubic* or *solid foot.* If the cube is 3 feet long, 3 feet wide, and 3 feet thick, it is called a *cubic* or *solid yard.*



3ft. = 1yd.

3ft. = 1 yd.

Now, since each side of a cubic yard, as represented in the diagram, contains 9 sq. ft. of surface (Art. 135), it is plain, if a block be cut off from one side, *one* foot thick, it can be divided into 9 solid blocks, with sides 1 foot in length, breadth, and thickness, and therefore will contain 9 solid feet; and since the whole block or cube is *three* feet thick, it must contain 3 solid feet $\times$ 9 = 27 solid feet; for 3 solid feet $\times$ 3 $\times$ 3 = 27 solid feet. Hence,

To find the contents of a cubic body, multiply the numbers denoting its length, breadth, and thickness together.

NOTE 3.— A cubic foot of distilled water at the maximum density, at the level of the sea, and the barometer at 30 inches, is equal in weight to $62\frac{1}{2}$lb. or 1000oz. avoirdupois.

NOTE 4.—A cubic foot of lead weighs $708\frac{3}{4}$lb.; of brass, $534\frac{3}{4}$lb.; of copper, 555lb.; of wrought-iron, $486\frac{3}{4}$lb.; of cast-iron, $450\frac{1}{2}$lb.; of marble, 171lb.; of granite, 165lb.; of clay, 130lb.; of common soil, 124lb.; of bricks, 124lb.; of sand, 95lb.; of sea-water, $64\frac{3}{10}$lb.; of oak wood, 55lb.; of Anthracite coal, 54lb.; of Bituminous coal, 50lb.; of red-pine wood, 42lb.; and of white-pine wood, 30lb.

EXAMPLES.

1. In 29 cords of wood how many solid inches?
2. In 6414336 cubic inches how many cords?
3. In 19 tons of timber how many solid inches?
4. How many tons of timber in 1313280 cubic inches?
5. How many cubic feet of wood in 128 cords?
6. How many cords of wood in 16384 cubic feet?
7. How many cubic feet in a pile of wood, 40 feet long, 4 feet wide, and 7 feet high?
8. How many cords of wood in 8650 cubic feet?
   Ans. 67 cords, 74 cubic feet.
9. How many cubic feet in a granite block, 17 feet long, 11 feet wide, and 9 feet high? Ans. 1683 cubic feet.

LIQUID OR WINE MEASURE.

138. Liquid or Wine Measure is used in measuring all kinds of liquids, except, in some places, beer, ale, porter, and milk.

TABLE.

4 Gills (gi.)	make	1 Pint,	pt.
2 Pints	"	1 Quart,	qt.
4 Quarts	"	1 Gallon,	gal.
63 Gallons	"	1 Hogshead,	hhd.
2 Hogsheads	"	1 Pipe, or Butt,	pi.
2 Pipes	"	1 Tun,	tun.

gi.		pt.		qt.		gal.		hhd.		pi.		tun.
4	=	1										
8	=	2	=	1								
32	=	8	=	4	=	1						
2016	=	504	=	252	=	63	=	1				
4032	=	1008	=	504	=	126	=	2	=	1		
8064	=	2016	=	1008	=	252	=	4	=	2	=	1

NOTE 1. — By laws of Massachusetts, 32 gallons make 1 *barrel.* In some States $31\frac{1}{2}$ gallons, and in others from 28 to 32 gallons, make 1 barrel. 42 gallons make 1 *tierce,* and 2 tierces make 1 *puncheon.*

NOTE 2. — The term *hogshead* is often applied to any large cask that may contain from 50 to 120 gallons, or more.

NOTE 3. — The *Standard Unit* of *Liquid Measure* adopted by the government of the United States is the *Winchester Wine Gallon,* which contains 231 cubic inches, and is of a capacity to hold $8\frac{339}{1000}$lb. Avoirdupois of distilled water, at its maximum density, weighed in air, the barometer being at 30 inches. It has the name Winchester, from its standard having been formerly kept at Winchester, England. The *Imperial Gallon,* now adopted in Great Britain, contains $277\frac{274}{1000}$ cubic inches; so that 6 Winchester gallons make about 5 Imperial gallons.

NOTE 4. — 1 gallon of alcohol weighs 7lb.; of camphene, $7\frac{1}{5}$lb.; of proof spirits, $7\frac{15}{16}$lb.; of spirits of turpentine, $7\frac{5}{16}$lb.; of sperm oil, $7\frac{1}{4}$lb.; of olive oil, $7\frac{1}{2}$lb.; of linseed oil, $7\frac{3}{4}$lb.; and of molasses, $11\frac{3}{8}$lb.

NOTE 5. — The *fluid measure* of apothecaries, used by them in measuring liquids of medical prescriptions, divides the gallon (marked Cong.) into 8 pints (O.); the pints into 16 fluid ounces (f℥); the fluid ounces into 8 fluid drams (fʒ); and the fluid drams into 60 minims (m) or drops. The abbreviation Cong. stands for *congiarium,* the Latin for gallon, and the O. is the initial of *octans,* the Latin for an eighth, the pint being an eighth of a gallon.

EXAMPLES.

1. In 57T. 3hhd. 50gal. 3qt. how many pints?
2. In 116830 pints how many tuns?
3. Reduce 96hhd. 47gal. 2qt. to gills.
4. How many hogsheads in 195056 gills?
5. What cost 40 hogsheads of wine at \$0.37½ per pint?
6. How much may be gained by buying 2 hogsheads of molasses, at 40 cents a gallon, and selling it at 12 cents a quart? Ans. \$10.08.

BEER MEASURE.

139. Beer Measure is used in measuring beer, ale, porter, and milk.

TABLE.

2 Pints (pt.)	make	1 Quart,	qt.
4 Quarts	"	1 Gallon,	gal.
54 Gallons	"	1 Hogshead,	hhd.

pt.		qt.		gal.		hhd.
2	=	1				
8	=	4	=	1		
432	=	216	=	54	=	1

NOTE 1. — The gallon of beer measure contains 282 cubic inches; and has been usually reckoned, 36 gallons equal 1 barrel; 2 hogsheads, or 108 gallons, 1 butt; 2 butts, or 216 gallons, 1 tun.

NOTE 2. — Beer Measure is becoming obsolete. Milk and malt liquors, at the present time, are bought and sold, very generally, by wine or liquid measure.

EXAMPLES.

1. How many pints in 46hhd. 49gal.?
2. In 20264 pints how many hogsheads?
3. In 368hhd. how many pints?
4. In 158976 pints how many hogsheads?
5. At 29 cents per gallon, what cost 76 hogsheads of ale? Ans. $ 1190.16.
6. How much may be obtained by selling 47hhd. 36gal. of lager-bier at 5 cents a quart? Ans. $ 514.80.

DRY MEASURE.

140. This measure is used in measuring grain, fruit, salt, &c.

TABLE.

2 Pints (pt.)	make	1 Quart,	qt.
8 Quarts	"	1 Peck	pk.
4 Pecks	"	1 Bushel,	bu.

pts.		gal.				
8	=	1		pk.		
16	=	2	=	1		bu.
64	=	8	=	4	=	1

NOTE 1. — The *Standard Unit* of *Dry Measure* adopted by the United States government is the *Winchester* bushel, which is in form a cylinder, $18\frac{1}{2}$ inches in diameter, and 8 inches deep, containing $2150\frac{42}{100}$ cubic inches. The *Standard Imperial* bushel of Great Britain contains $2218\frac{192}{1000}$ cubic inches, so that 32 Imperial bushels equal about 33 Winchester bushels. The gallon in Dry Measure contains $268\frac{4}{5}$ cubic inches.

NOTE 2. — Of wheat a standard bushel is 60lb.; of shelled corn, 56lb.; of corn on the cob, 70lb.; of rye, 56lb.; of barley, 48lb.; of buckwheat in Pa., 50lb.; in Kentucky, 52lb.; in Mass., 48lb.; of oats in Ohio, Ill., Mass., &c., 32lb.; of oats in Ky., $33\frac{1}{2}$lb.; of oats in Me., 30lb.; of oats in Pa., 30lb.; of clover-seed, 60lb.; of flax-seed, 56lb.; of Timothy-seed, 45lb.; of bran, 20lb.; of beans, 60lb.; of onions, in Pa., Ky., &c., 57lb.; of onions in Mass., 52lb.; of salt in Ky., 56lb.; of salt in Ill., 50lb.; of dried apples in Pa., 22lb.; of dried apples in Ill., 24lb.; of dried peaches in Pa., 33lb.; of dried peaches in Ill., 32lb.; of stove coal in Ill., 80lb.; of bituminous coal in the Western States, 76lb.; and of hard-wood charcoal, 30lb. The weight by law, of a few

of the articles named, to a bushel, is not uniform in all the States, and therefore may vary slightly from the above, in a few States not mentioned.

NOTE 3. — In some places it is customary, in measuring coal, potatoes, and like articles, to "heap" the bushel, as it is called, and in that case 5 even pecks are about equal to 1 "heaped bushel." The "coal bushel," as established by laws of Massachusetts, Ohio, and some other States, is of greater capacity than the Winchester bushel. In some parts of the United States a *chaldron*, a measure of coal, consists of 36 bushels; and in other parts of the country it consists of 32 bushels, or of 4 *quarters*, each consisting of 8 bushels. The *quarter*, however, in England is 8 Imperial bushels, a measure of grain equal to 560lb., or one quarter of a ton of 2240lbs.

EXAMPLES.

1. How many pints in 35bu. 3pk.?

2. In 2288 pints how many bushels?

3. In 676 chaldrons, of 36 bushels each, how many pecks?

4. How many chaldrons, of 36 bushels each, in 97344 pecks?

5. A grocer purchased 50 bushels of potatoes, by "heaped" measure, at 60 cents a bushel, and sold the same, by "even" measure, at 15 cents a peck; did he gain or lose by the operation? Ans. Gain $7.50.

6. If I purchase by measure 96 bushels of oats, weighing 2304 pounds, at 42 cents a bushel, and sell the same by weight in Ohio, at 45 cents a bushel, shall I gain or lose by so doing? Ans. Lose $7.92.

DRY, LIQUID, AND BEER MEASURES COMPARED.

141. The relative value of the gallon and its subdivisions, of the several measures, in cubic inches, and in denominations of each other, are shown in the following

TABLE.

	cu. in.		
1 gal. B. M. =	282	= 1gal. 1pt. $3\frac{5}{77}$gi. L. M.	= 1gal. $\frac{11}{28}$pt. D. M.
1 gal. D. M. =	$268\frac{4}{5}$	= 1gal. 1pt. $1\frac{13}{55}$gi. L. M.	= 3qt. $1\frac{147}{235}$pt. B. M.
1 gal. L. M. =	231	= 3qt. $\frac{7}{8}$pt. D. M.	= 3qt. $\frac{26}{47}$pt. B. M.
1 qt. B. M. =	$70\frac{1}{2}$	= 1qt. 0pt. $1\frac{59}{77}$gi. L. M.	= 1qt. $\frac{11}{112}$pt. D. M.
1 qt. D. M. =	$67\frac{1}{5}$	= 1qt. $1\frac{17}{55}$pt. L. M.	= 1pt. $3\frac{191}{470}$gi. B. M.
1 qt. L. M. =	$57\frac{3}{4}$	= $1\frac{23}{32}$pt. D. M.	= $1\frac{30}{47}$pt. B. M.
1 pt. B. M. =	$35\frac{1}{4}$	= $1\frac{68}{77}$pt. L. M.	= $1\frac{11}{224}$pt. D. M.
1 pt. D. M. =	$33\frac{3}{5}$	= 1pt. $\frac{17}{55}$gi L. M.	= $3\frac{661}{940}$gi. B. M.
1 pt. L. M. =	$28\frac{7}{8}$	= $\frac{55}{64}$pt. D. M.	= $\frac{77}{94}$pt. B. M.
1 gi. L. M. =	$7\frac{7}{32}$	= $\frac{55}{256}$pt. D. M.	= $\frac{77}{376}$pt. B. M.

NOTE 1. — By the table, it is evident that each of the measures of capacity is a species of cubic measure; and to change cubic measure, expressed in cubic inches, to any denomination of either dry, liquid, or beer measure, *divide by the number of cubic inches required to make a unit of the proposed denomination.* Thus, to reduce 14ft. 294in. cubic measure to gallons of liquid measure: 14 cu. ft. 294 cu. in. = 24486 cu. in.; 24486 cu. in. ÷ 231 = 106 gallons, liquid measure.

NOTE 2. — To change a quantity from one measure of capacity to its equivalent in another, *reduce the given quantity to cubic inches, and then find their value in denominations of the proposed measure.*

EXAMPLES.

1. Change 4hhd. 15 gal. beer measure to liquid measure?

2. Change 4hhd. 30 gal. liquid measure to beer measure?

3. If a milkman buy 2820 gallons of milk at 4 cents per quart, beer measure, and sell the same at 6 cents per quart, liquid measure, what will he gain?

Ans. $ 375.02$\frac{26}{77}$.

4. If 2538 gallons of milk have been purchased by liquid measure, at 4 cents per quart, and the same has been sold by beer measure at 6 cents per quart, what has been the gain? Ans. $ 92.88.

5. A merchant bought 385 bushels of seed peas at $ 4.00 per bushel, dry measure. He sold the same at 20 cents per quart, liquid measure. What did he gain by the purchase? Ans. $ 1327.20.

6. J. Day bought 1000 bushels of corn at $ 1.05 per bushel, dry measure, and sold the same at $ 1.12 per bushel, liquid measure. Did he gain or lose by the operation, and how much?

7. My hogshead contains 30 cubic feet. How many more gallons of dry measure will it contain, than of beer measure?

8. Bought of my neighbor, John Smith, 365 gallons of milk, at 5 cents per quart; but by mistake he measured it in his liquid measure. How much did I lose?

MEASURE OF TIME.

142. This measure is applied to the various divisions and subdivisions into which time is divided.

TABLE.

60	Seconds (sec.)	make	1 Minute,	m.
60	Minutes	"	1 Hour,	h.
24	Hours	"	1 Day,	d.
7	Days	"	1 Week,	w.
$365\frac{1}{4}$	Days, or 52 weeks $1\frac{1}{4}$ days,	"	1 Julian Year,	y.
12	Calendar Months (mo.)	"	1 Year,	y.

sec.		m.		h.		d.		w.		y.
60	=	1								
3600	=	60	=	1						
86400	=	1440	=	24	=	1				
604800	=	10080	=	168	=	7	=	1		
31557600	=	525960	=	8766	=	$365\frac{1}{4}$			=	1

NOTE 1. — The true *Solar* or *Tropical Year* is the time measured from the sun's leaving either equinox or solstice to its return to the same again, and is 365d. 5h. 48m. $49\frac{7}{10}$sec.

The *Julian Year*, so called from the calendar instituted by Julius Cæsar, contains $365\frac{1}{4}$ days, as a medium; three years in succession containing 365 days, and the fourth year 366 days; which, as compared with the true solar year, produces an average yearly error of 11m. $10\frac{3}{10}$sec., or a difference that would amount to 1 whole day in about 129 years.

The *Gregorian Year*, or that instituted by Pope Gregory XIII., in the year 1582, and which is now the *Civil* or *Legal Year* in use among most nations of the earth, contains, like the Julian year, 365 days for three years in succession, and 366 for the fourth, *excepting centennial years whose number cannot be exactly divided by* 400. The Gregorian year is so nearly correct as to err only 1 day in 3866 years, a difference so little as hardly to be worth taking into account.

The manner of reckoning time according to the Julian Calendar is termed *Old Style*, and that according to the calendar of Gregory, *New Style*. England did not adopt the new style till 1752, when, according to an act of Parliament, the difference between the two styles, which then amounted to 11 days, was removed, by the day following the 2d of September of that year being accounted the 14th day. The difference now between old and new style is 12 days.

A *Common Year* is one of 365 days, and a *Leap* or *Bissextile Year* is one of 366 days. Any year is Leap Year whose number can be divided by 4 without a remainder, except years whose number can be divided without a remainder by 100, but not by 400.

A *Sidereal Year* is the time in which the earth revolves round the sun, and is 365d. 6h. 9m. $9\frac{6}{10}$sec.

NOTE 2. — The names of the 12 calendar months, composing the civil year, are January, February, March, April, May, June, July, August, September, October, November, December, and the number of days in each may be readily remembered by the following lines: —

"Thirty days hath September,
April, June, and November;
And all the rest have thirty-one,
Save February, which alone
Hath twenty-eight; and this, in fine,
One year in four hath twenty-nine."

TABLE

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

FROM ANY DAY OF	TO THE SAME DAY OF NEXT											
	Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
JANUARY	365	31	59	90	120	151	181	212	243	273	304	334
FEBRUARY	334	365	28	59	89	120	150	181	212	242	273	303
MARCH	306	337	365	31	61	92	122	153	184	214	245	275
APRIL	275	306	334	365	30	61	91	122	153	183	214	244
MAY	245	276	304	335	365	31	61	92	123	153	184	214
JUNE	214	245	273	304	334	365	30	61	92	122	153	183
JULY	184	215	243	274	304	335	365	31	62	92	123	153
AUGUST	153	184	212	243	273	304	334	365	31	61	92	122
SEPTEMBER	122	153	181	212	242	273	303	334	365	30	61	91
OCTOBER	92	123	151	182	212	243	273	304	335	365	31	61
NOVEMBER	61	92	120	151	181	212	242	273	304	334	365	30
DECEMBER	31	62	90	121	151	182	212	243	274	304	335	365

For example, suppose we wish to find the number of days from April 4th to November 4th, we look for April in the left-hand vertical column, and for November at the top, and where the lines intersect is 214, the number sought. Again, if we wish the number of days from June 10th to September 16th, we find the difference between June 10th and September 10th to be 92 days, and add 6 days for the excess of the 16th over the 10th of September, and so we have 98 days as the exact difference.

If the end of February be included between the points of time, a day must be added in leap year.

When the time includes more than one year, there must be added 365 days for each year.

EXAMPLES.

1. How many seconds in a solar year?

2. In 31556929 seconds how many days?

3. How many seconds from the deluge, which took place 2348 years before Christ, to the year 1856, a year being $365\frac{1}{4}$ days? Ans. 132636592800.

4. In 74726807872 seconds how many solar years? Ans. 2368 years.

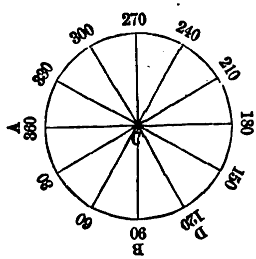
5. How long did the last war with England continue; it having commenced June 18th, 1812, and ended February 17th, 1815? Ans. 974 days = 2 years 244 days.

CIRCULAR ANGULAR MEASURE.

143. Circular Angular Measure is applied to the measurement of circles and angles, and is used in reckoning latitude and longitude, and the revolutions of the planets round the sun.

60 Seconds (″)	make	1 Minute,	′.
60 Minutes	"	1 Degree,	°.
30 Degrees	"	1 Sign,	S.
12 Signs, or 360 Degrees,		The Circle of the Zodiac,	C.

″		′		°		S.		C.
60	=	1						
3600	=	60	=	1				
108000	=	1800	=	30	=	1		
1296000	=	21600	=	360	=	12	=	1



NOTE 1. — A *Circle* is a plane figure bounded by a curve line, all parts of which are equally distinct from a point called its centre.

The *Circumference* of a circle is the line which bounds it, as shown by the diagram.

An *Arc* of a circle is any part of its circumference; as AB.

A *Radius* of a circle is a straight line drawn from its centre to its circumference; as CA, CB, or CD.

Every circle, large or small, is supposed to be divided into 360 equal parts, called degrees.

A *Quadrant* is one fourth of a circle, or an arc of 90°; as AB.

An *Angle*, as ACB, is the inclination or opening of two lines which meet at a point, as C. The point is the *vertex* of the angle. If a circle be drawn around the vertex of an angle as a centre, the two sides of the angle, as *radii* of the circle, will include an arc, which is the *measure* of the angle; as the arc AD = 120° is the measure of the angle ACD, and AB = 90°, the measure of the angle ACB; hence the one is called an angle of 120°, and the other an angle of 90°.

NOTE 2. — As the earth turns on its axis from west to east every 24 hours, the sun appears to pass from east to west $\frac{1}{24}$ of 360° of longitude every hour, or 15° of longitude in 1 hour's time, or 1° in 4 minutes of time, and 1′ in 4 seconds of time; so that, for instance, when it is noon at any place, it is 1 hour earlier for every 15° of longitude westward, and 1 hour later for every 15° of longitude eastward. Thus, Boston being 71° 4′ west of Greenwich, and San Francisco 51° 17′ west of Boston, when it is noon at Boston, it is 4h. 44m. 16sec. past noon at Greenwich, and wanting 3h. 25m. 8sec. of noon at San Francisco.

EXAMPLES.

1. How many minutes in 27S. 27° 43′?
2. In 50263 minutes how many signs?
3. How many seconds in 44S. 18° 57′ 23″?
4. How many signs in 4820243″?
5. How many seconds in 360°?
6. How many degrees in 1296000″?

MISCELLANEOUS TABLE.

144. The following denominations, frequently used, are not embraced in the preceding tables.

12	units	make	1 dozen.
12	dozen	"	1 gross.
12	gross	"	1 great gross.
20	units	"	1 score.
24	sheets of Paper	"	1 quire.
20	quires	"	1 ream.
2	reams	"	1 bundle.
5	bundles	"	1 bale.
14	pounds of Iron or Lead	"	1 stone.
$21\frac{1}{2}$	stone	"	1 pig.
8	pigs	"	1 fother
18	inches	"	1 cubit.
$24\frac{3}{4}$	cubic feet of Stone	"	1 perch.
60	pairs of Shoes	"	1 case.
25	pounds of Powder	"	1 keg.
56	pounds of Butter	"	1 firkin.
100	pounds of Fish	"	1 quintal.
196	pounds of Flour	"	1 barrel.
200	pounds of Beef	"	1 barrel.
200	pounds of Pork	"	1 barrel.
200	pounds of Shad or Salmon, in N. Y. & Ct.	"	1 barrel.
220	pounds of Fish, in Md.	"	1 barrel.
256	pounds of Soap	"	1 barrel.
300	pounds of Hydraulic Cement	"	1 barrel.
30	gallons of Fish, in Mass.	"	1 barrel.
5	bushels of Shelled Corn, in Southern States	"	1 barrel.
8	bushels of Salt	"	1 hogshead.

NOTE 1.—A sheet of paper folded into 2 leaves forms a folio; a sheet folded into 4 leaves, a quarto; a sheet folded into 8 leaves, an octavo; a sheet folded into 12 leaves, a 12mo; a sheet folded into 18 leaves, an 18mo; a sheet folded into 24 leaves, a 24mo.

NOTE 2.—Of shoemaker's measure, No. 1 of small size is $4\frac{1}{8}$ inches in length; and No. 1, large size, is $8\frac{11}{24}$ inches in length; and each succeeding number of either size is $\frac{1}{3}$ of an inch additional length.

EXAMPLES.

1. In 4 bales 4 bundles 1 ream 10 quires of paper how many sheets? Ans. 23760.
2. In 23760 sheets of paper how many bales?
3. In 10 fothers 6 pigs 8 stones of iron how many pounds?
4. In 25998 pounds of iron how many fothers?

5. At 23 cents a pound, what will 12 firkins of butter cost?

6. If $22 is paid for a barrel of pork, how much is that by the pound? Ans. 11 cents.

7. How much must be paid for 302 hogsheads of salt at $0.30 a bushel? Ans. $724.80.

8. At $4 per quintal, how many pounds of fish may be bought for $50.24? Ans. 1256 pounds.

9. If the wholesale price of one writing-book be $4\frac{1}{2}$ cents, what will be the cost of a great-gross of writing-books?

10. A dairyman sells 2 firkins of butter at 20 cents a pound, and takes in pay half a barrel of flour at 5 cents a pound, and the balance in cash. How much cash does he receive? Ans. $17.50.

MISCELLANEOUS EXAMPLES IN REDUCTION.

1. In 57£. 15s. how many half-pence? Ans. 27720.

2. In 59lb. 13pwt. 15gr. how many grains?

3. In 340167 grains how many pounds?

4. How many ells English in 761 yards? Ans. 608 E. E. 4qr.

5. How many yards in 61 ells Flemish? Ans. 45yd. 3qr.

6. How many bottles, that contain 3 pints each, will it take to hold a hogshead of wine? Ans. 168.

7. How many steps, of 2ft. 8in. each, will a man take in walking from Bradford to Newburyport, the distance being fifteen miles? Ans. 29700.

8. How many spoons, each weighing 2oz. 12pwt., can be made from 5lb. 2oz. 8pwt. of silver? Ans. 24.

9. How many times will the wheel of a coach revolve, whose circumference is 14ft. 9in., in passing from Boston to Washington, the distance being 436 miles? Ans. $156073\frac{39}{177}$.

10. I have a field of corn, consisting of 123 rows, and each row contains 78 hills, and each hill has 4 ears of corn; now if it takes 8 ears of corn to make a quart, how many bushels does the field contain? Ans. 149bu. 3pk. 5qt. 0pt.

11. If it take 5yd. 2qr. 3na. to make a suit of clothes, how many suits can be made from 182 yards?

12. A goldsmith wishes to make a number of rings, each

weighing 5pwt. 10gr., from 3lb. 1oz. 2pwt. 2gr. of gold; how many will there be? Ans. 137.

13. How many shingles will it take to cover the roof of a building, which is 60 feet long and 56 feet wide, allowing each shingle to be 4 inches wide and 18 inches long, and to lie one third to the weather? Ans. 20160.

14. There is a house 56 feet long, and each of the two sides of the roof is 25 feet wide; how many shingles will it take to cover it, if it require 6 shingles to cover a square foot?

Ans. 16800.

15. If a man can travel 22m. 3fur. 17rd. a day, how long would it take him to walk round the globe, the distance being about 25000 miles? Ans. $1114\frac{4822}{7177}$ days.

16. If a family consume 7lb. 10oz. of sugar in a week, how long would 10cwt. 3qr. 16lb. last them? Ans. $143\frac{5}{61}$ weeks.

17. What will 7 hogsheads of wine cost, at 9 cents a quart?

18. What will 15 hogsheads of beer cost, at 3 cents a pint? Ans. $ 194.40.

19. What will 73 bushels of meal cost, at 2 cents a quart? Ans. $ 46.72.

20. A merchant has 29 bales of cotton cloth; each bale contains 57 yards; what is the value of the whole at 15 cents a yard? Ans. $ 247.95.

21. There is a certain pile of wood 120 feet long, $4\frac{1}{2}$ feet high, and 4 feet wide; what is its value at $ 4.00 per cord? Ans. $ 67.50.

22. How much must be paid, at twenty cents a square yard, for plastering overhead in a room which is 33 feet long and 18 feet wide?

23. An apothecary, in compounding 20 boxes of pills, each box containing 25 pills, used 6 grains of aloes, 5 grains of rhubarb, and 4 grains of calomel in each pill. What was the entire quantity used? Ans. 7500 grains.

24. Purchased a cargo of molasses, consisting of 87 hogsheads; what is the value of it at 33 cents a gallon?

Ans. $ 1808.73.

25. If a cubic foot of white-oak wood weighs 880 ounces, and a cubic foot of white-pine wood weighs 480 ounces, how much will a load weigh, which is composed of half a cord of white-oak, and of a cord of white-pine? Ans. 7360lbs.

26. How many chests of tea, weighing 24 pounds, at 43 cents a pound, can be bought for \$ 1548 ?

27. Joseph Eldredge received \$ 10, as a birthday present, from his father, on every 29th day of February, from 1837 to 1857. How much less than \$ 200 did he receive, in all?

Ans. \$ 150.

28. If $25\frac{4}{5}$ grains of standard gold be worth \$ 1, how many pounds avoirdupois of standard gold will be worth \$ 1,000,000?

Ans. $3685\frac{5}{7}$ pounds.

29. A merchant, who had bought 188 gallons of molasses, at 40 cents a gallon, intended to have it sold at the rate of 50 cents a gallon; but his shop-boy retailed half of the quantity at $12\frac{1}{2}$ cents a quart, beer measure, when, finding he had made a blunder, he sold the balance at 14 cents a quart, wine measure, thereby expecting to exactly make up for the mistake. How much less did the whole bring than was intended?

Ans. \$ 2.86.

ADDITION OF COMPOUND NUMBERS.

145. ADDITION of Compound Numbers is the process of finding the amount of two or more compound numbers.

Ex. 1. Required the amount of 31£. 17s. 9d. 2far.; 16£. 16s. 6d. 1far.; 16£. 11s. 11d. 1far.; 19£. 19s. 9d. 3far.; 61£. 17s. 1d. 2far.

Ans. 147£. 3s. 2d. 1far.

OPERATION.

	£.	s.	d.	far.
	31	17	9	2
	16	16	6	1
	16	11	11	1
	19	19	9	3
	61	17	1	2
Ans.	147	3	2	1

Having written units of the same denomination in the same column, we find the sum of farthings in the right-hand column to be 9 farthings = 2d. 1far. We write the 1far. under the column of farthings, and carry the 2d. to the column of pence; the sum of which is 38d. = 3s. 2d. We write the 2d. under the column of pence, and carry the 3s. to the column of shillings; the sum of which is 83s. = 4£. 3s. Having written the 3s. under the column of shillings, we carry the 4£. to the column of pounds, and find the entire amount sought to be 147£. 3s. 2d. 1far.

The same result can be arrived at by *reducing the numbers as they are added* in their respective columns. Thus, beginning with farthings, we can add, in this way: 2far. + 3far. = 5far. = 1d. 1far.;

and 1far. = 1d. 2far., and 1far. = 1d. 3far., and 2far. = 2d. 1far. Writing the 1far. under the column of farthings, we carry the 2d. to the column of pence; and add, 2d. (carried) + 1 = 3d., and 9d. = 12d. = 1s., and 11d. = 1s. 11d., and 6d. = 2s. 5d., and 9d. = 3s. 2d. Writing the 2d. under the column of pence, we carry the 3s. to the column of shillings; and add, 3s. (carried) + 17s. = 20s. = 1£., and 19s. = 1£. 19s., and 11s. = 2£. 10s., and 16s. = 3£. 6s., and 17s. = 4£. 3s. Writing the 3s. under the column of shillings, we carry the 4£. to the column of pounds, and so find the whole amount to be, as before, 147£. 3s. 2d. 1far.

The last method of operation may be rendered more concise, as it should always be in practice, by merely naming results as the adding is performed (Art. 45).

From the illustrations given, it is evident that the adding of compound numbers is like that of simple numbers, except in carrying; which difference holds also in subtracting, multiplying, and dividing compound numbers.

RULE.—*Write all the given numbers, so that units of the same denomination may stand in the same column.*

Add as in addition of simple numbers; and carry, from column to column, one for as many units as it takes of the denomination added to make a unit of the denomination next higher.

Proof.—The proof is the same as in addition of simple numbers.

EXAMPLES.

2.

Ton.	cwt.	qr.	lb.	oz.	dr.
61	19	3	17	15	15
63	13	3	16	11	11
51	12	3	17	7	6
61	16	1	11	12	12
13	13	3	12	13	15
71	18	2	13	14	14
324	15	2	15	12	9

3.

cwt.	qr.	lb.	oz.	dr.
61	2	11	11	14
16	3	15	15	11
41	3	13	9	9
38	2	11	10	10
42	1	9	8	13
31	3	17	11	12

4.

lb.	oz.	pwt.	gr.
16	11	19	23
31	10	18	16
63	9	12	15
17	8	13	12
61	7	12	16
17	6	17	22
209	7	15	8

5.

lb.	oz.	pwt.	gr.
123	9	7	13
98	11	17	14
49	7	13	21
13	10	10	20
47	9	19	23
51	5	15	15

6.

℔	℥	ʒ	℈	gr.
27	11	7	2	19
16	10	6	1	13
41	9	3	2	16
38	10	5	2	14
41	4	4	1	11
16	6	6	2	6
183	6	3	1	19

7.

℔	℥	ʒ	℈	gr.
37	9	6	1	18
14	4	4	2	11
61	6	3	2	6
41	4	7	2	16
39	8	4	1	12
51	11	7	2	19

8.

Deg.	m.	fur.	rd.	ft.	in.
17	69	7	39	16	11
61	62	3	17	12	9
16	16	6	16	13	10
48	19	3	15	15	9
17	58	6	33	14	7
33	35	5	19	9	9
195	55½	1	24	½	7
	½=4			½=6	
195	55	5	24	1	1

9.

m.	fur.	rd.	yd.	ft.	in.
69	7	31	5	2	11
16	6	16	4	1	6
61	7	32	3	2	10
73	3	16	4	2	9
19	4	14	1	1	8
75	5	25	5	2	7

NOTE. — As half a mile is equal to 4 furlongs, we add them to the 1 furlong, which makes 5 furlongs. And as half a foot is equal to 6 inches, we add them to the 7 inches, which makes 13 inches; and these are equal to 1 foot 1 inch.

10. Add together 37yd. 3qr. 3na. 2in.; 61yd. 3qr. 1na. 1in.; 13yd. 2qr. 2na. 2in.; 32yd. 1qr. 1na. 1in.; 61yd. 2qr. 2na. 2in.; and 22yd. 1qr. 3na. Ans. 229yd. 3qr. 3na. 1¼in.

11. Add together 671E.E. 1qr. 1na. 1in.; 161E.E. 3qr. 3na. 2in.; 617E.E. 3qr. 1na. 2in.; 178E.E. 3qr. 2na. 1in.; 717E.E. 2qr. 1na. 2in.; and 166EE. 3qr. 2na. 1in.

12. Add together 761A. 3R. 37p. 260ft. 125in.; 131A. 2R. 16p. 135ft. 112in.; 613A. 1R. 14p. 116ft. 131in.; 161A. 3R. 13p. 116ft. 123in.; 321A. 2R. 31p. 97ft. 96in.; and 47A. 3R. 19p. 91ft. 48in. Ans. 2038A. 1R. 13p. 2ft. 95in.

13. Add together 38A. 1R. 39p. 272ft.; 61A. 3R. 38p. 167ft.; 35A. 3R. 19p. 198ft.; 47A. 3R. 16p. 271ft.; 86A. 2R. 13p. 198ft.; and 46A. 1R. 14p. 269ft.

14. Add together 17m. 7fur. 9ch. 3p. 24l.; 16m. 3fur. 4ch.

1p. 15l.; 27m. 4fur. 6ch. 2p. 17l.; 18m. 6fur. 3ch. 3p. 21l.; 61m. 7fur. 7ch. 2p. 16l.; and 17m. 1fur. 8ch. 2p. 19l.

Ans. 160m. 0fur. 1ch. 1p. 12l.

15. Required the sum of 27m. 4fur. 3ch. 1p. 21l.; 29m. 3fur. 1ch. 3p. 23l.; 67m. 3fur. 3ch. 1p. 19l.; 21m. 7fur. 1ch. 3p. 16l.; 16m. 7fur. 9ch. 3p. 13l.; and 31m. 4fur. 8ch. 1p. 20l.

16. Required the value of 29T. 36ft. 1279in. + 69T. 19ft. 1345in. + 67T. 18ft. 1099in. + 71T. 14ft. 1727in. + 43T. 35ft. 916in. + 53T. 17ft. 1719in.

Ans. 335T. 23ft. 1173in.

17. Add together 61C. 127ft. 1161in.; 37C. 89ft. 1711in.; 61C. 98ft. 1336in.; 43C. 56ft. 1678in. 91C. 119ft. 1357in. and 81C. 115ft. 1129in.

18. Required the value of 61 tuns 3hhd. 62gal. 3qt. 1pt. + 39 tuns 2hhd. 16gal. 1qt. 1pt. + 68 tuns 3hhd. 57gal. 2qt. 1pt. + 87 tuns 3hhd. 45gal. 3qt. 1pt. + 47 tuns 2hhd. 59gal. 3qt. 1pt. + 47 tuns 3hhd. 39gal. 2qt. 1pt.

Ans. 354 tuns 0hhd. 30gal. 1qt.

19. Required the value of 67hhd. 15gal. 3qt. 1pt. + 16hhd. 16gal. 3qt. + 39hhd. 16gal. 3qt. + 47hhd. 62gal. 1qt. 1pt. + 43hhd. 57gal. 3qt. + 71hhd. 61gal. 3qt. 1pt.

20. Add together of beer measure 161hhd. 53gal. 3qt. 1pt.; 371hhd. 52gal. 3qt. 1pt.; 98hhd. 19gal. 1qt.; 47hhd. 43gal. 1qt. 0pt.; 61hhd. 43gal. 1qt. 1pt.; and 42hhd. 27gal. 3qt. 1pt.

21. Find the amount of 37bu. 3pk. 5qt. 1pt. + 61bu. 2pk. 7qt. 1pt. + 32bu. 3pk. 2qt. + 71bu. 1pk. 6qt. 1pt. + 61bu. 1pk. 3qt. 1pt. + 32bu. 3pk. 3qt. 1pt.

Ans. 298bu. 0pk. 4qt. 1pt.

22. Required the value of 31bu. 3pk. 3qt. + 31bu. 3pk. 1qt. + 16bu. 3pk. 1qt. + 15bu. 3pk. + 17bu. 3pk. 1qt. + 14bu. 3pk. 1qt.

23. Required the value of 57y. 11mo. 27d. 23h. 29m. 55s. + 31y. 11mo. 18d. 19h. 19m. 39s. + 46y. 9mo. 23d. 17h. 28m. 56s. + 43y. 10mo. 16d. 18h. 17m. 48s. + 32y. 9mo. 19d. 16h. 23m. 28s. + 14y. 1mo. 29d. 21h. 28m. 16s.

Ans. 227y. 7mo. 16d. 21h. 28m. 2s.

24. Required the value of 23w. 6d. 23h. 59m. 58s. + 51w. 3d. 18h. 51m. 17s. + 29w. 5d. 21h. 47m. 49s. + 28w. 4d.

23h. 56m. 18s. + 19w. 6d. 10h. 18m. 53s. + 86w. 1d. 20h. 40m. 51s.

25. Add together 4S. 29° 59′ 59″; 6S. 17° 17′ 29″; 11S. 16° 56′ 58″; 9S. 13° 46′ 51″; 5S. 27° 16′ 42″; and 2S. 25° 17′ 17″. Ans. 5S. 10° 35′ 16″.

26. Add together 11S. 11° 16′ 51″; 6S. 6° 6′ 16″; 9S. 14° 56′ 56″; 3S. 29° 29′ 49″; 9S. 17° 18′ 58″; and 6S. 13° 13′ 52″.

NOTE. — We divide the sum of the signs, in the last two questions, by 12, and write down the remainder only. Since the circumference of a circle cannot exceed 12 signs (Art. 148).

27. Bought of a London tailor a vest for 1£. 13s. 4d., a coat for 7£. 12s. 9d., pantaloons for 2£. 3s. 9d., and surtout for 9£. 8s. 0d.; what was the whole amount?

Ans. 20£. 17s. 10d.

28. Bought a silver tankard, weighing 1lb. 8oz. 17pwt. 14gr., a silver can, weighing 1lb. 2oz. 12pwt., a porringer, weighing 11oz. 19pwt. 20gr.; and 3 dozen of spoons, weighing 1lb. 9oz. 15pwt. 10gr.; what was the whole weight?

Ans. 5lb. 9oz. 4pwt. 20 gr.

29. What is the weight of a mixture of 3℔ 4℥ 2ʒ 2℈ 14gr. of aloe, 2℔ 7℥ 6ʒ 1℈ 13gr. of picra, and 1℔ 10℥ 1ʒ 2℈ 17gr. of saffron? Ans. 7℔ 10℥ 3ʒ 1℈ 4 gr.

30. Sold 4 loads of hay; the first weighed 27cwt. 3qr. 18lb.; the second, 31cwt. 1qr. 15lb.; the third, 19cwt. 1qr. 15lb.; and the fourth, 38cwt. 2qr. 24lb.; what is the weight of the whole?

31. Bought 5 pieces of broadcloth; the first contained 17yd. 3qr. 2na.; second, 13yd. 2qr. 1na.; the third, 87yd. 1qr. 3na.; the fourth, 27yd. 1qr. 2na.; and the fifth, 29yd. 1qr. 2na.; what was the whole quantity purchased?

Ans. 175yd. 2qr. 2na.

32. A pedestrian travelled, the first week, 371m. 3fur. 37rd. 5yd. 2ft. 10in.; the second week, 289m. 2fur. 18rd. 3yd. 1ft. 9in.; and the third week, 399m. 7fur. 3ft. 11in.; how many miles did he travel? Ans. 1060m. 5fur. 16rd. 5yd. 1ft.

33. A man has three farms; the first contains 186A. 3R. 14p.; the second, 286A. 17p.; and the third, 115A. 2R; how much do they all contain? Ans. 588A. 1R. 31p.

34. The Moon is 5S. 18° 14′ 17″ east of the Sun; Jupiter is 7S. 10° 29′ 28″ east of the Moon; Mars is 11S. 12° 11′ 56″ east of Jupiter; and Herschel is 7S. 18° 38′ 15″ east of Mars; how far is Herschel east from the Sun?

Ans. 7S. 29° 33′ 56″.

SUBTRACTION OF COMPOUND NUMBERS.

146. Subtraction of Compound Numbers is the process of finding the difference between two compound numbers.

Ex. 1. From 617£. 11s. 8d. take 181£. 15s. 5d.

Ans. 435£. 16s. 3d.

OPERATION.

	£	s.	d.
Min.	617	11	8
Sub.	181	15	5
Rem.	435	16	3

Having placed the less number under the greater, pence under pence, shillings under shillings, &c., we begin with pence, thus: 5d. from 8d. leaves 3d., which we set under the column of pence. As we cannot take 15s. from 11s., we add 20s. = 1£. to the 11s., making 31s., and then subtract the 15s. from it, and set the remainder, 16s., under the column of shillings. Then, having added 1£. = 20s. to the 181£., to compensate for the 20s. added to the 11s. in the minuend, we subtract the pounds as in subtraction of simple numbers, and obtain 435£. for the remainder, and as the result complete, 435£. 16s. 3d.

Rule. — *Write the less compound number under the greater, so that units of the same denomination shall stand in the same column.*

Subtract as in subtraction of simple numbers.

If any number in the subtrahend is larger than that above it, add to the upper number as many units as make one of the next higher denomination before subtracting, and carry one to the next lower number before subtracting it.

Proof. — The proof is the same as in simple subtraction.

Examples.

2.

£	s.	d.
87	16	$3\frac{1}{4}$
19	17	$9\frac{1}{2}$
67	18	$5\frac{3}{4}$

3.

£	s.	d.
617	11	$5\frac{1}{2}$
181	15	$8\frac{1}{4}$

4.

T.	cwt.	qr.	lb.	oz.	dr.
71	18	1	13	1	13
19	19	2	16	8	5
51	18	2	21	9	8

5.

cwt.	qr.	lb.	oz.
73	1	15	13
19	1	19	15

6.

lb.	oz.	pwt.	gr.
71	3	12	15
16	10	17	20
54	4	14	19

7.

lb.	oz.	pwt.	gr.
58	5	12	10
19	9	17	21

8.

℔	℥	ʒ	℈	gr.
71	1	3	1	14
18	6	7	2	19
52	6	3	1	15

9.

℔	℥	ʒ	℈	gr.
15	2	2	0	15
9	9	1	1	18

10.

m.	fur.	rd.	ft.	in.
16	7	18	3	1
9	7	19	16	8
6	7	38	$2\frac{1}{2}$	5
			$\frac{1}{2}=$	6
6	7	38	2	11

11.

deg.	m.	fur.	rd.	yd.	ft.	in
38	41	3	29	2	1	7
29	36	5	31	3	1	9

NOTE. — As half a foot is equal to 6 inches, we add them to the 5 inches, which make 11 inches.

12. From 67yd. 1qr. 1na. 1in. take 18yd. 2qr. 2na. 2in.
Ans. 48yd. 2qr. 2na. $1\frac{1}{4}$in.

13. From 51E.E. 2qr. 3na. take 19E.E. 3qr. 1na.

14. From 56A. 1R. 19p. 119ft. 110in. take 17A. 3R. 13p. 127ft. 113in. Ans. 38A. 2R. 5p. 264ft. 33in.

15. Find the value of 13A. 1R. 15p. 19yd. 1ft. 17in. — 9A. 3R. 16p. 30yd. 5ft. 17in.

16. Subtract 19m. 2fur. 1ch. 3p. 21l. from 21m. 1fur. 3ch. 2p. 19l. Ans. 1m. 7fur. 1ch. 2p. 23l.

17. From 28m. 6fur. 1ch. 2p. 18l. take 15m. 7fur. 3ch. 1p. 19l.

18. Required the value of 49T. 13ft. 1611in. — 18T. 15ft. 1719in. Ans. 30T. 37ft. 1620in.

19. Required the value of 361C. 47ft. 1178in. — 197C. 121ft. 1617in.

20. Subtract 11tun 1hhd. 28gal. 2qt. 1pt. from 79tun 3hhd. 19gal. 1qt. 1pt. Ans. 68tun 1hhd. 53 gal. 3qt.

21. Subtract of beer measure 191hhd. 19gal. 3qt. from 769hhd. 18gal. 1qt.

22. From 56ch. 2bu. 1pk. take 38ch. 3bu. 1pk.
Ans. 17ch. 35bu.

23. Required the value of 25bu. 3pk. 1qt. — 12bu. 3pk. 5qt.
Ans. 12bu. 3pk. 4qt.

24. Required the difference between 6mo. 16d. 13h. 27m. 19s. and 1mo. 22d. 16h. 41m. 37s.
Ans. 4mo. 23d. 20h. 45m. 42s.

25. From 48y. 0mo. 15d. 19h. 27m. 31s. take 19y. 10mo. 29d. 21h. 38m. 56s.

26. From 6S. 11° 12′ 48″ subtract 9S. 8° 15′ 56″.
Ans. 9S. 2° 56′ 52″.

27. Take 1S. 22° 19′ 28″ from 4S. 19° 41′ 22″.

28. I have 73A. of land; if I should sell 5A. 3R. 1p. 7ft., how much should I have left? Ans. 67A. 0R. 38p. 265¼ft.

29. A owes B 100£.; what will remain due after he has paid him 3s. 6½d.? Ans. 99£. 16s. 5½d.

30. It is about 25,000 miles round the globe; if a man shall have travelled 43m. 17rd. 9in., how much will remain to be travelled? Ans. 24,956m. 7fur. 22rd. 15ft. 9in.

31. Bought 7 cords of wood; 2 cords 78ft. having been stolen, how much remained?

32. I have 15 yards of cloth; having sold 3yd. 2qr. 1na., what remains? Ans. 11yd. 1qr. 3na.

33. If a wagon loaded with hay weighs 43cwt. 2qr. 18lb., and the wagon is afterwards found to weigh 9cwt. 3qr. 23lb., what is the weight of the hay? Ans. 33cwt. 2qr. 20lb.

34. Bought a hogshead of wine, and by an accident 8gal. 3qt. 1pt. leaked out; what remains?

35. I had 10A. 3R. 10p. of land; and I have sold two house-lots, one containing 1A. 2R. 13p., the other 2A. 2R. 5p.; how much have I remaining? Ans. 6A. 2R. 32p.

36. The Moon moves 13° 10′ 35″ in a solar day, and the Sun 59′ 8″; now supposing them both to start from the

same point in the heavens, how far will the Moon have gained on the Sun in 24 hours? Ans. 12° 11′ 27″.

37. A farmer raised 136bu. of wheat; if he sells 49bu. 2pk. 7qt. 1pt., how much has he remaining?

Ans. 86bu. 1pk. 0qt. 1pt.

38. If from a stick of timber containing 2T. 18ft. 1410in. there be taken 38ft. 1720in., how much will be left?

Ans. 1T. 19ft. 1418in.

MULTIPLICATION OF COMPOUND NUMBERS.

147. MULTIPLICATION of Compound Numbers is the process of taking a compound number any proposed number of times.

Ex. 1. What will 6 bales of cloth cost, at 7£. 12s. 7d. per bale? Ans. 45£. 15s. 6d.

OPERATION.

	£.	s.	d.
Multiplicand	7	12	7
Multiplier			6
Product	45	15	6

Having written the multiplier under the lowest denomination of the multiplicand, we multiply thus: 7d. × 6 = 42d. = 3s. 6d. We write the 6d. under the number multiplied, and reserve the 3s. to be added to the product of the shillings. Then, 12s. × 6 = 72s., and 3s. (carried) = 75s. = 3£. 15s. We write the 15s. under the column of shillings, and reserve the 3£. to be added to the product of the pounds. Again, 7£. × 6 = 42£., and 3£. (carried) = 45£. This, placed under the column of pounds, gives 45£. 15s. 6d.

RULE. — *Multiply each denomination of the compound number as in multiplication of simple numbers, and carry as in addition of compound numbers.*

Proof. — Write down by themselves the several products obtained by multiplying each denomination of the multiplicand by the multiplier, and these partial products added together will equal the entire product, if the work be right. (Art. 60.)

NOTE. — Going a second time carefully over the work is a good way of testing its accuracy. On learning Division of Compound Numbers, the pupil will find that rule a better method of proving multiplication of compound numbers.

EXAMPLES.

2. Multiply 1£. 8s. 7d. 2far. by 7. Ans. 10£. 0s. 4d. 2far.

OPERATION.				PROOF BY ADDITION.					
	£.	s.	d.						
	2	8	$9\frac{1}{2}$	$9\frac{1}{2}$d. × 8	= 76d.	=	0£.	6s.	4d.
			8	8s. × 8	= 64s.	=	3£.	4s.	0d.
Ans.	19	10	4	2£. × 8	= 16£.	=	16£.	0s.	0d.
				2£. 8s. $9\frac{1}{2}$d. × 8		=	19£.	10s.	4d.

NOTE. — The answers to the following examples may be found in corresponding numbers of examples in Division of Compound Numbers.

3.

T.	cwt.	qr.	lb.	oz.	dr.
61	19	3	17	15	15
					9

4.

lb.	oz.	pwt.	gr.
7	11	14	15
			5

5.

lb.	oz.	pwt.	gr.
32	8	17	12
			8

6.

℔	℥	ʒ	℈	gr.
38	10	5	2	14
				11

7.

deg.	m.	fur.	rd.	ft.	in.
71	38	2	13	14	4
					12

8.

m.	fur.	ch.	p.	l.
17	7	9	3	23
				12

9. Multiply 16A. 2R. 4p. 19yd. 7ft. 79in. by 11.
10. Multiply 10yd. 3qr. 3na. by 5.
11. Multiply 17tun 2hhd. 50gal. 1qt. by 7.
12. Multiply 29hhd. 61gal. 3qt. 1pt. 3gi. by 7.
13. Multiply 19bu. 2pk. 7qt. 1pt. by 6.
14. What is the value of 13y. 316d. 15h. 27m. 39s. × 8?
15. Multiply 16deg. 39m. 3fur. 39rd. 5yd. 2ft. by 9.
16. If a man gives each of his 9 sons 23A. 3R. $19\frac{1}{3}$p., what do they all receive?
17. If 12 men perform a piece of labor in 7h. 24m. 30s., how long would it take 1 man to perform the same task?
18. If 1 bag contain 3bu. 2pk. 4qt., what quantity do 8 bags contain?

148. When the multiplier is a composite number, and none of its factors exceed 12.

Ex. 1. What will 35 loads of coal weigh, if each load weighs 2T. 1cwt. 2qr. 6lb.?

OPERATION.

T.	cwt.	qr.	lb.	
2	1	2	6	= weight of 1 load.
			7	
14	10	3	17	= weight of 7 loads.
			5	
72	14	2	10	= weight of 35 loads.

We find the number 35 equal to the product of 7 and 5; we therefore multiply the weight of 1 load by 7, and then that product by 5; and the last product is the answer. Hence, when the multiplier is a composite number,

Multiply by its factors in succession.

EXAMPLES.

2. Bought 90 hogsheads of sugar, each weighing 12cwt. 2qr. 11lb.; what was the weight of the whole?

3. What cost 18 sheep at 5s. 9½d. apiece?

4. What cost 21 yards of cloth at 9s. 11d. per yard?

5. What cost 22 hats at 11s. 6d. each?

6. If 1 share in a certain stock be valued at 13£. 8s. 9½d., what is the value of 96 shares?

7. If 1 spoon weighs 3oz. 5pwt. 15gr., what is the weight of 120 spoons?

8. If a man travel 24m. 7fur. 4rd. in 1 day, how far will he go in 1 month?

9. If the earth revolve 0° 15′ per minute, how far does it revolve per hour?

10. Multiply 39A. 3R. 17p. 30yd. 8ft. 100in. by 32.

11. If a man be 2d. 5h. 17m. 19sec. in walking 1 degree, how long would it take him to walk round the earth, allowing 365¼ days to a year?

149. When the multiplier is not a composite number, and exceeds 12; or when a composite number one of whose factors exceeds 12.

Ex. 1. What is the value of 453 tons of iron at 18£. 17s. 11d. a ton?

OPERATION.

	£.	s.	d.		£.	s.	d.	
1 ton =	18	17	11,	×3 =	56	13	9	= Value of 3 tons.
			10					
10 tons =	188	19	2,	×5 =	944	15	10	= Value of 50 tons.
			10					
100 tons =	1889	11	8,	×4 =	7558	6	8	= Value of 400 tons.
				Ans.	8559	16	3	= Value of 453 tons.

Since 453 is not a composite number, we cannot resolve it into factors; but we may separate it into parts, and find the value of each part separately: Thus, 453 = 400 + 50 + 3. In the operation, we first multiply by 10, and obtain the value of 10 tons, and this product we multiply by 10, and obtain the value of 100 tons. Then, to find the value of 400 tons, we multiply the last product by 4; and to find the value of 50 tons, we multiply the value of 10 tons by 5; and to find the value of 3 tons, we multiply the value of 1 ton by 3. Adding the several products, we obtain 8559£. 16s. 3d. for the answer. Hence,

Having resolved the multiplier into any convenient parts, as of units, tens, &c., multiply by these several parts, and add together the products thus obtained for the required result.

EXAMPLES.

2. Multiply 2hhd. 19gal. 0qt. 1pt. by 39.

3. Multiply 3bu. 1pk. 4qt. 1pt. 1gi. by 53.

4. Multiply 16ch. 7bu. 2pk. 0qt. 0pt. by 17.

5. What will 57 gallons of wine cost at 8s. 3¼d. per gallon?

6. Bought 29 lots of wild land, each containing 117A. 3R. 27p.; what were the contents of the whole?

7. Bought 89 pieces of cloth, each containing 37yd. 3qr. 2na. 2in.; what was the whole quantity?

8. Bought 59 casks of wine, each containing 47gal. 3qt. 1pt.; what was the whole quantity?

9. If a man travel 17m. 3fur. 13rd. 14ft. in one day, how far will he travel in a year?

10. If a man drink 3gal. 1qt. 1pt. of beer in a week, how much will he drink in 52 weeks?

11. There are 17 sticks of timber, each containing 37ft. 978in.; what is the whole quantity?

12. There are 17 piles of wood, each containing 7 cords 98 cubic feet; what is the whole quantity?

DIVISION OF COMPOUND NUMBERS.

150. DIVISION of Compound Numbers is the process of dividing compound numbers into any proposed number of equal parts.

Ex. 1. Divide 139£. 13s. 11d. 2far. equally between 5 persons. Ans. 27£. 18s. 9d. 2far.

OPERATION.

	£.	s.	d.	far.
5)	139	13	11	2
	27	18	9	2

Having divided 139£. by 5, we find the quotient to be 27£., and 4£. remaining. We place the quotient 27£. under the 139£., and the remainder 4£. reduced to shillings = 80s.; 80s. + the 13s. in the dividend = 93s.; 93s. ÷ 5 = 18s. and a remainder of 3s. We write the quotient 18s. under the shillings in the dividend; and the remainder 3s. reduced to pence = 36d.; 36d. + 11d. in the dividend = 47d.; 47d. ÷ 5 = 9d. and a remainder of 2d. We write the quotient 9d. under the pence in the dividend; and the remainder 2d. reduced to farthings = 8far., + the 2far. in the dividend = 10far.; 10far. ÷ 5 = 2far. The quotient 2far. we write under the farthings in the dividend; and thus find the answer to be 27£. 18s. 9d. 2far.

RULE. — *Divide as in division of simple numbers, each denomination in its order, beginning with the highest.*

If there be a remainder, reduce it to the next lower denomination, adding in the number already contained in the dividend of this denomination, if any, and divide as before.

PROOF. — The same as in simple numbers.

NOTE. — When the divisor and dividend are both compound numbers, they must be reduced to the same denomination, and the division then is that of simple numbers.

EXAMPLES.

NOTE. — The answers to the following examples are found in the corresponding numbers of examples in Multiplication of Compound Numbers.

2.

	£.	s.	d.
8)	19	10	4

3.

	T.	cwt.	qr.	lb.	oz.	dr.
9)	557	19	1	11	15	7

4.

	lb.	oz.	pwt.	gr.
5)	39	10	13	3

5.

	lb.	oz.	pwt.	gr.
8)	261	11	0	0

6.

	℔	℥	ʒ	℈	gr.
11)	427	10	0	2	14

7.

	deg.	m.	fur.	rd.	ft.	in.
12)	858	44	4	6	7	0

8.

	m.	fur.	ch.	p.	l.
12)	215	7	9	3	1

9.

	A.	R.	p.	yd.	ft.	in.
11)	181	3	11	6	4	41

10. Divide 54yd. 2qr. 3na. equally among 5 persons.
11. Divide 123tun 3hhd. 36gal. 3qt. by 7.
12. Divide 209hhd. 55gal. 3qt. 0pt. 1gi. by 7.
13. What is the value of 118bu. 1pk. 5qt. ÷ 6?
14. What is the value of 110y. 343d. 3h. 41m. 12s. ÷ 8?
15. Divide 149deg. 9m. 5fur. 13rd. 3yd. 1ft. by 9.
16. A man divides his farm of 214A. 3R. 12p. equally among his 9 sons; how much does each receive?
17. If one man perform a certain piece of labor in 3d. 16h. 54m., how long would it take 12 men to perform the same work?
18. A farmer has 29 bushels of oats which he wishes to put in 8 sacks; how much must each sack contain?

151. When the divisor is a composite number, and none of its factors exceed 12.

Ex. 1. If 35 loads of coal weigh 72T. 14cwt. 2qr. 10lb., what will 1 load weigh?

OPERATION.

	T.	cwt.	qr.	lb.	
5)	72	14	2	10	= weight of 35 loads.
7)	14	10	3	17	= weight of 7 loads.
	2	1	2	6	= weight of 1 load.

We find the factors of 35 to be 5 and 7. We therefore divide the weight of 35 loads by 5, and obtain the weight of 7 loads; and the weight of 7 loads we divide by 7, and thus find the weight of 1 load. Hence, when the divisor is a composite number,

Divide by its factors in succession.

Examples.

2. If 90 hogsheads of sugar weigh 56T. 14cwt. 3qr. 15lb., what is the weight of 1 hogshead?
3. What will be the price of 1 sheep, if 18 cost 5£. 4s. 3d.?

4. If 21 yards of cloth cost 10£. 8s. 3d., what is the price of 1 yard?

5. What is the value of 1 hat, when 22 cost 12£. 13s. 0d.?

6. When 96 shares of a certain stock are valued at 1290£. 4s. 0d., what would be the cost of 1 share?

7. If 120 spoons weigh 32lb. 9oz. 15pwt., what does 1 weigh?

8. If a man in 1 month travels 746m. 5fur., how far does he go in 1 day?

9. If the earth revolves 15° on its axis in 1 hour, how far does it revolve in 1 minute?

10. Divide 1275A. 2R. 16p. 22yd. 8ft. 32in. equally among 32 men.

11. If a man walk round the earth in 2y. 68d. 19h. 54m., how long would it take him to walk 1 degree, allowing 365¼ days to a year?

152. When the divisor is not a composite number, and exceeds 12, or when a composite number one of whose factors exceeds 12, *the whole operation can be written*, as in the following example.

Ex. 1. Divide 360£. 8s. 4d. by 173. Ans. 2£. 1s. 8d.

OPERATION.

```
      £.   s.  d.
173)360   8   4 (2£.
    346
    ---
     14
     20
    ---
173)288 (1s.
    173
    ---
    115
     12
    ----
173)1384 (8d.
    1384
    ----
```

We divide the pounds by 173, and obtain 2£. for the quotient, and 14£. remaining, which we reduce to shillings, and add the 8s., and again divide by 173, and obtain 1s. for the quotient. The remainder, 115s., we reduce to pence, and add the 4d., and again divide by 173, and obtain 8d. for the quotient. Thus, the method is the same as by general rule (Art. 150). By uniting the several quotients, we obtain 2£. 1s. 8d. for the answer.

2. Divide 89hhd. 52gal. 3qt. 1pt. by 39.

3. Divide 179bu. 3pk. 5qt. 0pt. 1gi. by 53.

4. Divide 275ch. 19bu. 2pk. equally among 17 men.

5. If 57 gallons of wine cost 23£. 11s. $5\frac{1}{4}$d., what cost 1 gallon?

6. Divide 3419A. 2R. 23p. by 29.

7. If 89 pieces of cloth contain 3375yd. 3qr. 1na. $0\frac{1}{4}$in., how much does 1 piece contain?

8. If 59 casks contain 44hhd. 52gal. 2qt. 1pt. of wine, what are the contents of 1 cask?

9. If a man travel in 1 year (365 days) 6357m. 5fur. 14rd. $11\frac{1}{2}$ft., how far is that per day?

10. When 175gal. 2qt. of beer are drunk in 52 weeks, how much is consumed in 1 week?

11. When 17 sticks of timber measure 15T. 38ft. 1074in., how many feet does 1 contain?

12. Divide 132 cords 2ft. by 17.

13. Divide 697T. 18cwt. 3qr. 14lb. by 146.
Ans. 4T. 15cwt. 2qr. $10\frac{129}{146}$lb.

14. Divide 916m. 3fur. 30rd. 10ft. 6in. by 47.
Ans. 19m. 3fur. 39rd. 13ft. $2\frac{26}{47}$in.

15. Divide 718A. 3R. 37p. by 29. Ans. 24A. 3R. $6\frac{23}{29}$p.

16. Divide 815A. 1R. 17p. 200ft. by 87.
Ans. 9A. 1R. 19p. $139\frac{86}{87}$ft.

17. Divide 144A. 3R. 18p. 3yd. 1ft. 36in. by 11.
Ans. 13A. 0R. 27p. 3yd. 0ft. $45\frac{9}{11}$in.

PRINCIPLES AND APPLICATIONS.

DIFFERENCE BETWEEN DATES.

153. To find the time between two different dates.

Ex. 1. What is the difference of time between May 16, 1819, and March 4, 1857? Ans. 37y. 9mo. 18d.

FIRST OPERATION.

	y.	mo.	d.
Min.	1857	2	4
Sub.	1819	4	16
Rem.	37	9	18

Commencing with January, the first month in the year, and counting the months and days in the later date up to March 4th, we find that 2mo. and 4d. have elapsed. We therefore write the numbers for subtraction as in the first operation.

SECOND OPERATION.

Min.	1857	3	4
Sub.	1819	5	16
Rem.	37	9	18

The same result, however, could be obtained, as some prefer, by reckoning the *number* of the given months instead of the *number of months* that have elapsed since the beginning of the year, which would require the numbers to be written as in the second operation. Written either way, *the earlier date, being placed under the later, is subtracted from it.*

NOTE. — In finding the difference between two dates, and in computing interest for less than a month, 30 days are considered a month. In *legal* transactions, however, a month is reckoned from any day in one month to the corresponding day of the following month, if it has a corresponding day, otherwise to its end. The above process, which is that ordinarily used by business men, does not give always the exact time between two different dates. The result obtained by it may deviate sometimes a day, and, less often, two days, from the exact difference. But for practical purposes it is generally regarded as sufficiently accurate.

EXAMPLES.

2. What is the time from June 3d, 1854, to April 19th, 1857?

3. A note was given October 26th, 1856, and paid June 12th, 1857; how long was it on interest?

4. The Pilgrims landed at Plymouth December 22d, 1620, N. S., and the Declaration of Independence was made July 4th, 1776; what is the difference of time between these events?

5. General Washington was born February 22d, 1732, and died December 14th, 1799; how long did he live?

Ans. 67y. 9mo. 22d.

154. To find the exact number of days between two different dates.

Ex. 1. How many days from January 28 to July 30, common year? Ans. 183 days.

OPERATION.

January to July = 6mo.	$6 \times 31 =$	186 days.
For Feb. 3d., April 1d., June 1d.,	$3 + 1 + 1 =$	5 "
		181
For difference between 30 and 28,	$30 - 28 =$	2
	Ans.	183 days.

The difference between January and July we find to be 6mo., which, multiplied by 31, the greatest number of days in any month in the year, gives 186 days. But since in the interval of time included between the given dates several months end that do not con-

tain 31 days, we make deductions for these, which, in all, amount to 5 days, and have left 181 days; and as the difference between the given dates is the difference between 30 and 28 more than exactly 6mo., we add 2 to the 281 days, thus obtaining 183 days, the difference of time required. Hence, to find the number of days between two different dates,

Find the number of months ending between the given dates, and multiply that number by 31, and from the product make the necessary deduction for the months counted that do not contain 31 days, if any. Should the later date end later in the month than the earlier, add the difference of days; but should it end earlier, subtract the same.

NOTE. — The exact difference in days between two different dates can also be obtained by use of the table in Note 2, Art. 142.

EXAMPLES.

2. How many days has a note to run dated November 15, 1856, and made payable February 13, 1857? Ans. 90 days.

3. How many days from June 18, 1855, to May 1, 1856?

4. How many days from March 4 to May 3 of the same year?

5. From November 4, 1856, to April 4, 1857, how many days? Ans. 151 days.

6. In a leap year, how many days are there from the 7th of January to the 11th of December? Ans. 339 days.

155. To find the day of the week corresponding to any given day of the month, when the day of the week of some other date is given.

Ex. 1. If the 16th day of May be on Saturday, what day of the week will the next 25th of December be?

Ans. Friday.

OPERATION.

From May 16 to December 25 = 223 days.
223 days $\div$ 7 = 31 weeks and 6 days.
6 days after Saturday = Friday, Ans.

Having found the difference of time in days between the given dates, we bring the days to weeks by dividing by 7, and obtain 31 weeks and 6 days. The 25th of December, therefore, must come 6 days after Saturday, or on Friday. Hence, we

Reduce the days between the given dates to weeks. Should there be no remainder, the day given will be the same as that sought, but should there be a remainder, it will indicate the number of days that the day sought is after the day given.

Examples.

2. If the 2d day of April be on Wednesday, what day of the week will the following 4th of July be? Ans. Friday.

3. If a leap year commence on Tuesday, on what day will the 17th of June, the anniversary of the battle of Bunker Hill, happen?

4. If in a common year the 25th of December, or Christmas, be on Tuesday, on what day did the year commence?

Ans. Monday.

5. If the 4th day of November be on Tuesday, what day of the next February will be the second Monday of that month? Ans. The 9th.

6. A bill was dated on Thursday, December 20th, 1855, and made payable 90 days after date. In what year and month, and on what day of the month and week, did it become due? Ans. Wednesday, March 19, 1856.

DIFFERENCE OF LATITUDE.

156. Latitude is the distance of any place from the equator, north or south. It is reckoned in degrees, minutes, and seconds, from the equator to either pole of the earth; and cannot exceed 90 degrees, or one fourth of the earth's circumference. Places north of the equator are said to be in north latitude, and those south of the equator, in south latitude.

Note. — The shape of the earth not being that of a perfect sphere, but somewhat flattened toward the poles, the degrees of latitude differ a little from each other in length toward either pole. Thus the 1st degree is about $68\frac{8}{10}$ statute miles in length; the 40th degree, about $68\frac{93}{100}$ miles; and the 89th degree about $69\frac{38}{100}$ miles.

157. To find the difference of latitude of any two places.

Ex. 1. The latitude of London is 51° 31′ north, and that of Boston is 42° 23′ north. What is their difference of latitude? Ans. 9° 8′.

OPERATION.

Lat. of London	=	51°	31′ N.
Lat. of Boston	=	42°	23′ N.
Dif. of Latitude	=	9°	8′ N.

The two places being both in north latitude, we subtract the less latitude from the greater. If the latitude of the one place had been north and that of

the other south, we should have added the two latitudes together for the difference required. Hence

When the latitudes of two places are either both north or both south, subtract the less latitude from the greater for their difference; and when the latitude of the one place is north and the other south, add the latitudes together for their difference.

NOTE.—In *north* latitude, when the sailing is *southerly*, or in *south* latitude, when the sailing is *northerly*, if the difference of latitude be subtracted from the latitude LEFT, the remainder will be the latitude IN; or, in *north* latitude, when the sailing is *northerly*, or in *south* latitude, when the sailing is *southerly*, if the difference of latitude be added to the latitude LEFT, the sum will give the latitude IN.

EXAMPLES.

2. The latitude of Quebec is 46° 48′ north, and that of New Orleans 29° 57′ north. What is their difference of latitude? Ans. 16° 51′.

3. The latitude of Washington City is 38° 53′ north, and that of Cape Horn 55° 58′ south. What is their difference of latitude? Ans. 94° 51′.

4. Valparaiso is in latitude 33° 2′ south, and San Francisco 37° 48′ north. What is their difference of latitude?

5. Captain James Francis, sailing southerly from New York City, whose latitude is 40° 42′ north, found on reaching Havana that his latitude differed 17° 33′ from that he left. What latitude was he then in? Ans. 23° 9′ north.

6. Philadelphia is 9° 15′ of latitude north of Mobile, whose latitude is 30° 41′ north; what is the latitude of Philadelphia? Ans. 39° 56′ north.

DIFFERENCE OF LONGITUDE.

158. LONGITUDE is the distance of any place from a given meridian, east or west. It is reckoned in degrees, minutes, and seconds, and cannot exceed 180 degrees, or one half of a circle.

NOTE 1.—A degree of longitude on the equator is about $69\frac{1}{6}$ statute miles, and, in general, a degree is $\frac{1}{360}$ of any circle of latitude. The meridians all converge from the equator to the poles to a point, so that the degrees of longitude under different parallels of latitude vary, diminishing with the circles of latitude, till at the poles the longitude becomes nothing. (Art. 133, Note 4.)

NOTE 2. — The mariners of Great Britain and the United States reckon longitude from the meridian of Greenwich in England, as do the nautical books of both countries for the most part. American maps, however, very generally have longitude reckoned both from the meridian of Greenwich, and from the meridian of Washington.

159. To find the difference of longitude of any two places.

Ex. 1. What is the difference of longitude between Boston, which is 71° 4′ west, and Detroit, which is 82° 58′ west.

Ans. 11° 54′.

OPERATION.

Long. of Detroit	=	82°	58′
Long. of Boston	=	71°	4′
Dif. of Longitude	=	11°	54′

The two places being both in west longitude, we subtract the less longitude from the greater. If, however, one of the places had been in east longitude, and the other in west, we should have added the two longitudes together for the difference required. Hence,

When the longitudes of two places are either both east or both west, subtract the less longitude from the greater for their difference; and when the longitude of the one place is east, and that of the other west, add the longitudes together for their difference.

NOTE. — In adding together two longitudes, should their sum exceed 180 degrees, subtract it from 360 degrees, and the remainder will be the correct difference of longitude.

EXAMPLES.

2. What is the difference of longitude between the city of Washington, whose longitude is 77° 16′ west, and Paris, whose longitude is 2° 20′ east? Ans. 79° 36′.

3. The United States extend from the St. Croix River, longitude 67° 2′ west, to Cape Flattery, longitude 124° 43′ west. Over how many degrees of longitude does the Union extend?

4. What is the difference of longitude between Raleigh, whose longitude is 78° 48′ west, and Sacramento City, whose longitude is 120° west? Ans. 41° 12′.

5. What is the difference of longitude between Hartford, whose longitude is 72° 40′ west, and Fort Leavenworth whose longitude is 94° 44′ west?

6. The longitude of Honolulu is 157° 52′ west, and that of Canton 113° 14′ east. What is their difference of longitude? Ans. 88° 54′.

LONGITUDE AND TIME.

160. To find the time corresponding to degrees and minutes of longitude.

Ex. 1. The difference of longitude between Boston and London being 71° 4′, what is their difference of time?

Ans. 4h. 44m. 16sec.

OPERATION.

Dif. of longitude	=	71° 4′
		4
Dif. of time	=	284m. 16sec.

284m. 16sec. = 4h. 44m. 16sec. Ans.

Since 1′ of longitude corresponds to 4sec. of time, and 1° of longitude to 4m. of time (Art. 143, Note 2), 4′ corresponds to 16sec., and 71° to 284m.; and 284m. 16sec. = 4h. 44m. 16sec., the answer required. The apparent motion of the sun being west, the time at Boston is as much earlier than that of London as the difference of time between them. Thus, when it is noon at London, it wants 4h. 44m. 16sec. of noon at Boston (Art. 143, Note 2). Therefore, to find the time corresponding to degrees and minutes of longitude,

Multiply the minutes of longitude by 4, for seconds of time.
Multiply the degrees of longitude by 4, for minutes of time.

NOTE 1.— Should the seconds of time, when found, be 60 or more, they may be reduced to minutes; and should, also, the minutes be 60 or more, they may be reduced to hours. The difference of time between two places is exactly as much as the true clock time of the one is fast or slow as compared with that of the other.

EXAMPLES.

2. Galveston in Texas is 14° 43′ west of Pittsburg. When it is 12 o'clock at Galveston, what is the time at Pittsburg?

3. The longitude of Valparaiso is 71° 37′ west, and the longitude of Rome is 20° 30′ east. When it is 11h. 15m. A. M. at Valparaiso, what is the time at Rome?

Ans. 23m. 28sec. past 5 o'clock, P. M.

4. The longitude of Jerusalem is 35° 32′ east, and the longitude of Baltimore 76° 37′ west. When it is 9 o'clock, A. M. at Jerusalem, what time is it at Baltimore?

Ans. 1h. 31m. 24sec. A. M.

161. To find the longitude corresponding to hours, minutes and seconds of time.

Ex. 1. The difference of time between Boston and London is 4h. 44m. 16sec.; what is the difference of longitude?

Ans. 71° 4′.

OPERATION.

$$15^\circ \times 4 = 60^\circ$$
$$44 \div 4 = 11^\circ$$
$$16 \div 4 = \ 4'$$
$$\text{Dif. of longitude} = 71^\circ\ 4'$$

Since 1h. of time corresponds to 15° of longitude, 4m. of time to 1° of longitude, and 4sec. of time to 1′ of longitude (Art. 143, Note 2), 4h. corresponds to 60°, 44m. to 11°, and 16sec. to 4′; and 60° + 11° + 4′ = 71° 4′, the answer required.

Hence, to find the longitude corresponding to hours, minutes, and seconds of time,

Multiply the hours of time by 15, *and divide the minutes of time by* 4, *for degrees of longitude; and divide the seconds of time by* 4, *for minutes of longitude. The several results added together will be the difference of longitude.*

EXAMPLES.

2. The difference of time between Washington and Cincinnati is 29m. 36sec.; what is the difference of longitude?

3. A ship-captain sailing from New York to Europe, after being at sea some days, on taking an observation, found that the sun at noon was 2h. 20m. 40sec. earlier than the New York time, as shown by his chronometer. How many degrees east had he sailed? Ans. 35° 10′.

4. A gentleman travelling west from Philadelphia, whose longitude is 75° 10′ west, found, on arriving at St. Louis, that his watch, an accurate timekeeper, which was right when he left Philadelphia, was 1h. 20sec. earlier than the time at St. Louis. What, then, is the longitude of St. Louis? Ans. 90° 15′ west.

5. The difference of time between Baltimore and New Orleans is 53m. 30sec.; what is the difference of longitude?

6. When it is noon at St. Paul's, longitude 93° 5′ west, it is at Bangor 1h. 37m. 12sec. P.M.; what is the longitude of Bangor? Ans. 68° 47′ west.

7. When it is 11 A.M. at a place 30° east of Greenwich, it is 3h. 44m. 20sec. A.M. at Buffalo; what is the longitude of Buffalo? Ans. 78° 55′ west.

8. The difference of time between Cambridge Observatory and Greenwich is 4h. 44m. 32sec.; what is the difference of longitude? Ans. 71° 8′.

MISCELLANEOUS EXAMPLES.

1. If the population of the world be as follows: America, 57,650,000; Europe, 263,517,496; Asia, 626,400,000; Africa, 100,000,000; Australia, 1,445,000; Polynesia, 1,500,000; and the average length of life be 33 years, what must be the average number of deaths annually? Ans. 31,833,712.

2. A farmer has in 3 bins 755 bushels of grain; there being in the first 125 bushels, and in the second 96 bushels, more than in the third; how many bushels in the second and third? Ans. 363 in the second, 267 in the third.

3. There is a certain island 30 miles in circumference. If A and B commence travelling round it, A at the rate of 3 miles an hour, and B at the rate of 5 miles an hour, how far apart will they be at the end of 30 hours?

4. Having money to invest, I purchased two farms at $ 1,750 each, and 19 shares of bank stock at $ 103 per share, and have left $ 113; how much money had I? Ans. $ 5,570.

5. It has been agreed by 12 men to gather 960 bushels of cranberries, and receive for their labor one half of the quantity gathered; after one half was gathered, one third of the men withdrew, leaving the others to complete the job. How many bushels should each man receive? Ans. Those who left, 20 bushels each; those who remained, 50 bushels each.

6. A drover made $ 652.00 by selling a lot of sheep, at a profit of 50 cents each; how many did he sell?

7. What will it cost to carpet a floor that is 18 feet wide and 27 feet long, provided the carpeting cost $ 2.25 per sq. yd.?

8. If a young man, by early rising and economy of time, can save for study and improvement of mind two and a half hours a day, how many years' study, of 12 hours per day, can thus be gained in 20 years? Ans. 4y. 60d. 10h.

9. From 4 piles of wood, the first containing 7c. 76ft. 1671in., the second 16c. 28ft. 56in., the third 29c. 127ft. 1000in., the fourth 29c. 10ft. 1216in., I have sold 45 cords and 6 cord feet; how much remains? Ans. 37c. 19ft. 487in.

10. Boston is in north latitude 42° 21′; Portland is in latitude 1° 15′ north of Boston; and Charleston is in latitude

10° 40′ south of Portland. What is the latitude of Charleston? Ans. 32° 56′ north.

11. If 1 cubic foot of anthracite coal weighs 54 pounds, how many cubic feet of space are required to stow 2 tons of 2000 pounds each? Ans. $74\frac{2}{27}$ cubic feet.

12. Two engineers, A and B, surveyed a certain house-lot. A made its contents 3R. 18p. 0yd. 6ft. 64sq. in., but B made its contents 3R. 17p. 30yd. 8ft. 100sq. in. How much did the one differ from the other?

13. The products of the industry of 250,000 persons in Massachusetts, during the year 1855, amounted to $ 295,300,000. What was the average amount to each individual, and how much was added to the capital of the State, if one fourth of the whole amount was saved? Ans. $ 1,181.20 to each individual; $ 73,825,000 added to the capital.

14. The capacity of a certain cistern is 216 cubic feet; how many hogsheads of water will it contain?

15. What day of the month and what day of the year is the second Monday of May, in a common year commencing on Thursday? Ans. 11th day of May; 131st day of the year.

16. Purchased 18T. 17cwt. 3qr. 20lb. of copperas, at 4 cents per pound. I sold 4T. 6cwt. 1qr. 14lb. at 5 cents per pound, and 7T. 1cwt. 3qr. 10lb. at 6 cents per pound. Moses Atwood purchased one fourth of the remainder at 6 cents per pound. One half of what then remained I sold to J. Gale at 10 cents per pound. The remainder I sold to J. Smith at 12 cents per pound; but he has become a bankrupt, and I lose half my debt. What have I gained by my purchase? Ans. $ 894.07½.

17. The distance between Boston and San Francisco is 2691 miles. If Nathan Swift of San Francisco and Oliver Fleet of Boston, on Thursday, the first day of January, 1857, set out to meet each other, Swift travelling 3 miles 7 furlongs 29 rods 15 feet per hour, and Fleet 5 miles 10 rods and 1½ feet per hour, both travelling 6½ hours per day, commencing at 8 o'clock, A. M., provided they rest on the Sabbath, in what year and month, and on what day of the month, and at what time of the day, will they meet, and how far will each have travelled?

Ans. On Monday, February 23, 1857, 2h. 30m. P. M. Swift, 1186m. 4fur. 22rd. 13ft. 6in.; Fleet, 1504m. 3fur. 17rd. 3ft.

EXAMPLES BY ANALYSIS.

1. If 7 pairs of shoes cost $ 8.75, what will one pair cost? what will 20 pairs cost? Ans. $ 25.00.

2. If 5 tons of hay cost $ 85, what will 1 ton cost? what will 17 tons cost? Ans. $ 289.00.

3. When $ 0.75 are paid for 3gal. of molasses, what is the value of 1gal.? What cost 37 gal.?

4. Gave $ 1.92 for 4lb. of tea; what cost 1lb.? what cost 37lb.? Ans. $ 17.76.

5. For 12lb. of rice I paid $ 1.08; what was paid for 1lb.; and what must I give for 25lb.? Ans. $ 2.25.

6. Gave S. Smith $ 63.00 for 9 tubs of butter; what was the cost of 1 tub? What cost 27 tubs? Ans. $ 189.00.

7. T. Swan can walk 20 miles in 5 hours; how far can he walk in 1 hour? How long would it take him to walk from Bradford to Boston, the distance being in a straight line 28 miles?

8. If a hungry boy would eat 49 crackers in 1 week, how many would he eat in 1 day? how many would be sufficient to last him 19 days? Ans. 133 crackers.

9. Gave $ 20 for 5 barrels of flour; what cost 1 barrel? what cost 40 barrels? Ans. $ 160.00.

10. For 3lb. of lard there were paid 36 cents; what was the cost of 37lb.?

11. Paid F. Johnson 72 cents for 9 nutmegs; how many cents were paid for 1 nutmeg; and what should be charged for 37 nutmegs? Ans. $ 2.96.

12. Paid 2£. 17s. 5d. for 52lb. of sugar; what cost 1lb.? what cost 76lb.?

13. Paid 4£. 3s. 11d. for 76lb. of sugar; what cost 52lb.?

14. If a man walk 17m. 4fur. 28rd. in 6 days, how far will he walk in 100 days? Ans. 293m. 1fur.

15. If a farmer feed to his stock in 7 months 41bu. 3pk. 4qt. 1pt. of grain, how much is required for 1 month? how much for 7 years? Ans. 502bu. 2pk. 6qt.

16. A field containing 39A. 2R. 5p. 8yd. 6ft. 108in. will pasture 8 cows during the season. How large a field will pasture 1 cow? How large a field 72 cows?

17. If 4 casks of vinegar contain 63gal. 3qt., what are the contents of one cask? What are the contents of 37 casks? Ans. 589gal. 2qt. 1pt. 2gi.

18. When 5yd. 3qr. 1na. of cloth cost $ 4, how much cloth can be bought for $ 1? How much for $ 36?
Ans. 52yd. 1qr. 1na.

19. If 11T. 3cwt. 2qr. of hay be sufficient to keep 4 horses $7\frac{3}{11}$ months, how much will keep 1 horse the same time? How much 23 horses? Ans. 64T. 5cwt. 12lb. 8oz.

20. If 12 men can dig a certain ditch in 286 days 4h. 33m., how long will it require 1 man to do the same labor? How long 72 men? Ans. 47 days 16h. 45m. 30sec.

21. If 27yd. 1qr. of cloth be required to make 21 coats, how many yards will be required to make 11 coats?

22. If a train of cars move at the rate of 174m. 26rd. in 7 hours, how far will it move in 1 hour? How far in 10 hours? Ans. 248m. 5fur. 20rd.

23. If 4 cases of shoes, containing 60 pairs each, cost $ 192, what will 1 pair cost? What will 25 cases cost?

24. When 3A. 2R. 20rd. of land will buy 4 hogsheads of molasses, how much land will buy 1 hogshead? How much 30 hogsheads? Ans. 27A. 0R. 30rd.

25. If a man can travel 20deg. 49m. 5fur. 35rd. 5yd. 3in. in 9 weeks, how far would he travel in 1 week? How far in 90 weeks? Ans. 207deg. 13m. 1fur. 25rd. 5yd.

PROPERTIES OF NUMBERS.

DEFINITIONS.

162. An integer is a whole number; as 1, 7, 16.
All whole numbers are either *prime* or *composite*.

163. A *prime* number is a number which can be ex- divided only by itself and 1; as 1, 3, 5, 7, 11.

A *composite* number is a number which can be exactly divided by some number besides itself and 1; as 6, 9, 14, 18.

164. A factor of a number is such a number as will, by being taken an entire number of times, produce it; as, 3 is a factor of 9, and 4 a factor of 16.

165. A prime factor of a number is a prime number that will exactly divide it; thus the prime factors of 10 are the prime numbers 1, 2, and 5.

NOTE. — Unity or 1 is not generally regarded as a prime factor, since multiplying or dividing any number by 1 does not alter its value. It therefore will be omitted when speaking of the prime factors of numbers.

A composite factor of a number is a composite number that will exactly divide it; thus, 6 and 8 are composite factors of 48.

166. Numbers are prime to each other when they have no factor in common; thus, 4, 9, and 23 are prime to each other.

167. An aliquot part of a number is such a part as will exactly divide it; as, 1, 3, and 5 are aliquot parts of 15.

NOTE. — The aliquot parts of a number include all its factors, prime and composite.

An aliquant part of a number is such a part as will not exactly divide it; as 2, 4, 5, 7, and 8 are aliquant parts of 9.

168. The *reciprocal* of a number is the quotient arising from dividing 1 by the number; thus, the reciprocal of 2 is $\frac{1}{2}$.

169. The *power* of a number is the product obtained by taking the number a certain number of times as a factor; thus 25 is a power of 5.

NOTE. — When the number is taken once, it is called its first power; when taken twice, as a factor, the product is called its second power; and so on. The second power of a number is sometimes termed its *square*, and the third power, its *cube*.

170. The *exponent* of a power is a figure written at the right of a number, and a little above it, to show how many times it is taken as a factor; thus, in the expression 4^2, the exponent is the 2, and the whole is read 4 second power; and in 7^3, it is the 3, and the whole read 7 third power.

NOTE. — The first power of a number being always the number itself, its exponent is not expressed.

PROPERTIES OF PRIME NUMBERS.

171. No direct process of detecting prime numbers has been discovered.

NOTE. — A few facts, such as are given below, if kept in mind, will aid somewhat in ascertaining whether a number is prime or not.

172. *The only even prime number is* 2; since all other even numbers, as 4, 6, 8, and 10, it is evident, can be exactly divided by 2, and therefore must be composite.

173. *The only prime number having* 5 *for a unit or right-hand figure is* 5; since every other whole number thus terminating, as 15, 25, 35, and 45, can be exactly divided by 5, and therefore must be composite.

174. *Every prime number, except* 2 *and* 5, *must have* 1, 3, 7, *or* 9 *for the right-hand figure;* since all other numbers are composite.

175. *Every prime number above* 3, *when divided by* 6, *must leave* 1 *or* 5 *for a remainder;* since every prime number above 3 is either 1 greater or 1 less than 6, or some exact number of times 6.

176. *In a series of odd numbers written in their proper or natural order, if beginning with* 3 *every* THIRD *number, with* 5 *every* FIFTH, *with* 7 *every* SEVENTH, *be cancelled, as composite, the remaining numbers, with* 2, *will be the prime numbers of the natural series.* Thus, in the series 1, 3, 5, 7, $\cancel{9}$, 11, 13, $\cancel{15}$, 17, 19, $\cancel{21}$, 23, $\cancel{25}$, $\cancel{27}$, 29, 31, $\cancel{33}$, $\cancel{35}$, 37, $\cancel{39}$, 41, 43, $\cancel{45}$, 47, $\cancel{49}$, every third number from the 3, every fifth from the 5, every seventh from the 7, every ninth from the 9, and so on, being cancelled, the remaining numbers, with 2, are all the prime numbers under 50.

NOTE 1. — In the series, every third number from the 3 contains that number as a factor; every fifth number from the 5, that number as a factor; and so on.

NOTE 2. — The whole number of prime numbers from 1 to 100,000 is 9,593. Although all of these, except 2 and 5, end in 1, 3, 7, or 9, there are, within the same range, no less than 30,409 composite numbers terminating with some one of the same figures.

177. All the prime numbers not larger than 4057 are included in the following

TABLE OF PRIME NUMBERS.

1	233	557	883	1249	1613	2017	2399	2801	3253	3643
2	239	563	887	1259	1619	2027	2411	2803	3257	3659
3	241	569	907	1277	1621	2029	2417	2819	3259	3671
5	251	571	911	1279	1627	2039	2423	2833	3271	3673
7	257	577	919	1283	1637	2053	2437	2837	3299	3677
11	263	587	929	1289	1657	2063	2441	2843	3301	3691
13	269	593	937	1291	1663	2069	2447	2851	3307	3697
17	271	599	941	1297	1667	2081	2459	2857	3313	3701
19	277	601	947	1301	1669	2083	2467	2861	3319	3709
23	281	607	953	1303	1693	2087	2473	2879	3323	3719
29	283	613	967	1307	1697	2089	2477	2887	3329	3727
31	293	617	971	1319	1699	2099	2503	2897	3331	3733
37	307	619	977	1321	1709	2111	2521	2903	3343	3739
41	311	631	983	1327	1721	2113	2531	2909	3347	3761
43	313	641	991	1361	1723	2129	2539	2917	3359	3767
47	317	643	997	1367	1733	2131	2543	2927	3361	3769
53	331	647	1009	1373	1741	2137	2549	2939	3371	3779
59	337	653	1013	1381	1747	2141	2551	2953	3373	3793
61	347	659	1019	1399	1753	2143	2557	2957	3389	3797
67	349	661	1021	1409	1759	2153	2579	2963	3391	3803
71	353	673	1031	1423	1777	2161	2591	2969	3407	3821
73	359	677	1033	1427	1783	2179	2593	2971	3413	3823
79	367	683	1039	1429	1787	2203	2609	2999	3433	3833
83	373	691	1049	1433	1789	2207	2617	3001	3449	3847
89	379	701	1051	1439	1801	2213	2621	3011	3457	3851
97	383	709	1061	1447	1811	2221	2633	3019	3461	3853
101	389	719	1063	1451	1823	2237	2647	3023	3463	3863
103	397	727	1069	1453	1831	2239	2657	3037	3467	3877
107	401	733	1087	1459	1847	2243	2659	3041	3469	3881
109	409	739	1091	1471	1861	2251	2663	3049	3491	3889
113	419	743	1093	1481	1867	2267	2671	3061	3499	3907
127	421	751	1097	1483	1871	2269	2677	3067	3511	3911
131	431	757	1103	1487	1873	2273	2683	3079	3517	3917
137	433	761	1109	1489	1877	2281	2687	3083	3527	3919
139	439	769	1117	1493	1879	2287	2689	3089	3529	3923
149	443	773	1123	1499	1889	2293	2693	3109	3533	3929
151	449	787	1129	1511	1901	2297	2699	3119	3539	3931
157	457	797	1151	1523	1907	2309	2707	3121	3541	3943
163	461	809	1153	1531	1913	2311	2211	3137	3547	3947
167	463	811	1163	1543	1931	2333	2713	3163	3557	3967
173	467	821	1171	1549	1933	2339	2719	3167	3559	3989
179	479	823	1181	1553	1949	2341	2729	3169	3571	4001
181	487	827	1187	1559	1951	2347	2731	3181	3581	4003
191	491	829	1193	1567	1973	2351	2741	3187	3583	4007
193	499	839	1201	1571	1979	2357	2749	3191	3593	4013
197	503	853	1213	1579	1987	2371	2753	3203	3607	4019
199	509	857	1217	1583	1993	2377	2767	3209	3613	4021
211	521	859	1223	1597	1997	2381	2777	3217	3617	4027
223	523	863	1229	1601	1999	2383	2789	3221	3623	4049
227	541	877	1231	1607	2003	2389	2791	3229	3631	4051
229	547	881	1237	1609	2011	2393	2797	3251	3637	4057

FACTORING.

178. FACTORING is the process of resolving a quantity into its factors.

179. *Every number that is not prime is composed of prime factors*, since all numbers are either prime or composite; and, if composite, can be separated into factors, which, if themselves composite, can be further separated into those that shall be prime.

180. To resolve a composite number into its prime factors.

Ex. 1. It is required to find the prime factors of 42.

Ans. 2, 3, 7.

OPERATION.

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

We divide by 2, the least prime number greater than 1, and obtain the quotient 21; and, since 21 is a composite number, we divide this by 3, and obtain for a quotient 7, which is a prime number. The several divisors and the last quotient, all being prime, constitute all the prime factors of 42, which, multiplied together, they equal. Hence

Divide the given number by any prime number that will exactly divide it, and the quotient, if a composite number, in the same manner; and so continue dividing, until a prime number is obtained for a quotient. The several divisors and the last quotient will be the prime factors required.

NOTE 1. — The composite factors of any number may be found by multiplying together two or more of its prime factors.

NOTE 2. — Such prime factors as two or more numbers may have alike, are termed prime factors common to them; and these may be readily determined after the numbers are resolved into their prime factors.

EXAMPLES.

2. What are the prime factors of 105? Ans. 3, 5, 7.
3. Resolve 220 into its prime factors.
4. What are the prime factors of 936?

   Ans. 2, 2, 2, 3, 3, 13.
5. What are the prime factors of 1953?
6. Resolve 12462 into its prime factors. Ans. 2, 3, 31, 67.
7. Resolve 19987 into its prime factors. Ans. 11, 23, 79.
8. What are the prime factors common to 225, 435, and 540? Ans. 3, 5.

9. What are the prime factors common to 960, 1568, and 5824?

10. What are the prime factors common to 2340, 11934, 12987, and 14859? Ans. 3, 3, 13.

11. A man has 105 apples, which he wishes to distribute into small parcels, each of equal numbers; what are the smallest whole numbers, greater than 1, into which they may be exactly divided? Ans. 3, 5, and 7.

DIVISIBILITY OF NUMBERS.

181. One number is said to be *divisible* by another, when the latter will divide the former without a remainder. Thus, 9 is divisible by 3.

182. *One number is divisible by another, when it contains all the prime factors of that number. Thus,* 12, *which contains all the factors of* 4, *is divisible by* 4.

183. *All even numbers, or such as terminate with* 0, 2, 4, 6, *or* 8, *are divisible by* 2, *since each of them contains* 2 *as a factor. Thus,* 10, 24, 36, 58, *are each divisible by* 2.

184. *All numbers which terminate with* 0 *or* 5 *are divisible by* 5, since each of them contains 5 as a factor. Thus, 20, 25, 50, are each divisible by 5.

185. *Every number is divisible by* 4, *or any other number that will exactly divide* 100, *when its two right-hand figures are divisible by the same.* For any figure on the left of the two right-hand figures must express one or more hundreds, and a factor of one hundred is a factor of any number of hundreds; so, if the sum exactly divides the units and tens of a number, the entire number will be divisible by it. Thus, 116 is divisible by 4; 140, by 20; 225, by 25; and 450, by 50.

186. *Every number is divisible by* 8, *or any other number that will exactly divide* 1000, *when its three right-hand figures are divisible by the same.* For any figure on the left of the three right-hand figures must express one or more thousands, and a factor of one thousand is a factor of any number of thousands; so, if the sum exactly divides the units, tens, and

hundreds of a number, the entire number will be divisible by it. Thus, 1824 is divisible by 8; 1840, by 40; 3375, by 125; 2750, by 250; and 4500, by 500.

187. *Every number the sum of whose digits 3 or 9 will exactly divide, is divisible by 3 or 9.* For 10, or any power of 10, less 1, gives a number, as 9, 99, 999, &c., which is divisible by 3 and by 9. Hence, any number of tens, hundreds, thousands, &c., less as many units, must be divisible by 3 and by 9; and if the excess of units denoted by the significant figures, in the aggregate, is likewise divisible by 3 and by 9, it follows that the entire number is thus divisible. For example, 7542 is a number, the sum of whose digits is divisible by 3 and by 9; and separated into tens, hundreds, and thousands, it is equal to $7000 + 500 + 40 + 2$. Now, $7000 = 7 \times 1000 = 7 \times (999 + 1) = 7 \times 999 + 7$; $500 = 5 \times 100 = 5 \times (99 + 1) = 5 \times 99 + 5$; and $40 = 4 \times 10 = 4 \times (9 + 1) = 4 \times 9 + 4$. Therefore, $7542 = 7 \times 999 + 5 \times 99 + 4 \times 9 + 7 + 5 + 4 + 2$. The remainders $7 + 5 + 4 + 2$, corresponding with the significant figures of the number, added together, equal 18, which sum being divisible by 3 and by 9, it is evident that 7542 is divisible in like manner.

NOTE.—Upon the property of 9 now explained depends the method of proving, by excess of nines, multiplication (Art. 63), and division (Art. 75).

The same method of proof may be resorted to in addition and in subtraction. Thus,

To prove Addition. Find the excess of nines in each number added, and then the excess of nines in the sum of these results; which, if the work be right, will equal the excess of nines in the answer.

To prove Subtraction. Find the excess of nines in the subtrahend, and also in the remainder, and then the excess of nines in the sum of these results; which, if the work be right, will equal the excess of nines in the minuend.

188. *Every number occupying four places, in which two like significant figures have two ciphers between them, is divisible by 7, 11, and 13. Thus,* 9009, 1001, 3003, 4004, &c., are each divisible by 7, 11, and 13.

189. *Every number is divisible by* 11, *in which the sum of the digits in the odd places is equal to the sum of the digits in the even places, or in which the difference of their sums can be exactly divided by* 11. Thus, 8305, in which $3 + 5 = 8 + 0$, and 628001, in which $2 + 0 + 1$ and $6 + 8 + 0$ differ by 11, are each divisible by 11.

190. *Every number divisible by two or more numbers, which are prime to each other, is divisible by their product.* For, being prime to each other, dividing by one of the numbers does not cancel the others as factors. Thus, 770, being divisible by 2, 5, and 7, which are prime to each other, is divisible by 70, their product.

191. *Every even number, the sum of whose digits* 6 *will exactly divide, is divisible by* 6. For being even, it is divisible by 2 (Art. 183), and its digits being divisible by twice 3, or 6, are evidently divisible by once 3, so that the number is also divisible by 3; and as the 2 and the 3 are prime to each other, the number is divisible by their product, or 6 (Art. 190). Thus, 174, 6312, are each divisible by 6.

192. *Every number terminating with* 0 *or* 5 *that* 3 *will exactly divide, is divisible by* 15, *and every number that* 9 *will exactly divide, is divisible by* 45. For, terminating with 0 or 5, it is divisible by 5 (Art. 184), and, as 3 and 9 are each prime to 5, if it can be exactly divided by 3 or by 9, it must be divisible by $5 \times 3 = 15$, or by $5 \times 9 = 45$. Thus, 75, which 3 will exactly divide, is divisible by 15; and 90, which 9 will exactly divide, is divisible by 45.

Divisors or Measures.

193. A *divisor* or measure of a number is any number that will divide it without a remainder. Thus, 3 is a divisor or measure of 6, and 5 a divisor or measure of 10.

194.° To find all the divisors or measures of a number.

Ex. 1. Required all the divisors of 60.

Ans. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

OPERATION.

$$60 = 2 \times 2 \times 3 \times 5.$$

$$\text{Divisors}\begin{cases} 1 \quad 2 \quad 4 = 2 \times 2 \\ 3 \quad 6 \quad 12 = 2 \times 2 \times 3 \\ 5 \quad 10 \quad 20 = 2 \times 2 \times 5 \\ 15 \quad 30 \quad 60 = 2 \times 2 \times 3 \times 5. \end{cases}$$

Ans. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

Resolving the number into its prime factors, we find $60 = 2 \times 2 \times 3 \times 5$. Now, any number is divisible by 1, and every composite number by its prime factors, and every product these factors can form. Now 1, being a divisor of every number, is a divisor of 60, and, since 2 enters twice as a factor into 60, it is evident that 2, and $4 = 2 \times 2$, are also divisors of 60. These divisors we arrange on a horizontal line, and determine the other divisors by multiplying those on this line by the factor 3, for the second line of divisors, by 5 for the third line, and by 3×5 for the fourth line, and thus obtain all the possible divisors of the given number.

The whole number of divisors is 12, which corresponds to the product arising from multiplying together the exponents, each increased by 1, of the different prime factors of 60; thus, of the different prime factors, since 2 enters twice, its exponent is $2, + 1 = 3$; 3 enters once, its exponent is $1, + 1 = 2$; 5 enters once, its exponent is $1, + 1 = 2$; and $3 \times 2 \times 2 = 12$, the number of divisors. The same holds true in all cases. Hence, in any composite number,

TO FIND THE NUMBER OF DIVISORS. — *Multiply together the exponents, each increased by* 1, *of the different prime factors of the given number, and the product will be the number of divisors required.* And

TO FIND THE SEVERAL DIVISORS. — *Form from the prime factors of the number all the products possible, and these factors (including* 1) *and products will be the divisors required.*

EXAMPLES.

2. What are the divisors of 72?
   Ans. 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.
3. Required the divisors of 105?
4. How many divisors has 1764? Ans. 27.
5. How many divisors has 3528?
6. How many divisors has 5880? Ans. 48.

COMMON DIVISORS OR MEASURES.

195. A common divisor or measure of two or more numbers is any number that will divide them without a remainder; thus, 2 is a common divisor of 4, 8, and 10.

196. A common divisor of two numbers is a divisor of their *sum*, and also of their *difference*. Thus, 6, a common divisor of 12 and 18, is a divisor of their sum, 30, and of their difference, 6.

197. A common divisor of the *remainder* and the *divisor* is a divisor of the dividend. Thus, in a division having 8 for a remainder, 16 for divisor, and 24 for dividend, 8, a common divisor of 8 and 16, is also a divisor of the 24.

198. To find all the divisors common to two or more numbers.

Ex. 1. Required all the common divisors of 45 and 135.

Ans. 1, 3, 5, 9, 15, and 45.

OPERATION.

$$45 = 3 \times 3 \times 5.$$
$$135 = 3 \times 3 \times 3 \times 5.$$

$$\text{Common Divisors}\begin{cases} 1 \quad 3 \quad 9 = 3 \times 3 \\ 5 \quad 15 \quad 45 = 3 \times 3 \times 5. \end{cases}$$

Ans. 1, 3, 5, 9, 15, and 45.

Resolving the given numbers into their prime factors, we find they have of these 3, 3, and 5 in common, and *these* COMMON *prime factors with* 1, *and all the products we are able to form from them* (Art. 194), give all the common divisors required. When only the *number* of common divisors is required, it may readily be found by *multiplying together the exponents, each increased by* 1, *of the different* COMMON *prime factors*. (Art. 194.)

EXAMPLES.

2. What are the common divisors of 51, 153, and 255?
3. Required the several common divisors of 180 and 360.
   Ans. 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90, and 180.
4. How many common divisors have 2025, 6075, and 8100?
   Ans. 15.
5. How many common divisors have 4500 and 9000?
   Ans. 36.

THE GREATEST COMMON DIVISOR OR MEASURE.

199. The greatest common divisor or measure of two or more numbers is the greatest number that will divide each of

them without a remainder. Thus, 4 is the greatest common divisor of 8, 12, and 16.

200. To find the greatest common divisor of two or more numbers.

Ex. 1. Required the greatest common divisor or measure of 24 and 88. Ans. 8.

FIRST OPERATION.

$$24 = 2 \times 2 \times 2 \times 3$$
$$88 = 2 \times 2 \times 2 \times 11.$$
$$2 \times 2 \times 2 = 8. \quad \text{Ans.}$$

Resolving the numbers into their prime factors, thus, $24 = 2 \times 2 \times 2 \times 3$, and $88 = 2 \times 2 \times 2 \times 11 = 88$, we find the factors $2 \times 2 \times 2$ are common to both. Since only these common factors, or the product of two or more of such factors, will exactly divide both numbers, it follows that *the product of all their common prime factors must be the greatest factor that will exactly divide both of them.* Therefore, $2 \times 2 \times 2 = 8$, the greatest common divisor required.

The same result may be obtained by a sort of trial process, as by the second operation.

SECOND OPERATION.

```
24)88(3
   72
   --
   16)24(1
      16
      --
       8)16(2
         16
         --
```

It is evident, since 24 cannot be exactly divided by a number greater than itself, if it will also exactly divide 88, it will be the *greatest common divisor* sought. But, on trial, we find 24 will not exactly divide 88, there being a remainder, 16. Therefore 24 is not a common divisor of the two numbers.

We know that a common divisor of 16 and 24 will, also, be a common divisor of 88 (Art. 197). We next try to find that divisor. It cannot be greater than 16. But 16 will not exactly divide 24, there being a remainder, 8; therefore 16 is not the greatest common divisor.

As before, the common divisor of 8 and 16 will be the common divisor of 24 and 88 (Art. 197); we make trial to find that divisor, knowing that it cannot be greater than 8, and find 8 will exactly divide 16. Therefore 8 is the greatest common divisor required.

THIRD OPERATION.

```
24)88(3
   72
   --
   1̸6̸
    8)24(3
      24
      --
```

The last method may be often contracted, if there should be observed to be any prime factor in a remainder which is not common to the preceding divisor, by canceling said factor. Thus, in the third operation, the factor 2 being found in the remainder 16 once more than in the divisor 24, we cancel one 2 from 16, and, having left the composite factor 8, we divide 24 by that factor. There being no remainder, 8 is the greatest common divisor, as before obtained.

RULE 1. — *Resolve the given numbers into their prime factors. The product of all the factors common to the several numbers will be the greatest common divisor.* Or,

RULE 2. — *Divide the greater number by the less, and if there be a remainder divide the preceding divisor by it, and so continue dividing until nothing remains. The last divisor will be the greatest common divisor.*

NOTE. — When the greatest common divisor is required of *more* than *two* numbers, find it of two of them, and then of that common divisor and of one of the other numbers, and so on for all the given numbers. The last common divisor will be the greatest common divisor required.

Another method is to *divide the numbers by any factor common to them all; and so continue to divide till there are no longer any common factors; and the product of all the common factors will be the greatest common divisor required.*

EXAMPLES.

2. What is the greatest common divisor of 56 and 168?

3. What is the greatest common divisor of 96 and 128?

4. What is the greatest common measure of 57 and 285? Ans. 57.

5. What is the greatest common measure of 169 and 175?

6. What is the greatest common measure of 175 and 455? Ans. 35.

7. What is the greatest common divisor of 169 and 866? Ans. 1.

8. What is the greatest common measure of 47 and 478? Ans. 1.

9. What is the greatest common measure of 84 and 1068? Ans. 12.

10. What is the greatest common divisor of 75 and 165? Ans. 15.

11. What is the greatest common measure of 78, 234, and 468? Ans. 78.

12. I have three fields; one containing 16 acres; the second, 20 acres; and the third, 24 acres. Required the largest-sized lots, containing each an exact number of acres, into which the whole can be divided. Ans. 4 acre lots.

13. A farmer has 12 bushels of oats, 18 bushels of rye, 24 bushels of corn, and 30 bushels of wheat. Required the largest bins, of uniform size, and containing an exact number of bushels, into which the whole can be put, each kind by itself, and all the bins be full.

LEAST COMMON MULTIPLE.

201. A *common multiple* of two or more numbers is a number that can be divided by each of them without a remainder; thus, 14 is a common multiple of 2 and 7.

The *least common multiple* of two or more numbers is the *least* number that can be divided by each of them without a remainder; thus, 12 is the least common multiple of 4 and 6.

202. A *multiple* of a number contains all the prime factors of that number; the *common multiple* of two or more numbers contains all the prime factors of each of the numbers; and the *least common multiple* of two or more numbers contains only each prime factor taken the greatest number of times it is found in any of the several numbers. Hence,

1. The least common multiple of two or more numbers must be the *least* number that will contain all the prime factors of them, and none others.

2. The least common multiple of two or more numbers, which are prime to each other, must equal their product.

3. The least common multiple of two or more numbers must equal the product of their greatest common divisor, by the factors of each number not common to all the numbers.

4. The least common multiple of two or more numbers, divided by any one of them, must equal the product of those factors of the others not common to the divisor.

203. To find the least common multiple of two or more numbers.

Ex. 1. What is the least common multiple of 8, 16, 24, 32, 44. Ans. 1056.

FIRST OPERATION.

$$\begin{aligned} 8 &= 2 \times 2 \times 2 \\ 16 &= 2 \times 2 \times 2 \times 2 \\ 24 &= 2 \times 2 \times 2 \times 3 \\ 32 &= 2 \times 2 \times 2 \times 2 \times 2 \\ 44 &= 2 \times 2 \times 11 \end{aligned}$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 11 = 1056 \text{ Ans.}$$

Resolving the numbers into their prime factors, we find their different prime factors to be 2, 3, and 11. The greatest number of times the 2 occurs as a factor in any of the given numbers is 5 times; the greatest num-

ber of times 3 occurs in any of the numbers is once; and the greatest number of times the 11 occurs in any of the numbers is once. Hence, 2, 2, 2, 2, 2, 3, and 11 must be all the prime factors necessary in composing 8, 16, 24, 32, and 44; and consequently, 1056, the product of these factors, is the least common multiple required (Art. 202).

SECOND OPERATION.

$$\begin{array}{r|rrrrr}
2) & 8 & 16 & 24 & 32 & 44 \\ \hline
2) & 4 & 8 & 12 & 16 & 22 \\ \hline
2) & 2 & 4 & 6 & 8 & 11 \\ \hline
2) & 1 & 2 & 3 & 4 & 11 \\ \hline
 & 1 & 1 & 3 & 2 & 11
\end{array}$$

$2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 11 = 1056$ Ans.

Having arranged the numbers on a horizontal line, we divide by 2, a prime number that will divide two or more of them without a remainder, and write the quotients in a line below; and we continue to divide by a prime number as before, till the divisor and remainders are all prime to each other. Then, these, since they include all the factors necessary to form the given numbers and no others, we multiply together for the required least common multiple, and obtain 1056, as before.

The least common multiple of two or more numbers may be found generally by a process much shorter than either of the above methods, by canceling any number that is a factor of any other of the given numbers, and also by dividing the numbers by such a composite number as may be observed to be their common or greatest common divisor.

THIRD OPERATION.

$$\begin{array}{rrrrr}
4)\ \cancel{8} & \cancel{16} & 24 & 32 & 44 \\ \hline
 & & 2)\ 6 & 8 & 11 \\ \hline
 & & 3 & 4 & 11
\end{array}$$

$4 \times 2 \times 3 \times 4 \times 11 = 1056$ Ans.

Thus, in the third operation, 8 being a factor of several of the numbers, and 16 being a factor of one other number, we cancel them; and observing that 4 is the greatest common divisor of the remaining numbers, we divide them by it. We next divide by 2, as in the second operation. The numbers in the lower line then being prime to each other, we multiply them and the divisors together, and obtain 1056 as the least common multiple.

FOURTH OPERATION.

$$\begin{array}{rrr|rr}
\cancel{8} & \cancel{16} & 24 & 32 & 44 \\ \cline{4-5}
 & & & 4 & 11
\end{array}$$

$24 \times 4 \times 11 = 1056$ Ans.

The fourth operation exhibits a process yet more contracted. The 8 and 16 being factors each of one or more of the other numbers, we cancel them, as in the third operation. Of the remaining numbers we cut off 24 by a short vertical line from the rest as a factor of the least common multiple sought. We then strike out of the two remaining numbers the largest factor each has in common with the 24, by dividing each of them by the greatest common divisor

between it and 24, and write the result beneath. The numbers in the lower line having no factor in common, we carry the process no further. The continued product of the number cut off by the numbers in the lower line gives 1056, the least common multiple, as by the other methods. In this instance we cut off the 24, but either the 32 could have been separated from the rest, or the 44 cut off, and the needless factors striken out with like result. If, however, we had cut off the 44, the numbers placed in the second line would have contained factors common to each other, so that it would have been necessary in that line to have cut off and stricken out factors as before. The reason for this abridged process is, that by the separating off, and by the striking out of factors, we get rid, in an expeditious way, of the factors not required to form the least common multiple sought.

RULE 1. — *Resolve the given numbers into their prime factors. The product of these factors, taking each factor only the greatest number of times it occurs in any of the numbers, will be the least common multiple.* Or,

RULE 2. — *Having arranged the numbers on a horizontal line, cancel such of them as are factors of any of the others, and separate some convenient one from the rest. Reject from each of the numbers remaining the greatest factor common to it and that number, and write the result in a line below. Should there be in the second line numbers having factors in common, proceed as before; and so continue until the numbers written below are prime to each other. The continued product of the number or numbers separated from the others with those in the last line will be the least common multiple.*

NOTE 1. — Some give a preference to the following rule for finding the least common multiple: *Having arranged the numbers on a horizontal line, divide by such a prime number as will exactly divide two or more of them, and write the quotients and undivided numbers in a line beneath. So continue to divide until the quotients shall be prime to each other. Then the product of the divisors and the numbers of the last line will be the least common multiple.*

NOTE 2. — The least common multiple of two or more numbers that are prime to each other is found by multiplying them together (Art. 202).

NOTE 3. — When a single number alone is prime to all the rest, it may be separated off, and used only as a factor of the least common multiple sought.

NOTE 4. — When the least common multiple of several numbers, and all the numbers except one, which is prime to the others, are given, to find the unknown number, divide the least common multiple given by that of the known numbers (Art. 202).

EXAMPLES.

2. What is the least common multiple of 3, 13, 37, and 91.

3. What is the least common multiple of 9, 14, 30, 35, and 47? Ans. 29610.

OPERATION.

$$\begin{array}{l}
\underline{9 \quad 14 \quad 30}\,|\,35 \quad 47. \\
\underline{9 \quad \cancel{2}}\,|\;6 \\
3
\end{array}$$

$47 \times 35 \times 6 \times 3 = 29610$ Ans.

4. What is the least common multiple of 6, 8, 10, 18, 20, and 24?

5. What is the least common multiple of 14, 19, 38, and 57? Ans. 798.

6. What is the least common multiple of 20, 36, 48, and 50? Ans. 3600.

7. What is the least common multiple of 15, 25, 35, 45, and 100? Ans. 6300.

8. What is the least common multiple of 100, 200, 300, 400, and 575? Ans. 27600.

9. The least common multiple of 1, 2, 3, 4, 5, 6, 8, 9, and one other number prime to them, is 2520. What is that other number? Ans. 7.

10. What is the least common multiple of 18, 24, 36, 126, 20, and 48?

11. I have four different measures; the first contains 4 quarts, the second 6 quarts, the third 10 quarts, and the fourth 12 quarts. How large is a vessel, that may be filled by each one of these, taken a certain number of times full? Ans. 60 quarts.

12. What is the smallest sum of money with which I can purchase a number of oxen at \$ 50 each, cows at \$ 40 each, or horses at \$ 75 each? Ans. \$ 600.

MISCELLANEOUS EXAMPLES.

1. How many times does 7 occur as a factor of 6174? Ans. 3 times.

2. Required the largest prime factor of 5775.

3. Required the largest composite factor of 19929. Ans. 6643.

4. Required the quotients of 2338 divided by its two prime factors next larger than 1. Ans. 1169; 334.

5. Required all the prime numbers that will divide 17385 without a remainder.

6.° A farmer has 3000 bushels of grain; which are the three smallest-sized bags, and the three largest-sized bins, holding an exact number of bushels, that will each measure the same without a remainder?

Ans. Bags of 1, 2, or 3 bushels each; and bins of 1500, 1000, or 750 bushels each.

7. A teacher having a school consisting of 152 ladies and 136 gentlemen, divided it in such a manner that each class of ladies equalled each class of gentlemen, and the classes were the largest the school would admit of, and have them all of the same size. Required the number of classes, and the number in each class.

Ans. 19 classes of ladies, 17 classes of gentlemen, and 8 pupils in a class.

8. At noon the second, minute, and hour hand of a clock are together; how long after will they be again, for the first time, in the same position?

9. J. Porter has a four-sided garden, the first side of which is 348 feet in length; the second, 372 feet; the third, 444 feet; and the fourth, 492 feet. Required the length of the longest rails that can be used in fencing it, allowing the end of each rail to lap by the other 9 inches, and all the panels to be of equal length; also, the number of rails, if 5 rails be allowed to each panel. Ans. Length 12ft. 9in.; and 690 rails.

10. L. Ford has 5 pieces of land, the first containing 3A. 2R. 1p.; the second, 5A. 3R. 15p.; the third, 8A. 29p.; the fourth, 12A. 3R. 17p.; and the fifth, 15A. 31p. Required the largest sized house-lots, containing each an exact number of square rods, into which the whole can be divided.

Ans. 1A. 27p. each.

11.° What three numbers between 30 and 140 have 12 for their greatest common divisor, and 2772 for their least common multiple. Ans. 36, 84, and 132.

12. Four men, A, B, C, and D, are engaged in making regular excursions into the country, between which each stays at home just 1 day; and A is always absent exactly 3 days, B 5 days, and C and D 7 days. Provided they all start off on the same day, how many days must elapse before they can all be at home again on the same day? Ans. 23 days.

COMMON FRACTIONS.

204. A FRACTION is an expression denoting one or more equal *parts* of a unit.

205. *A fractional unit* is one of the equal parts into which the whole thing or integral unit has been divided. Thus halves, thirds, &c., being equal parts of integral units or whole things, are fractional units.

206. *The unit of a fraction* is the unit or whole thing from which its fractional parts have been derived.

207. A COMMON FRACTION is expressed by two numbers one above the other, with a line between them.

208. The number below the line is called the *denominator*. It shows into how many parts the whole number has been divided. It gives *name* to the fraction and value to the fractional unit. Thus, in the expression $\frac{2}{7}$, the denominator is 7, indicating that the unit of the fraction has been divided into 7 equal parts, and that the value of the fractional unit is one seventh.

The number above the line is called the *numerator*. It shows how many parts have been taken, or *numbers* the fractional units expressed by the fraction. Thus, in the expression $\frac{2}{7}$, the numerator is 2, indicating that the fractional unit, which is one seventh, has been taken 2 times.

209. The *terms* of a fraction are its numerator and denominator. Thus, the terms of the fraction $\frac{2}{3}$ are the numerator 2 and the denominator 3.

210. A *proper* fraction is one whose numerator is *less* than the denominator; as $\frac{3}{5}$, $\frac{4}{3}$, $\frac{7}{9}$.

211. An *improper* fraction is one whose numerator is equal to, or greater than, the denominator; as $\frac{7}{7}$, $\frac{11}{10}$, $\frac{15}{8}$.

212. A *mixed* number is a whole number with a fraction; as $3\frac{1}{2}$, $16\frac{1}{3}$, $90\frac{7}{8}$.

213. A *simple* or *single* fraction has but one numerator and one denominator. It may be either proper or improper; as $\frac{7}{9}$, $\frac{10}{11}$, $\frac{13}{12}$.

214. A *compound* fraction is a fraction of a fraction, or two or more fractions connected by the word *of;* as $\frac{3}{4}$ of $\frac{1}{3}$ of $\frac{7}{10}$, $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{3}$.

215. A *complex* fraction is a fraction having a fraction or a mixed number for its numerator or denominator, or both; as $\frac{\frac{1}{4}}{\frac{3}{7}}, \frac{7}{4\frac{1}{5}}, \frac{3\frac{1}{6}}{13}, \frac{2\frac{1}{3}}{4\frac{1}{4}}$.

216. *A fraction is an expression of division; the numerator answering to the dividend, and the denominator to the divisor* (Art. 67); *and the value of a fraction is the quotient arising from the division of the numerator by the denominator* (Art. 80). Thus, in the fraction $\frac{15}{7}$, the numerator 15 is the dividend, the denominator 7 is the divisor, and the value expressed $2\frac{1}{7}$, or the quotient arising from the division of the 15 by the 7.

217. Since a fraction is an expression of division, it follows,

1. *That, if the numerator be multiplied, or the denominator be divided, by any number, the fraction is multiplied by the same number* (Art. 81).

2. *That, if the numerator be divided, or the denominator multiplied, by any number, the fraction is divided by the same number* (Art. 82).

3. *That, if the numerator and denominator be both multiplied, or both divided, by the same number, the fraction will not be changed in value* (Art. 83).

REDUCTION OF COMMON FRACTIONS.

218. Reduction of fractions is the process of changing their *form* of expression without altering their value.

219. A fraction is in its lowest terms, when its numerator and denominator are prime to each other (Art. 166).

220. To reduce a fraction to its lowest terms.

Ex. 1. Reduce $\frac{16}{48}$ to its lowest terms. Ans. $\frac{1}{3}$.

FIRST OPERATION.

$4\,)\,\frac{16}{48} = \frac{4}{12}$

$4\,)\,\frac{4}{12} = \frac{1}{3}$ Ans.

By dividing both terms of the fraction by 4, a factor common to them both, it is reduced to $\frac{4}{12}$. Dividing both terms of $\frac{4}{12}$ by 4, a factor common to them both, it is re-

duced to $\frac{1}{3}$. Now, as 1 and 3 are prime to each other, the fraction $\frac{1}{3}$ is in its lowest terms.

SECOND OPERATION.

$16\ )\ \frac{16}{48} = \frac{1}{3}$ Ans.

The same result is often more readily obtained by dividing the terms of the fraction by their greatest common divisor, as by the second operation.

Since dividing the numerator and denominator of a fraction by the same number, or cancelling equal factors in both, changes only the form of the fraction, while the value expressed remains unchanged (Art. 217).

RULE. — *Divide the numerator and denominator by any number greater than* 1 *that will divide them both without a remainder, and thus proceed until they are prime to each other.* Or,

Divide both the numerator and denominator by their greatest common divisor.

EXAMPLES.

2. Reduce $\frac{16}{84}$ to its lowest terms. Ans. $\frac{4}{21}$.
3. Reduce $\frac{24}{32}$ to its lowest terms. Ans. $\frac{3}{4}$.
4. Reduce $\frac{36}{84}$ to its lowest terms. Ans. $\frac{3}{7}$.
5. Reduce $\frac{14}{108}$ to its lowest terms.
6. Reduce $\frac{15}{175}$ to its lowest terms. Ans. $\frac{3}{35}$.
7. Reduce $\frac{162}{844}$ to its lowest terms. Ans. $\frac{81}{422}$.
8. Reduce $\frac{617}{4319}$ to its lowest terms.
9. Reduce $\frac{416}{875}$ to its lowest terms. Ans. $\frac{416}{875}$.
10. Reduce $\frac{811}{1116}$ to its lowest terms. Ans. $\frac{811}{1116}$.
11. Reduce $\frac{615}{915}$ to its lowest terms.
12. Reduce $\frac{816}{1018}$ to its lowest terms. Ans. $\frac{408}{509}$.
13. Reduce $\frac{123}{456}$ to its lowest terms. Ans. $\frac{41}{152}$.

221. To reduce an improper fraction to an equivalent whole or mixed number.

Ex. 1. How many yards in $\frac{117}{19}$ of a yard? Ans. $6\frac{3}{19}$.

OPERATION.

$$\begin{array}{r} 19\ )\ 117\ (\ 6\frac{3}{19} \text{ Ans.} \\ 114 \\ \hline 3 \end{array}$$

Since 19 nineteenths make one yard, it is evident there will be as many yards in 117 nineteenths as 19 is contained times in 117, which is $6\frac{3}{19}$ times. Therefore, $6\frac{3}{19}$ yards is the answer required.

RULE. — *Divide the numerator by the denominator.*

NOTE. — Should there a remainder occur, write it over the denominator, and make this fraction a part of the answer.

EXAMPLES.

2. Reduce $\frac{167}{15}$ to a mixed number. Ans. $11\frac{2}{15}$.
3. Reduce $\frac{1631}{116}$ to a mixed number. Ans. $14\frac{7}{116}$.
4. Reduce $\frac{131}{17}$ to a mixed number.
5. Reduce $\frac{647}{161}$ to a mixed number. Ans. $3\frac{164}{161}$.
6. Change $\frac{1000}{9}$ to a mixed number. Ans. $111\frac{1}{9}$.
7. Change $\frac{4123}{45}$ to a mixed number Ans. $91\frac{28}{45}$.
8. Change $\frac{125}{1}$ to a whole number. Ans. 125.
9. Change $\frac{37}{37}$ to a whole number.

222. To reduce a whole or mixed number to an improper fraction.

Ex. 1. Reduce 19 to a fraction whose denominator shall be 7.

OPERATION.

$19 \times 7 = 133.$

133 sevenths $= \frac{133}{7}$, Ans.

Since there are 7 sevenths in 1 whole one, 19 whole ones = 133 sevenths = $\frac{133}{7}$.

2. Reduce $17\frac{3}{5}$ to an improper fraction? Ans. $\frac{88}{5}$.

OPERATION.

$$\begin{array}{r} 17\frac{3}{5} \\ 5 \\ \hline 85 \\ 3 \\ \hline 88 \end{array}$$

88 fifths $= \frac{88}{5}$, Ans.

Since there are 5 fifths in 1 whole one, in 17 whole ones there are 85 fifths, and adding 3 fifths for the fraction, we have $\frac{88}{5}$ as the equivalent of $17\frac{3}{5}$. Hence the

RULE. — *Multiply the whole number by the given denominator, and to the product add the numerator of the fractional part, if any; and write the result over the denominator.*

NOTE. — A whole number may be expressed in its simplest fractional form, by *taking it for a numerator with* 1 *for a denominator.* Thus, 4 may be written $\frac{4}{1}$, and read 4 ones.

EXAMPLES.

3. Reduce 15 to fourths. Ans. $\frac{60}{4}$.
4. Reduce $161\frac{1}{16}$ to sixteenths.
5. Reduce $171\frac{57}{59}$ to an improper fraction. Ans. $\frac{10146}{59}$.
6. Change 11 to a fractional form. Ans. $\frac{11}{1}$.
7. Change 100 to an improper fraction.

8. Change 5 to a fraction whose denominator shall be 17. Ans. $\frac{85}{17}$.
9. Reduce $98\frac{48}{97}$ to an improper fraction. Ans. $\frac{9554}{97}$.
10. Reduce $116\frac{31}{61}$ to an improper fraction. Ans. $\frac{7107}{61}$.
11. $718\frac{45}{97}$ equal how many ninety-sevenths? Ans. $\frac{69691}{97}$.
12. Reduce $100\frac{100}{199}$ to an improper fraction. Ans. $\frac{20000}{199}$.
13. Reduce 7 to an improper fraction.
14. Reduce 19 to a fraction whose denominator shall be 13. Ans. $\frac{247}{13}$.
15. $116\frac{1}{4}$ yards equal how many fourths of a yard? Ans. 465 fourths.

223. To reduce a compound fraction to a simple fraction.

Ex. 1. Reduce $\frac{3}{4}$ of $\frac{7}{8}$ to a simple fraction. Ans. $\frac{21}{32}$.

OPERATION.

$\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$, Ans.

By multiplying the denominator of $\frac{7}{8}$ by 4, the denominator of $\frac{3}{4}$, it is evident, we obtain $\frac{1}{4}$ of $\frac{7}{8} = \frac{7}{32}$, since the parts into which the number is divided are 4 times as many, and consequently only $\frac{1}{4}$ as large as before; and since $\frac{1}{4}$ of $\frac{7}{8} = \frac{7}{32}$, $\frac{3}{4}$ of $\frac{7}{8}$ will be 3 times $\frac{7}{32} = \frac{21}{32}$.

RULE. — *Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.*

NOTE 1. — All whole and mixed numbers in the compound fraction must be reduced to improper fractions, before multiplying.

NOTE 2. — When there are factors common to both numerator and denominator, they may be cancelled in the operation.

EXAMPLES.

2. Reduce $\frac{3}{7}$ of $\frac{7}{15}$ of $\frac{15}{29}$ of $\frac{29}{33}$ to a simple fraction. Ans. $\frac{1}{11}$.

OPERATION.

$$\frac{\overset{1}{\cancel{3}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{15}} \times \frac{\cancel{15}}{\cancel{29}} \times \frac{\cancel{29}}{\underset{11}{\cancel{33}}} = \frac{1}{11}, \text{ Ans.}$$

3. What is $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{11}{12}$? Ans. $\frac{770}{1728} = \frac{385}{864}$.
4. What is $\frac{6}{7}$ of $\frac{11}{15}$ of $\frac{3}{5}$ of $\frac{1}{18}$?
5. Reduce $\frac{4}{7}$ of $\frac{5}{8}$ of $\frac{3}{8}$ of $\frac{11}{13}$ to a simple fraction. Ans. $\frac{165}{1456}$.
6. What is the value of $\frac{6}{11}$ of $\frac{6}{7}$ of $\frac{1}{4}$ of 21? Ans. $\frac{756}{308} = 2\frac{5}{11}$.

7. What is the value of $\frac{7}{11}$ of $15\frac{7}{8}$ of $5\frac{7}{10}$ of 100?

Ans. $5758\frac{13}{44}$.

8. What is $\frac{3}{4}$ of $\frac{4}{7}$ of $\frac{11}{24}$?

9. What is the value of $\frac{7}{11}$ of $\frac{11}{25}$ of $\frac{25}{31}$ of $\$7\frac{3}{4}$?

Ans. $1.75.

10. What is the value of $\frac{4}{9}$ of $\frac{9}{17}$ of $\frac{17}{18}$ of $3\frac{3}{5}$ gallons?

Ans. $\frac{4}{5}$ gal.

11. What part of a ship is $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{3}{4}$?

12. What is the value of $\frac{4}{7}$ of $\frac{9}{20}$ of $\frac{15}{16}$ of $\frac{14}{17}$ of $34?

Ans. $ 6.75.

A COMMON DENOMINATOR.

224. Fractions have a common denominator when all their denominators are alike.

225. A common denominator of two or more fractions is a common multiple of their denominators; and their least common denominator is the least common multiple of their denominators.

226. To reduce fractions to a common denominator.

Ex. 1. Reduce $\frac{7}{8}$, $\frac{5}{12}$, and $\frac{11}{16}$ to other fractions of equal value, having a common denominator.

FIRST OPERATION.

$$\left.\begin{array}{rl} 7 \times 12 \times 16 = 1344 & \text{new numerator.} \\ 5 \times 8 \times 16 = 640 & \text{“} \quad \text{“} \\ 11 \times 8 \times 12 = 1056 & \text{“} \quad \text{“} \\ \hline 8 \times 12 \times 16 = 1536 & \text{common denominator.} \end{array}\right. \quad \left.\begin{array}{l} \frac{7}{8} = \frac{1344}{1536} \\ \frac{5}{12} = \frac{640}{1536} \\ \frac{11}{16} = \frac{1056}{1536} \end{array}\right\} \text{Ans.}$$

We first multiply the numerator of $\frac{7}{8}$ by the denominators 12 and 16, and obtain 1344 for a new numerator. We next multiply the numerator of $\frac{5}{12}$ by the denominators 8 and 16, and obtain 640 for a new numerator; and then we multiply the numerator of $\frac{11}{16}$ by the denominators 8 and 12, and obtain 1056 for a new numerator. Finally, we multiply all the denominators together for a *common denominator*, and write it under the several numerators, as in the operation.

By this process, since the numerator and denominator of each fraction are multiplied by the same numbers, their relation to each other is not changed, and the value of the fraction remains the same. (Art. 217.)

SECOND OPERATION.

		48	least common denominator.
$\cancel{8}$ 12 | 16	8	$6 \times 7 = 42$, new numerator.	$\frac{7}{8} = \frac{42}{48}$
3	12	$4 \times 5 = 20$, " "	$\frac{5}{12} = \frac{20}{48}$ Ans.
	16	$3 \times 11 = 33$, " "	$\frac{11}{16} = \frac{33}{48}$

$16 \times 3 = 48$, least common multiple, and least common denominator.

Having first obtained the least common multiple of all the denominators of the given fractions, we assume this to be their least common denominator. We then take such a part of this number, 48, as is expressed by each of the fractions separately for their respective new numerators. Thus, to get a new numerator for $\frac{7}{8}$, we take $\frac{7}{8}$ of 48, the least common denominator, by dividing it by 8, and multiplying the quotient 6 by 7. We proceed in like manner with each of the fractions, and write the numerators thus obtained over the least common denominator. In this process the value of each fraction remains unchanged, as both terms are multiplied by the same number. (Art. 217.)

The method used in the second operation, it will be perceived, expresses the fractions of the result in lower terms than that used in the first. On this account it is often to be preferred to the other.

RULE. — *Find the least common multiple of the denominators for the* LEAST COMMON *denominator.*

Divide the least common denominator by the denominator of each of the given fractions, and multiply the quotients by their respective numerators, for the new numerators. Or,

Multiply each numerator by all the denominators except its own, for the new numerators; and all the denominators together for A COMMON *denominator.*

NOTE 1. — Compound fractions must be reduced to simple ones, whole and mixed numbers to improper fractions, before finding a common denominator, and all to their lowest terms, before finding the least common denominator.

NOTE 2. — Fractions may sometimes be reduced to a common denominator most readily by multiplying both terms of one or more of them by such a number as will make all the denominators alike. Thus $\frac{1}{2}$ and $\frac{1}{4}$ may be brought to a common denominator simply by multiplying both terms of the $\frac{1}{2}$ by 2, and changing in that way its form to $\frac{2}{4}$.

NOTE 3. — Fractions may often be reduced to lower terms, without destroying their common denominator, by dividing all their numerators and denominators by a common divisor.

EXAMPLES.

Reduce the following fractions to their least common denominator: —

2. Reduce $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{8}$, and $\frac{5}{12}$. Ans. $\frac{54}{72}$, $\frac{40}{72}$, $\frac{63}{72}$, $\frac{30}{72}$.
3. Reduce $\frac{6}{11}$, $\frac{8}{15}$, $\frac{19}{20}$, and $\frac{5}{24}$. Ans. $\frac{720}{1320}$, $\frac{704}{1320}$, $\frac{1254}{1320}$, $\frac{275}{1320}$.

4. Reduce $\frac{1}{7}$, $\frac{3}{14}$, $\frac{8}{21}$, and $\frac{5}{28}$.

5. Reduce $\frac{16}{21}$, $\frac{4}{7}$, $\frac{11}{14}$, and $\frac{1}{2}$. Ans. $\frac{32}{42}$, $\frac{24}{42}$, $\frac{33}{42}$, $\frac{21}{42}$.

6. Reduce $\frac{15}{22}$, $\frac{3}{11}$, $\frac{4}{7}$, and $\frac{3}{4}$. Ans. $\frac{210}{308}$, $\frac{84}{308}$, $\frac{176}{308}$, $\frac{231}{308}$.

7. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

8. Reduce $\frac{6}{7}$, $\frac{4}{9}$, $\frac{5}{11}$, and $\frac{7}{11}$. Ans. $\frac{594}{693}$, $\frac{308}{693}$, $\frac{315}{693}$, $\frac{441}{693}$.

9. Reduce $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{6}$, and $3\frac{4}{5}$.

10. Reduce $\frac{5}{12}$, $\frac{3}{8}$, $\frac{4}{7}$, and $4\frac{3}{4}$. Ans. $\frac{70}{168}$, $\frac{63}{168}$, $\frac{96}{168}$, $\frac{798}{168}$.

11. Reduce $\frac{4}{7}$, $\frac{5}{14}$, $\frac{11}{21}$, and $\frac{1}{2}$. Ans. $\frac{24}{42}$, $\frac{15}{42}$, $\frac{22}{42}$, $\frac{21}{42}$.

12. Reduce $\frac{8}{9}$, $\frac{7}{12}$, $\frac{17}{18}$, and $\frac{2}{21}$. Ans. $\frac{224}{252}$, $\frac{147}{252}$, $\frac{238}{252}$, $\frac{24}{252}$.

13. Reduce $\frac{19}{20}$, $\frac{7}{30}$, $\frac{11}{40}$, and $\frac{1}{50}$. Ans. $\frac{570}{600}$, $\frac{140}{600}$, $\frac{165}{600}$, $\frac{12}{600}$.

14. Reduce $\frac{7}{8}$, $\frac{7}{12}$, $\frac{7}{16}$, and $\frac{7}{20}$. Ans. $\frac{210}{240}$, $\frac{140}{240}$, $\frac{105}{240}$, $\frac{84}{240}$.

15. Reduce $\frac{2}{9}$, 7, 8, and $5\frac{1}{7}$.

Reduce the following fractions to a common denominator: —

16. Reduce $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{8}$ to fractions having a common denominator. Ans. $\frac{80}{120}$, $\frac{96}{120}$, $\frac{105}{120}$.

17. Reduce $\frac{4}{7}$, $\frac{8}{9}$, and $\frac{3}{10}$. Ans. $\frac{360}{630}$, $\frac{560}{630}$, $\frac{189}{630}$.

18. Reduce $\frac{6}{11}$, $\frac{4}{7}$, and $\frac{8}{13}$. Ans. $\frac{546}{1001}$, $\frac{572}{1001}$, $\frac{616}{1001}$.

19. Reduce $\frac{7}{12}$, $\frac{4}{13}$, and $7\frac{3}{4}$.

20. Reduce $\frac{11}{17}$, $\frac{4}{9}$, and $\frac{4}{15}$. Ans. $\frac{1485}{2295}$, $\frac{1020}{2295}$, $\frac{612}{2295}$.

21. Reduce $\frac{8}{9}$, $\frac{4}{17}$, and $11\frac{7}{15}$. Ans. $\frac{2040}{2295}$, $\frac{540}{2295}$, $\frac{26316}{2295}$.

22. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{7}$, and 8. Ans. $\frac{21}{42}$, $\frac{28}{42}$, $\frac{24}{42}$, $\frac{336}{42}$.

23. Reduce $\frac{4}{9}$, $\frac{7}{11}$, and $\frac{2}{3}$ of $7\frac{5}{8}$. Ans. $\frac{528}{1188}$, $\frac{756}{1188}$, $\frac{6039}{1188}$.

24. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{6}$, and 17.

25. Reduce $\frac{11}{12}$, $\frac{4}{5}$ of 6, and $21\frac{1}{2}$. Ans. $\frac{110}{120}$, $\frac{576}{120}$, $\frac{2580}{120}$.

26. Reduce $\frac{6}{7}$, $\frac{4}{11}$, $\frac{5}{13}$, $\frac{4}{7}$, and $\frac{1}{2}$.
Ans. $\frac{12012}{14014}$, $\frac{5096}{14014}$, $\frac{5390}{14014}$, $\frac{8008}{14014}$, $\frac{7007}{14014}$.

27. Reduce $\frac{27}{169}$, $\frac{121}{611}$, and $\frac{7}{1728}$.
Ans. $\frac{28506816}{17843152}$, $\frac{37088064}{17843152}$, $\frac{722813}{17843152}$.

ADDITION OF COMMON FRACTIONS.

227. Addition of fractions is the process of finding the value of two or more fractions in one sum.

NOTE. — Only units of the same kind, whether integral or fractional, can be collected into one sum; if, therefore, the fractions to be added do not express the same fractional unit, they require to be brought to the same, by being reduced to a common or the least common denominator.

228. To add together two or more fractions.

Ex. 1. Add $\frac{3}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, and $\frac{11}{12}$ together. Ans. $\frac{26}{12} = 2\frac{1}{6}$.

OPERATION.

$$\tfrac{3}{12} + \tfrac{5}{12} + \tfrac{7}{12} + \tfrac{11}{12} = \tfrac{26}{12} = \tfrac{13}{6} = 2\tfrac{1}{6}.$$

These fractions all being *twelfths*, that is, having 12 for a common denominator, we add their numerators together, and write their sum, 26, over the common denominator, 12. Thus we obtain $\frac{26}{12}$, which, being reduced, $= 2\frac{1}{6}$, the sum required.

2. What is the sum of $\frac{7}{8}$, $\frac{5}{12}$, $\frac{11}{16}$, and $\frac{13}{20}$? Ans. $2\frac{151}{240}$.

OPERATION.

$$\begin{array}{l|lll}
4 & \cancel{8} & 12 & 16 \;|\; 20 \\
\hline
 & & 3 & 4
\end{array}$$

$$20 \times 4 \times 3 = 240$$

$$\begin{array}{r|l}
 & 240 = \text{least common denominator.} \\
8 & 30 \times 7 = 210 \\
12 & 20 \times 5 = 100 \\
16 & 15 \times 11 = 165 \\
20 & 12 \times 13 = 156
\end{array}
\Bigg\} \text{ new numerators.}$$

$$\begin{array}{ll}
\text{Sum of numerators,} & 631 \\
\text{Least com. denom.,} & 240
\end{array} = 2\tfrac{151}{240}, \text{ Ans.}$$

The given fractions not expressing the same kind of fractional unit, we reduce them to their least common denominator, and thus make the fractional parts all of the same kind. The fractions now all expressing *two-hundred-fortieths*, we add their numerators, and write the result, 631, over the least common denominator, 240, and obtain $\frac{631}{240} = 2\frac{151}{240}$, the answer required.

RULE. — *Reduce the fractions, if necessary, to a common, or the least common denominator, and write the sum of the numerators over their common denominator.*

NOTE 1. — Mixed numbers must be reduced to improper fractions, and compound fractions to simple fractions, and each fraction to its lowest terms, before attempting to obtain the common denominator.

NOTE 2. — In adding mixed numbers, the fractional parts may be added separately, and their sum added to the amount of the whole numbers.

EXAMPLES.

3. Add $\frac{5}{17}$, $\frac{6}{17}$, $\frac{9}{17}$, $\frac{11}{17}$, $\frac{14}{17}$, and $\frac{16}{17}$ together. Ans. $3\frac{10}{17}$.
4. Add $\frac{4}{23}$, $\frac{9}{23}$, $\frac{11}{23}$, $\frac{13}{23}$, and $\frac{19}{23}$ together. Ans. $2\frac{10}{23}$.
5. What is the sum of $\frac{4}{87}$, $\frac{17}{87}$, $\frac{29}{87}$, and $\frac{77}{87}$?
6. What is the sum of $\frac{11}{144}$, $\frac{19}{144}$, $\frac{29}{144}$, and $\frac{107}{144}$? Ans. $1\frac{11}{72}$.
7. What is the sum of $\frac{619}{631}$, $\frac{491}{631}$, $\frac{567}{631}$, and $\frac{11}{631}$? Ans. $2\frac{426}{631}$.
8. Add $\frac{1}{4}$, $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{5}{9}$ together.
9. Add $\frac{7}{11}$, $\frac{9}{22}$, $\frac{5}{3}$, and $\frac{1}{6}$ together. Ans. $2\frac{29}{33}$.
10. Add $\frac{3}{10}$, $\frac{5}{18}$, $\frac{11}{10}$, and $\frac{7}{8}$ together. Ans. $2\frac{199}{360}$.

11. Add $\frac{5}{17}$, $\frac{3}{34}$, $\frac{16}{136}$, and $\frac{1}{2}$ together. Ans. 1.
12. Add $\frac{14}{15}$, $\frac{17}{20}$, $\frac{19}{30}$, and $\frac{9}{10}$ together. Ans. $3\frac{19}{60}$.
13. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$ together. Ans. $1\frac{83}{140}$.
14. Add $\frac{8}{9}$, $\frac{3}{7}$, and $5\frac{3}{8}$ together. Ans. $6\frac{349}{504}$.
15. Add $\frac{4}{11}$, $\frac{7}{22}$, and $9\frac{3}{11}$ together.
16. Add $\frac{3}{4}$, $\frac{5}{6}$, and $4\frac{3}{4}$ together. Ans. $6\frac{1}{3}$.
17. Add $\frac{4}{5}$, $7\frac{1}{2}$, and $8\frac{3}{4}$ together. Ans. $17\frac{1}{20}$.
18. Add $\frac{7}{9}$, $3\frac{1}{4}$, and $5\frac{3}{7}$ together. Ans. $9\frac{115}{252}$.
19. Add $6\frac{3}{5}$, $7\frac{8}{9}$, and $4\frac{3}{8}$ together. Ans. $18\frac{311}{360}$.
20. What is the sum of $17\frac{2}{9}$, $14\frac{1}{2}$, and $13\frac{2}{7}$?
21. What is the sum of $16\frac{2}{3}$, $8\frac{7}{8}$, $9\frac{3}{5}$, $3\frac{1}{4}$, and $1\frac{7}{8}$?
Ans. $40\frac{4}{15}$.
22. What is the sum of $371\frac{11}{12}$, $614\frac{19}{20}$, and $81\frac{3}{4}$?
Ans. $1068\frac{37}{60}$.
23. Add $\frac{4}{7}$ of $18\frac{3}{11}$, and $\frac{11}{12}$ of $\frac{4}{9}$ of $6\frac{3}{11}$ together.
Ans. $12\frac{691}{693}$.
24. Add $\frac{5}{8}$ of 18, and $\frac{4}{11}$ of $\frac{11}{12}$ of $7\frac{4}{11}$ together.

229. To add any two fractions, whose numerators are alike.

Ex. 1. Add $\frac{1}{4}$ to $\frac{1}{5}$. Ans. $\frac{9}{20}$.

OPERATION.

Sum of the denominators, $5 + 4 = 9$
Product of the denominators, $4 \times 5 = 20$

We first find the sum of the denominators, which is 9, and then their product, which is 20; and the 9 being written as a numerator of a fraction, and the 20 as its denominator, the result, $\frac{9}{20}$, is the answer required. The reason of the operation is, that the process *reduces the fractions to a common denominator, and then adds their numerators.* Hence, to add two fractions whose numerators are a unit,

Write the sum of the given denominators over their product.

2. Add $\frac{3}{4}$ to $\frac{3}{5}$. Ans. $1\frac{7}{20}$.

OPERATION.

$$\frac{\text{Sum of the denominators} \times \text{by one of the numerators,}\ (4 + 5) \times 3}{\text{Product of the denominators,}\ 4 \times 5} = \frac{27}{20} = 1\tfrac{7}{20}, \text{ Ans.}$$

By multiplying the sum of the denominators by one of the numerators for a new numerator, and the denominators together for a new denominator, we reduce the fractions to a common denominator, and add their numerators, and thus obtain $\frac{27}{20} = 1\frac{7}{20}$, the answer required. Hence, to add fractions whose numerators are alike, and greater than a unit,

Write the product of the sum of the given denominators by one of the numerators over the product of the denominators.

EXAMPLES.

3. Add $\frac{1}{12}$ to $\frac{1}{4}$, $\frac{1}{3}$ to $\frac{1}{2}$, $\frac{1}{3}$ to $\frac{1}{3}$, $\frac{1}{3}$ to $\frac{1}{4}$, $\frac{1}{3}$ to $\frac{1}{5}$, $\frac{1}{3}$ to $\frac{1}{6}$, $\frac{1}{3}$ to $\frac{1}{7}$.
4. Add $\frac{1}{11}$ to $\frac{1}{2}$, $\frac{1}{11}$ to $\frac{1}{3}$, $\frac{1}{11}$ to $\frac{1}{4}$, $\frac{1}{11}$ to $\frac{1}{5}$, $\frac{1}{11}$ to $\frac{1}{6}$, $\frac{1}{11}$ to $\frac{1}{7}$.
5. Add $\frac{1}{10}$ to $\frac{1}{2}$, $\frac{1}{10}$ to $\frac{1}{3}$, $\frac{1}{10}$ to $\frac{1}{4}$, $\frac{1}{10}$ to $\frac{1}{5}$, $\frac{1}{10}$ to $\frac{1}{6}$, $\frac{1}{10}$ to $\frac{1}{7}$.
6. Add $\frac{1}{4}$ to $\frac{1}{2}$, $\frac{1}{4}$ to $\frac{1}{3}$, $\frac{1}{4}$ to $\frac{1}{4}$, $\frac{1}{4}$ to $\frac{1}{5}$, $\frac{1}{4}$ to $\frac{1}{6}$, $\frac{1}{4}$ to $\frac{1}{7}$, $\frac{1}{4}$ to $\frac{1}{8}$.
7. Add $\frac{1}{5}$ to $\frac{1}{6}$, $\frac{1}{5}$ to $\frac{1}{7}$, $\frac{1}{5}$ to $\frac{1}{8}$, $\frac{1}{5}$ to $\frac{1}{9}$, $\frac{1}{5}$ to $\frac{1}{2}$, $\frac{1}{5}$ to $\frac{1}{3}$, $\frac{1}{5}$ to $\frac{1}{4}$.
8. Add $\frac{1}{7}$ to $\frac{1}{2}$, $\frac{1}{7}$ to $\frac{1}{3}$, $\frac{1}{7}$ to $\frac{1}{4}$, $\frac{1}{7}$ to $\frac{1}{5}$, $\frac{1}{7}$ to $\frac{1}{6}$, $\frac{1}{7}$ to $\frac{1}{8}$, $\frac{1}{7}$ to $\frac{1}{9}$.
9. Add $\frac{1}{9}$ to $\frac{1}{2}$, $\frac{1}{9}$ to $\frac{1}{3}$, $\frac{1}{9}$ to $\frac{1}{4}$, $\frac{1}{9}$ to $\frac{1}{5}$, $\frac{1}{9}$ to $\frac{1}{6}$, $\frac{1}{9}$ to $\frac{1}{7}$, $\frac{1}{9}$ to $\frac{1}{8}$.
10. Add $\frac{8}{9}$ to $\frac{8}{11}$, $\frac{8}{9}$ to $\frac{8}{13}$, $\frac{8}{9}$ to $\frac{8}{15}$, $\frac{8}{9}$ to $\frac{8}{17}$, $\frac{8}{9}$ to $\frac{8}{19}$, $\frac{8}{9}$ to $\frac{8}{21}$.
11. Add $\frac{4}{5}$ to $\frac{4}{7}$, $\frac{4}{5}$ to $\frac{4}{9}$, $\frac{4}{5}$ to $\frac{4}{11}$, $\frac{2}{5}$ to $\frac{2}{7}$, $\frac{2}{5}$ to $\frac{2}{9}$, $\frac{2}{5}$ to $\frac{2}{11}$, $\frac{3}{5}$ to $\frac{3}{7}$.
12. Add $\frac{3}{5}$ to $\frac{3}{11}$, $\frac{3}{5}$ to $\frac{3}{5}$, $\frac{5}{6}$ to $\frac{5}{7}$, $\frac{5}{6}$ to $\frac{5}{8}$, $\frac{5}{6}$ to $\frac{5}{9}$, $\frac{5}{6}$ to $\frac{5}{11}$.
13. Add $\frac{6}{7}$ to $\frac{6}{8}$, $\frac{6}{7}$ to $\frac{6}{11}$, $\frac{6}{7}$ to $\frac{6}{13}$, $\frac{6}{11}$ to $\frac{6}{13}$, $\frac{6}{11}$ to $\frac{6}{17}$, $\frac{6}{11}$ to $\frac{6}{19}$.
14. Add $\frac{8}{9}$ to $\frac{8}{11}$, $\frac{8}{11}$ to $\frac{8}{11}$, $\frac{8}{11}$ to $\frac{8}{13}$, $\frac{8}{13}$ to $\frac{8}{15}$, $\frac{8}{13}$ to $\frac{8}{17}$, $\frac{3}{13}$ to $\frac{8}{19}$.
15. Add $\frac{9}{10}$ to $\frac{9}{11}$, $\frac{9}{10}$ to $\frac{9}{14}$, $\frac{9}{10}$ to $\frac{9}{16}$, $\frac{9}{10}$ to $\frac{9}{17}$, $\frac{9}{10}$ to $\frac{9}{18}$.

SUBTRACTION OF COMMON FRACTIONS.

230. SUBTRACTION of Fractions is the process of finding the difference between two fractions.

NOTE. — When the fractions express different fractional units, they require to be brought to those of the same kind before the subtraction can be performed.

To subtract one fraction from another.

Ex. 1. From $\frac{11}{12}$ take $\frac{5}{12}$. Ans. $\frac{6}{12} = \frac{1}{2}$.

OPERATION.

$$\frac{11}{12} - \frac{5}{12} = \frac{6}{12}.$$

The fractions both being *twelfths*, having 12 for a common denominator, we subtract the less numerator from the greater, and write the difference, 6, over the common denominator, 12. Thus, we have $\frac{6}{12}$ as the difference required.

2. From $\frac{10}{11}$ take $\frac{4}{7}$. Ans. $\frac{26}{77}$.

OPERATION.

$$\begin{array}{r|l}
 & 77 \text{ common denominator.} \\
\hline
11 & \left.\begin{array}{r} 7 \times 10 = 70 \\ 11 \times 4 = 44 \end{array}\right\} \text{new numerators.} \\
7 & \\
 & \ \overline{26} \quad \text{dif. of numerators.} \\
 & \ \overline{77} \quad \text{common denominator.}
\end{array}$$

The given fractions not expressing the same kind of fractional unit, we reduce them to a common denominator, and thus make the fractional parts all of the same kind. We next find the difference of the new numerators, which we write over the common denominator, and obtain $\frac{26}{77}$, *the answer* required.

RULE. — *Reduce the fractions, if necessary, to a common, or the least common denominator. Write the difference of the numerators over their common denominator.*

NOTE. — If the minuend or subtrahend, or both, are compound fractions, they must be reduced to simple ones.

EXAMPLES.

3. Subtract $\frac{6}{17}$ from $\frac{13}{17}$. Ans. $\frac{7}{17}$.
4. Subtract $\frac{4}{19}$ from $\frac{13}{19}$. Ans. $\frac{9}{19}$.
5. From $\frac{23}{37}$ take $\frac{8}{37}$.
6. From $\frac{19}{39}$ take $\frac{17}{39}$. Ans. $\frac{2}{39}$.
7. From $\frac{23}{29}$ take $\frac{18}{29}$. Ans. $\frac{5}{29}$.
8. From $\frac{29}{37}$ take $\frac{14}{37}$.
9. Subtract $\frac{14}{24}$ from $\frac{20}{24}$. Ans. $\frac{1}{4}$.
10. Subtract $\frac{16}{144}$ from $\frac{36}{144}$. Ans. $\frac{5}{36}$.
11. Subtract $\frac{19}{160}$ from $\frac{47}{160}$. Ans. $\frac{7}{40}$.
12. Subtract $\frac{327}{1728}$ from $\frac{596}{1728}$. Ans. $\frac{269}{1728}$.
13. Subtract $\frac{48}{1000}$ from $\frac{100}{1000}$. Ans. $\frac{13}{250}$.
14. From $\frac{12}{17}$ take $\frac{5}{34}$.
15. From $\frac{7}{10}$ take $\frac{5}{12}$. Ans. $\frac{17}{60}$.
16. From $\frac{11}{17}$ take $\frac{5}{19}$. Ans. $\frac{124}{323}$.
17. From $\frac{18}{19}$ take $\frac{3}{38}$. Ans. $\frac{33}{38}$.
18. From $\frac{16}{21}$ take $\frac{4}{11}$. Ans. $\frac{92}{231}$.
19. From $\frac{31}{50}$ take $\frac{3}{10}$.
20. From $\frac{17}{25}$ take $\frac{7}{20}$. Ans. $\frac{33}{100}$.
21. From $\frac{10}{27}$ take $\frac{5}{18}$. Ans. $\frac{5}{54}$.
22. From $\frac{3}{28}$ take $\frac{3}{56}$. Ans. $\frac{3}{56}$.
23. From $7\frac{3}{4}$ take $\frac{5}{7}$ of 9. Ans. $1\frac{9}{28}$.
24. What is the value of $\frac{3}{5}$ of $8\frac{3}{4}$ — $\frac{2}{3}$ of 5?
25. What is the value of $\frac{1}{4}$ of 3 — $\frac{1}{3}$ of 2? Ans. $\frac{1}{12}$.

231. To subtract a proper fraction or a mixed number from a whole number.

Ex. 1. From 7 take $3\frac{5}{8}$. Ans. $3\frac{3}{8}$.

OPERATION.
From 7
Take $3\frac{5}{8}$
Rem. $3\frac{3}{8}$

Since we have no fraction from which to subtract the $\frac{5}{8}$, we must add 1, or its equal, $\frac{8}{8}$, to the minuend, and say $\frac{5}{8}$ from $\frac{8}{8}$ leaves $\frac{3}{8}$. We write the $\frac{3}{8}$ below the line, and carry 1 to the 3 in the subtrahend, and subtract as in subtraction of simple whole numbers. The result will be obtained, if we

Subtract the number denoting the numerator from that denoting the denominator, under the remainder write the denominator, and adding one to the whole number in the subtrahend, subtract the sum from the minuend.

NOTE. — When the subtrahend is a mixed number, we may reduce it to an improper fraction, and change the whole number in the minuend to a fraction having the same denominator, and then proceed as in Art. 230.

EXAMPLES.

	2.	3.	4.	5.	6.	7.
From	32	16	671	385	16	18
Take	$5\frac{3}{8}$	$4\frac{3}{4}$	$0\frac{41}{50}$	$16\frac{17}{39}$	$0\frac{117}{237}$	$1\frac{3}{4}$
Rem.	$26\frac{5}{8}$	$11\frac{1}{4}$	$670\frac{9}{50}$	$368\frac{22}{39}$	$15\frac{120}{237}$	$16\frac{1}{4}$

	8.	9.	10.	11.	12.	13.
From	19	27	169	711	46	81
Take	$13\frac{3}{7}$	$8\frac{3}{7}$	$91\frac{14}{23}$	$30\frac{15}{79}$	$15\frac{8}{9}$	$49\frac{11}{19}$

232. To subtract one mixed number from another mixed number.

Ex. 1. From $8\frac{3}{7}$ take $4\frac{4}{5}$. Ans. $3\frac{22}{35}$.

FIRST OPERATION.

$$\begin{array}{lrl} \text{From} & 8\frac{3}{7} = & 8\frac{15}{35} \\ \text{Take} & 4\frac{4}{5} = & 4\frac{28}{35} \\ \hline \text{Rem.} & & 3\frac{22}{35} \end{array}$$

We first reduce the fractional parts to a common denominator, and obtain as their equivalents $\frac{15}{35}$ and $\frac{28}{35}$. Now, since we cannot take $\frac{28}{35}$ from $\frac{15}{35}$, we add 1, equal to $\frac{35}{35}$, to the $\frac{15}{35}$ in the minuend, and obtain $\frac{50}{35}$. From $\frac{50}{35}$ taking $\frac{28}{35}$, we have left $\frac{22}{35}$, which we write below the line, and carry 1 to the 4 in the subtrahend, and subtract from the 8 above as in subtraction of simple whole numbers.

SECOND OPERATION.

$$\begin{array}{lrl} \text{From} & 8\frac{3}{7} = \frac{59}{7} = & \frac{295}{35} \\ \text{Take} & 4\frac{4}{5} = \frac{24}{5} = & \frac{168}{35} \\ \hline \text{Rem.} & & \frac{127}{35} = 3\frac{22}{35} \end{array}$$

In this operation, we reduce the mixed numbers to improper fractions, and these fractions to a common denominator. We then subtract the less fraction from the greater, and, reducing the remainder to a mixed number, obtain $3\frac{22}{35}$, as before. Hence, in performing like examples, we may

Reduce the fractional parts, if necessary, to a common denominator, and subtract the fractional parts and the whole numbers separately. Increase the fractional part of the minuend, when otherwise it would be less than the subtrahend, before subtracting, by as many parts as it takes to make a unit of the fraction (Art. 208), *and carry* 1 *to the whole number of the subtrahend before subtracting it.* Or,

Reduce the mixed numbers to improper fractions, then to a common denominator, and subtract the less fraction from the greater.

EXAMPLES.

	2.	3.	4.	5.	6.	7.
	cwt.	Tuns.	$	lb.	oz.	Miles.
From	$18\frac{3}{8}$	$73\frac{4}{5}$	$67\frac{1}{3}$	$29\frac{18}{19}$	$144\frac{3}{4}$	$171\frac{15}{22}$
Take	$9\frac{8}{9}$	$16\frac{11}{12}$	$16\frac{2}{3}$	$15\frac{4}{9}$	$99\frac{11}{13}$	$91\frac{9}{10}$

	8.	9.	10.	11.	12.	13.
	Furlongs.	Rods.	Inches.	Feet.	Bushels.	Pecks.
From	$101\frac{7}{8}$	$165\frac{1}{2}$	$77\frac{11}{12}$	$84\frac{23}{24}$	$671\frac{111}{721}$	$17\frac{123}{457}$
Take	$93\frac{4}{5}$	$98\frac{7}{8}$	$19\frac{19}{20}$	$15\frac{11}{12}$	$183\frac{69}{83}$	$8\frac{234}{711}$

14. From a hogshead of wine there leaked out $7\frac{9}{11}$ gallons; what quantity remained? Ans. $55\frac{2}{11}$gal.

15. A man engaged to labor 30 days, but was absent $5\frac{7}{12}$ days; how many days did he work?

16. From 144 pounds of sugar there were taken at one time $17\frac{5}{8}$ pounds, and at another $28\frac{7}{12}$ pounds; what quantity remains? Ans. $97\frac{19}{24}$lb.

17. A man sells $9\frac{7}{8}$ yards from a piece of cloth containing 34 yards; how many yards remain? Ans. $24\frac{1}{8}$yd.

18. The distance from Boston to Providence is 40 miles. A, having set out from Boston, has travelled $\frac{2}{17}$ of the distance; and B, having set out at the same time from Providence, has gone $\frac{2}{11}$ of the distance; how far is A from B?

Ans. $28\frac{4}{187}$m.

19. From $\frac{1}{4}$ of a square yard take $\frac{1}{6}$ of a yard square.

233. To subtract one fraction from another, when their numerators are alike.

Ex. 1. From $\frac{1}{3}$ take $\frac{1}{7}$. Ans. $\frac{4}{21}$.

OPERATION.

$7 - 3 = \underline{4}$, difference of the denominators.

$7 \times 3 = 21$, product of the denominators.

We first find the product of the denominators, which is 21, and then their difference, which is 4, and write the former for the denominator of the required fraction, and the latter for the numerator. By this process the fractions are reduced to a common denominator, and their difference found. Hence, to subtract one fraction from another, whose numerators are a unit, we may

Write the difference of the denominators over their product.

2. Take $\frac{2}{7}$ from $\frac{2}{3}$. Ans. $\frac{8}{21}$.

OPERATION.

Difference of the denominators multiplied by one of the numerators,
Product of the denominators,

$$\frac{(7-3)\times 2}{3\times 7}=\frac{8}{21}, \text{ Ans.}$$

We multiply the difference of the denominators by one of the numerators for a new numerator, and the denominators together for a new denominator, by which process the fractions are reduced to a common denominator, and the difference of their numerators is found. Hence, when the given fractions have their numerators alike and greater than a unit, we may

Write the product of the difference of the given denominators, by one of the numerators, over the product of the denominators.

EXAMPLES.

3. Take $\frac{1}{9}$ from $\frac{1}{6}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{7}$; $\frac{1}{19}$ from $\frac{1}{18}$, $\frac{1}{16}$, $\frac{1}{15}$.
4. Take $\frac{1}{6}$ from $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; $\frac{1}{13}$ from $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{8}$.
5. Take $\frac{1}{5}$ from $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; $\frac{1}{7}$ from $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.
6. Take $\frac{1}{11}$ from $\frac{1}{10}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$.
7. Take $\frac{1}{3}$ from $\frac{1}{2}$; $\frac{1}{4}$ from $\frac{1}{2}$, $\frac{1}{3}$; $\frac{1}{9}$ from $\frac{1}{8}$.
8. Take $\frac{1}{12}$ from $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$.
9. Take $\frac{1}{16}$ from $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$.
10. Take $\frac{1}{10}$ from $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$.
11. From $\frac{1}{4}$ take $\frac{1}{9}$; $\frac{1}{10}$ take $\frac{1}{20}$; $\frac{1}{12}$ take $\frac{1}{16}$; $\frac{1}{17}$ take $\frac{1}{25}$.
12. Take $\frac{3}{5}$ from $\frac{3}{4}$; $\frac{3}{8}$ from $\frac{3}{7}$; $\frac{3}{10}$ from $\frac{3}{4}$; $\frac{3}{11}$ from $\frac{3}{5}$.
13. Take $\frac{4}{9}$ from $\frac{4}{7}$; $\frac{4}{11}$ from $\frac{4}{5}$; $\frac{4}{11}$ from $\frac{4}{7}$; $\frac{4}{11}$ from $\frac{4}{10}$.
14. Take $\frac{5}{7}$ from $\frac{5}{6}$; $\frac{5}{11}$ from $\frac{5}{6}$; $\frac{5}{11}$ from $\frac{5}{7}$; $\frac{5}{11}$ from $\frac{5}{10}$.
15. Take $\frac{6}{11}$ from $\frac{6}{7}$; $\frac{6}{11}$ from $\frac{6}{8}$; $\frac{6}{13}$ from $\frac{6}{7}$; $\frac{6}{13}$ from $\frac{6}{11}$.
16. Take $\frac{4}{15}$ from $\frac{4}{5}$; $\frac{4}{15}$ from $\frac{4}{7}$; $\frac{4}{15}$ from $\frac{4}{9}$; $\frac{4}{15}$ from $\frac{4}{11}$.
17. Take $\frac{8}{11}$ from $\frac{8}{9}$; $\frac{8}{11}$ from $\frac{8}{7}$; $\frac{8}{11}$ from $\frac{8}{5}$; $\frac{8}{11}$ from $\frac{8}{3}$.
18. Take $\frac{2}{11}$ from $\frac{2}{5}$; $\frac{2}{11}$ from $\frac{2}{9}$; $\frac{2}{11}$ from $\frac{2}{3}$; $\frac{2}{11}$ from $\frac{2}{7}$.
19. Take $\frac{10}{13}$ from $\frac{10}{7}$; $\frac{10}{19}$ from $\frac{10}{17}$; $\frac{10}{29}$ from $\frac{10}{19}$; $\frac{10}{17}$ from $\frac{10}{11}$.
20. Take $\frac{12}{7}$ from $\frac{12}{5}$; $\frac{12}{7}$ from $\frac{12}{1}$; $\frac{12}{5}$ from $\frac{12}{3}$; $\frac{12}{13}$ from $\frac{12}{7}$.

MISCELLANEOUS EXAMPLES IN ADDITION AND SUBTRACTION OF FRACTIONS.

1. A benevolent man has given to one poor family $\frac{3}{4}$ of a cord of wood, to another $\frac{1}{2}$ of a cord, and to a third $\frac{7}{8}$ of a cord; how much has he given to them all? Ans. $2\frac{1}{8}$ cords.

2. I have paid for a knife $\$\frac{5}{8}$, for a Common School A. metic $\$\frac{1}{2}$, for a slate $\$\frac{1}{3}$, and for stationery $\$\frac{5}{6}$; what did pay for the whole?

3. R. Howland travelled one day $20\frac{7}{10}$ miles, another day $19\frac{1}{2}$ miles, and a third day $22\frac{1}{16}$ miles; what was the whole distance travelled? Ans. $62\frac{21}{80}$ miles.

4. I have bought $6\frac{1}{5}$ tons of anthracite coal, $19\frac{4}{5}$ tons of Cumberland coal, and $3\frac{3}{4}$ tons of cannel coal; what is the whole quantity purchased? Ans. $30\frac{1}{20}$ tons.

5. There is a pole standing $\frac{1}{5}$ in the mud, $\frac{1}{3}$ in the water, and the remainder above the water; what portion of it is above the water?

6. F. Adams, having a lot of sheep, sold at one time $\frac{2}{3}$ of them, and at another time $\frac{1}{4}$ of the remainder; what portion of the original number had he then left? Ans. $\frac{1}{4}$.

7. From a piece of calico containing $31\frac{1}{4}$ yards there have been sold $11\frac{5}{8}$ yards, $9\frac{1}{2}$ yards, and $3\frac{3}{4}$ yards; how much remains?

8. From a cask of molasses containing $84\frac{3}{8}$ gallons, there were drawn at one time $4\frac{3}{7}$ gallons, at another time 11 gallons; at a third time $26\frac{1}{2}$ gallons were drawn, and $\frac{1}{2}$ of $7\frac{1}{2}$ gallons returned to the cask; and at a fourth time $13\frac{8}{11}$ gallons were drawn, and $3\frac{1}{2}$ gallons of it returned to the cask. How much then remained in the cask? Ans. $35\frac{527}{616}$gal.

9. A merchant had 3 pieces of cloth, containing, respectively, $19\frac{3}{4}$ yards, $36\frac{1}{3}$ yards, and $33\frac{5}{8}$ yards. After selling several yards from each piece, he found he had left in the aggregate $71\frac{3}{8}$ yards. How many yards had he sold? Ans. $18\frac{1}{3}$.

MULTIPLICATION OF COMMON FRACTIONS.

234. MULTIPLICATION of Fractions is the process of multiplying when the multiplier, or multiplicand, or both, are fractional numbers.

NOTE. — If the multiplier is less than 1, only such a part of the multiplicand is taken as the multiplier is of 1. Therefore, the product resulting from multiplying a number by a proper fraction is not larger, but less, than the multiplicand.

235. To multiply when one or both of the factors are fractions.

Ex. 1. Multiply $\frac{7}{18}$ by 9. Ans. $\frac{63}{18} = 3\frac{1}{2}$.

FIRST OPERATION.

$\frac{7}{18} \times 9 = \frac{63}{18} = \frac{7}{2} = 3\frac{1}{2}$ Ans.

It is evident that the fraction $\frac{7}{18}$ is multiplied by 9 by multiplying its numerator by 9, since the parts taken, 63, are 9 times as many as before, while the parts into which the unit of the fraction is divided remain the same.

SECOND OPERATION.

$\frac{7}{18} \times 9 = \frac{7}{2} = 3\frac{1}{2}$ Ans.

It is evident, also, that the fraction $\frac{7}{18}$ is multiplied by 9 by dividing its denominator by 9, since the parts into which the unit of the fraction is divided are only $\frac{1}{9}$ as many, and consequently 9 times as large, as before, while the parts taken remain the same. Therefore,

Multiplying the numerator or dividing the denominator of a fraction by any number multiplies the fraction by that number (Art. 217).

2. Multiply 14 by $\frac{3}{7}$. Ans. 6.

FIRST OPERATION.

$$\begin{array}{r|l} 7 & 14 \\ \hline & 2 \times 3 = 6 \text{ Ans.} \end{array}$$

By dividing the *whole number*, 14, by 7, the denominator of the fraction, we obtain $\frac{1}{7}$ of 14 = 2, which multiplied by 3, the numerator of the fraction, gives $\frac{3}{7}$ of 14 = 6.

SECOND OPERATION.

$$\begin{array}{l} 14 \\ \underline{\;\;3} \\ 42 \div 7 = 6 \text{ Ans.} \end{array}$$

By multiplying the *whole number*, 14, by 3, the numerator of the fraction, we obtain 42, a product 7 times as large as it should be, as the multiplier was not 3, a whole number, but $\frac{3}{7}$, or $3 \div 7$; hence, we divide the 42 by 7; and thus obtain $\frac{3}{7}$ of 14 = 6, as before. Therefore,

Multiplying by a fraction is taking the part of the multiplicand denoted by the multiplier.

3. Multiply $\frac{7}{8}$ by $\frac{3}{5}$. Ans. $\frac{21}{40}$.

OPERATION.

$\frac{7}{8} \times \frac{3}{5} = \frac{21}{40}$ Ans.

To multiply $\frac{7}{8}$ by $\frac{3}{5}$ is to take $\frac{3}{5}$ of the multiplicand, $\frac{7}{8}$. Now, to obtain $\frac{3}{5}$ of $\frac{7}{8}$, we multiply the numerators together for a new numerator, and the denominators together for a new denominator (Art. 226). Therefore,

Multiplying one fraction by another is the same as reducing compound fractions to simple ones.

When either of the factors is not a fraction, as in examples first and second, it may be reduced to a fractional form, and then the operation may be like that in the last example. Hence the general

RULE. — *Reduce whole or mixed numbers, if any, to improper fractions. Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

NOTE. — When there are common factors in the numerators and denominators, the operation may be shortened by cancelling those factors.

Examples.

4. Multiply $\frac{7}{11}$ by $\frac{11}{21}$. Ans. $\frac{1}{3}$.

OPERATION.

$$\frac{\cancel{7}}{\cancel{11}} \times \frac{\cancel{11}}{\underset{3}{\cancel{21}}} = \frac{1}{3}.$$

5. Multiply 12 by $\frac{5}{7}$. Ans. $8\frac{4}{7}$.
6. Multiply $\frac{2}{7}$ by 12.
7. Multiply $\frac{14}{17}$ by $\frac{17}{42}$. Ans. $\frac{1}{3}$.
8. Multiply $\frac{3}{7}$ by $\frac{19}{20}$. Ans. $\frac{57}{140}$.
9. Multiply $\frac{4}{5}$ by $\frac{6}{11}$. Ans. $\frac{24}{55}$.
10. Multiply $\frac{7}{12}$ by $\frac{11}{16}$. Ans. $\frac{77}{192}$.
11. Multiply $\frac{8}{11}$ by $\frac{4}{7}$. Ans. $\frac{32}{77}$.
12. Multiply $\frac{3}{4}$ by $\frac{15}{16}$.
13. Multiply $\frac{16}{17}$ by $\frac{11}{12}$. Ans. $\frac{44}{51}$.
14. Multiply $\frac{9}{10}$ by 14. Ans. $12\frac{3}{5}$.
15. Multiply 13 by $\frac{4}{7}$. Ans. $7\frac{3}{7}$.
16. Multiply 16 by $\frac{3}{19}$ Ans. $2\frac{10}{19}$.
17. Multiply 11 by $\frac{4}{7}$. Ans. $6\frac{2}{7}$.
18. Multiply $\frac{9}{10}$ by 14.
19. Multiply $\frac{8}{9}$ by 19. Ans. $16\frac{8}{9}$.
20. Multiply $\frac{9}{11}$ by $\frac{22}{27}$. Ans. $\frac{2}{3}$.
21. Multiply $\frac{4}{9}$ by $\frac{18}{19}$. Ans. $\frac{8}{19}$.
22. Multiply $\frac{3}{19}$ by $\frac{38}{39}$. Ans. $\frac{2}{13}$.
23. Multiply $\frac{7}{9}$ by $\frac{9}{11}$.
24. Multiply $\frac{1}{99}$ by $\frac{99}{100}$. Ans. $\frac{1}{100}$.
25. Multiply $\frac{3}{7}$ of $\frac{7}{11}$ of $\frac{11}{24}$ by 100. Ans. $12\frac{1}{2}$.
26. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{8}$ by 11. Ans. $3\frac{5}{24}$.
27. What cost $\frac{7}{12}$ of a ton of hay, at \$17 per ton? Ans. \$$9\frac{11}{12}$.
28. What cost $\frac{9}{20}$ of an acre of land, at \$37 per acre?
29. At $\frac{5}{8}$ of a dollar per foot, what cost 7 cords of wood?
30. Multiply $161\frac{11}{15}$ by $19\frac{11}{28}$. Ans. $3136\frac{33}{70}$.
31. Multiply $\frac{3}{7}$ by $8\frac{3}{4}$. Ans. $3\frac{3}{4}$.
32. Multiply $\frac{9}{10}$ by $17\frac{3}{11}$. Ans. $15\frac{6}{11}$.
33. Multiply $\frac{8}{9}$ by $71\frac{4}{5}$. Ans. $63\frac{37}{45}$.
34. Multiply $\frac{3}{4}$ of $9\frac{1}{4}$ by $\frac{2}{3}$ of 17. Ans. $78\frac{5}{8}$.
35. Multiply $\frac{9}{10}$ of 7 by $\frac{11}{15}$ of $87\frac{3}{11}$.
36. Multiply 8 by $\frac{7}{9}$. Ans. $6\frac{2}{9}$.

37. Multiply 12 by $\frac{5}{7}$. Ans. $8\frac{4}{7}$.

38. Multiply 15 by $\frac{6}{11}$. Ans. $8\frac{2}{11}$.

39. A merchant owning $\frac{7}{8}$ of a ship sells $\frac{4}{11}$ of his share to A. What part is that of the whole ship?

40. Multiply $3\frac{7}{9}$ by $10\frac{4}{7}$. Ans. $39\frac{59}{63}$.

41. Multiply $\frac{2}{3}$ of $7\frac{1}{4}$ by $\frac{7}{8}$ of $11\frac{3}{4}$. Ans. $49\frac{133}{192}$.

42. Multiply $\frac{2}{7}$ of 9 by $\frac{3}{5}$ of 17. Ans. $26\frac{8}{35}$.

43. Multiply $\frac{4}{7}$ of $8\frac{3}{10}$ by $\frac{4}{7}$ of $9\frac{1}{4}$. Ans. $25\frac{17}{245}$.

236. When one of the factors is a whole number, and the other a mixed number, we may

Multiply the fractional part and the whole number separately, and add together the products.

EXAMPLES.

1. Multiply $7\frac{5}{8}$ by 9.

OPERATION.

$$\begin{array}{lr} & 7\frac{5}{8} \\ & 9 \\ \hline \frac{5}{8}\times 9 = \frac{45}{8} = & 5\frac{5}{8} \\ 7 \times 9 = & 63 \\ \hline & 68\frac{5}{8} \text{ Ans.} \end{array}$$

2. Multiply 12 by $3\frac{3}{4}$.

OPERATION.

$$\begin{array}{lr} & 12 \\ & 3\frac{3}{4} \\ \hline \frac{3}{4} \text{ of } 12 = & 9 \\ 12 \times 3 = & 36 \\ \hline & 45 \text{ Ans.} \end{array}$$

3. Multiply $8\frac{3}{5}$ by 7. Ans. $60\frac{1}{5}$.

4. Multiply 17 by $3\frac{1}{6}$.

5. Multiply 13 by $8\frac{3}{7}$. Ans. $109\frac{4}{7}$.

6. Multiply 37 by $13\frac{8}{11}$. Ans. $507\frac{10}{11}$.

7. Multiply $11\frac{6}{7}$ by 8. Ans. $94\frac{6}{7}$.

8. What cost $7\frac{6}{11}$lb. of beef at 5 cents per pound?

9. What cost $23\frac{7}{12}$bbl. of flour at \$ 6 per barrel? Ans. \$ $141\frac{1}{2}$.

10. What cost $8\frac{3}{8}$yd. of cloth at \$ 5 per yard? Ans. \$ $41\frac{7}{8}$.

11. What cost 9 barrels of vinegar at \$ $6\frac{3}{8}$ per barrel? Ans. \$ $57\frac{3}{8}$.

12. What cost 12 cords of wood at \$ $6.37\frac{1}{2}$ per cord? Ans. \$ 76.50.

13. What cost 11cwt. of sugar at \$ $9\frac{3}{8}$ per cwt.?

14. What cost $4\frac{3}{8}$ bushels of rye at \$ 1.75 per bushel? Ans. \$ $7.65\frac{5}{8}$.

15. What cost 7 tons of hay at \$ $11\frac{7}{8}$ per ton? Ans. \$ $83\frac{1}{8}$.

16. What cost 9 dozen of adzes at $\$ 10\frac{5}{8}$ per dozen?
17. What cost 5 tons of timber at $\$ 3\frac{1}{8}$ per ton?

Ans. $\$ 15\frac{5}{8}$.

18. What cost 15cwt. of rice at $\$ 7.62\frac{1}{2}$ per cwt.?

Ans. $\$ 114.37\frac{1}{2}$.

19. What cost 40 tons of coal at $\$ 8.37\frac{1}{2}$ per ton?

Ans. $\$ 335$.

DIVISION OF COMMON FRACTIONS.

237. Division of Fractions is the process of dividing when the divisor or dividend, or both, are fractional numbers.

Note. — If the divisor is less than 1, the quotient arising from the division will be as many times the dividend as the divisor is contained times in 1. Therefore, the quotient arising from dividing a whole or mixed number by a proper fraction will always be larger than the dividend.

238. The *reciprocal* of a fraction is the number resulting from taking its numerator as denominator, and its denominator as numerator, since *any two numbers, whose product is* 1, *are the reciprocals of each other.* Thus, the reciprocal of $\frac{10}{9}$ is that fraction inverted, or $\frac{9}{10}$, since $\frac{10}{9} \times \frac{9}{10} = 1$.

239. To divide when the divisor or dividend, or both, are fractions.

Ex. 1. Divide $\frac{14}{17}$ by 7. Ans. $\frac{2}{17}$.

FIRST OPERATION.

$\frac{14}{17} \div 7 = \frac{2}{17}$ Ans.

It is evident that the fraction $\frac{14}{17}$ is divided by 7 by dividing its numerator by 7, since the size of the parts, as denoted by the denominator, remains the same, while the number of parts taken is only $\frac{1}{7}$ as large as before.

SECOND OPERATION.

$\frac{14}{17} \div 7 = \frac{14}{119} = \frac{2}{17}$ Ans.

It is evident the fraction is also divided by 7 by multiplying its denominator by 7, since the number of parts taken, as denoted by the numerator, remains the same, while the size of the parts is only $\frac{1}{7}$ as large as before. Therefore,

Dividing the numerator or multiplying the denominator of a fraction by any number divides the fraction by that number (Art. 217).

2. Divide $\frac{12}{13}$ by $\frac{6}{13}$. Ans. 2.

OPERATION.

$\frac{12}{13} \div \frac{6}{13} = 2$ Ans.

Since the fractional units of the two fractions are of the same kind, it is evident that 12 thirteenths contain 6

thirteenths as many times as 6 is contained in 12; $12 \div 6 = 2$, Ans. Therefore,

When the fractions have a common denominator, the division can be performed as in whole numbers, by dividing the numerator of the dividend by the numerator of the divisor.

3. Divide $\frac{4}{7}$ by $\frac{3}{8}$. Ans. $1\frac{11}{21}$.

FIRST OPERATION.

$$\frac{4}{7} \div \frac{3}{8} = \frac{32}{56} \div \frac{21}{56} = 1\frac{11}{21} \text{ Ans.}$$

Having reduced the fractions to a common denominator, we divide the numerator 32 of the dividend by the numerator 21 of the divisor, as in working the last example, and obtain as the required result $1\frac{11}{21}$.

SECOND OPERATION.

$$\frac{4}{7} \div \frac{3}{8} = \frac{4}{7} \times \frac{8}{3} = \frac{32}{21} = 1\frac{11}{21} \text{ Ans.}$$

In the second operation, we invert the divisor, and then proceed as in multiplication of fractions (Art. 235). The reason of this process, which in effect reduces the fractions to a common denominator, and divides the numerator of the dividend by that of the divisor, will be seen, if we consider that the divisor, $\frac{3}{8}$, is an expression denoting that 3 is to be divided by 8. Now, regarding 3 as a whole number, we divide the fraction $\frac{4}{7}$ by it, by multiplying the denominator; thus, $\frac{4}{7 \times 3} = \frac{4}{21}$. But the divisor 3 is 8 times as large as it ought to be, since it was to be divided by 8, as seen in the original fraction; then the quotient, $\frac{4}{21}$, is $\frac{1}{8}$ as large as it should be, and must be multiplied by 8; thus, $\frac{4 \times 8}{21} = \frac{32}{21} = 1\frac{11}{21}$, the answer, as before. By this operation we have multiplied the dividend by the reciprocal of the divisor, the denominator of the dividend having been multiplied by the numerator of the divisor, and the numerator of the dividend by the denominator of the divisor. Therefore,

Dividing by a fraction is the same as multiplying by its reciprocal.

When either divisor or dividend is not a fraction, it may be changed to a fractional form, and the division performed by the last method. Hence the general

RULE. — *Invert the divisor, and then proceed as in multiplication of fractions.*

NOTE 1. — When either divisor or dividend is a whole or mixed number, or a compound fraction, it must be reduced to the form of a simple fraction before dividing.

NOTE 2. — Factors common to both numerator and denominator should be cancelled.

NOTE 3. — When the given fractions have a *common denominator*, the answer may be obtained by dividing the numerator of the dividend by that of the divisor. Also, if the fractions have *numerators alike*, the answer may be obtained by dividing the denominator of the divisor by that of the dividend.

NOTE 4. — When the numerator of the divisor will exactly divide the numerator of the dividend, and the denominator of the divisor exactly divide the denominator of the dividend, the division can be effected in that way.

EXAMPLES.

4. Divide $\frac{7}{10}$ by $\frac{7}{62}$. Ans. $6\frac{1}{5}$.

OPERATION.

$$\frac{7}{10} \div \frac{7}{62} = \frac{\cancel{7}}{10} \times \frac{62}{\cancel{7}} = \frac{62}{10} = 6\frac{1}{5} \text{ Ans.}$$

$$\text{Or } \frac{7}{10} \div \frac{7}{62} = 62 \div 10 = 6\frac{1}{5} \text{ Ans.}$$

5. Divide $\frac{9}{28}$ by $\frac{3}{7}$. Ans. $\frac{3}{4}$.

OPERATION.

$$\frac{9}{28} \div \frac{3}{7} = \frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{28}}} \times \frac{\cancel{7}}{\cancel{3}} = \frac{3}{4} \text{ Ans.}$$

$$\text{Or } \frac{9}{28} \div \frac{3}{7} = \frac{9}{28} \frac{\div 3}{\div 7} = \frac{3}{4} \text{ Ans.}$$

6. Divide $\frac{7}{11}$ by 18. Ans. $\frac{7}{198}$.
7. Divide $\frac{4}{9}$ by $\frac{7}{8}$. Ans. $\frac{32}{63}$.
8. Divide 18 by $\frac{7}{11}$.
9. Divide $\frac{5}{27}$ by $\frac{5}{3}$. Ans. $\frac{1}{9}$.
10. Divide $\frac{16}{21}$ by $\frac{2}{3}$. Ans. $\frac{8}{7} = 1\frac{1}{7}$.
11. Divide $\frac{15}{22}$ by 28. Ans. $\frac{15}{616}$.
12. Divide $\frac{1}{17}$ by 27.
13. Divide $\frac{2}{15}$ by 128. Ans. $\frac{1}{960}$.
14. Divide $\frac{11}{17}$ by 98. Ans. $\frac{11}{1666}$.
15. Divide $\frac{14}{23}$ by 19. Ans. $\frac{14}{437}$.
16. Divide $\frac{5}{6}$ by 167. Ans. $\frac{5}{1002}$.
17. Divide $\frac{16}{29}$ by 49. Ans. $\frac{16}{1421}$.
18. Divide $\frac{1}{15}$ by 15. Ans. $\frac{1}{225}$.
19. Divide 27 by $\frac{1}{17}$.
20. Divide 128 by $\frac{2}{15}$. Ans. 960.
21. Divide 98 by $\frac{11}{17}$. Ans. $151\frac{5}{11}$.
22. Divide 19 by $\frac{14}{23}$. Ans. $31\frac{3}{14}$.
23. Divide 167 by $\frac{15}{18}$. Ans. $200\frac{2}{5}$.
24. Divide 49 by $\frac{16}{29}$.
25. Divide 15 by $\frac{1}{15}$. Ans. 225.
26. Divide $\frac{36}{51}$ by $\frac{3}{17}$. Ans. 4.
27. Divide $\frac{31}{97}$ by $\frac{31}{301}$. Ans. $3\frac{10}{97}$.
28. Divide $\frac{5}{6}$ by $\frac{8}{9}$. Ans. $\frac{15}{16}$.
29. Divide $\frac{11}{15}$ by $\frac{7}{11}$. Ans. $1\frac{16}{105}$.

30. Divide $\frac{8}{25}$ by $\frac{14}{17}$.
31. Divide $\frac{16}{21}$ by $\frac{1}{15}$. Ans. $11\frac{3}{7}$.
32. Divide $\frac{9}{25}$ by $7\frac{3}{4}$. Ans. $\frac{36}{775}$.
33. Divide $\frac{8}{11}$ by $16\frac{4}{9}$. Ans. $\frac{18}{407}$.
34. Divide $11\frac{3}{7}$ by $\frac{4}{7}$.
35. Divide $21\frac{4}{5}$ by $18\frac{4}{7}$. Ans. $1\frac{113}{650}$.
36. Divide $17\frac{3}{11}$ by $28\frac{11}{26}$. Ans. $\frac{4940}{8129}$.
37. Divide $161\frac{3}{17}$ by $14\frac{3}{5}$. Ans. $11\frac{49}{1241}$.
38. Divide $\frac{7}{11}$ of $\frac{4}{5}$ by $\frac{3}{5}$ of $\frac{8}{11}$. Ans. $1\frac{1}{6}$.
39. Divide $\frac{5}{9}$ of $7\frac{3}{11}$ by $\frac{4}{11}$ of $17\frac{3}{7}$. Ans. $\frac{350}{549}$.
40. Divide $\frac{6}{17}$ of 15 by $\frac{7}{15}$ of 22.

41. Bought $\frac{7}{8}$ of a coal-mine for \$3675, and having sold $\frac{5}{7}$ of my share, I gave $\frac{2}{3}$ of the remainder to a charitable society, and divided the residue among 7 poor persons; what was the share of each? Ans. \$50 for each poor person.

42. Of an estate valued at \$5000, the widow receives $\frac{1}{3}$, the oldest son $\frac{2}{3}$ of the remainder; the residue is equally divided among 7 daughters; what is the share of each daughter? Ans. $\$158\frac{46}{63}$.

240. When the dividend is a mixed number, and the divisor a whole number, we may

Divide the integral part of the mixed number as in division of whole numbers, and the remainder divide as in Art. 239; and add together the results for the quotient required.

Ex. 1. Divide $27\frac{3}{5}$ by 6. Ans. $4\frac{3}{5}$.

OPERATION.

$6\,)\,\underline{27\frac{3}{5}}$

4, rem. $3\frac{3}{5}$; $3\frac{3}{5} = \frac{18}{5}$; $\frac{18}{5 \times 6} = \frac{18}{30} = \frac{3}{5}$; $4 + \frac{3}{5} = 4\frac{3}{5}$, Ans.

2. Divide $29\frac{3}{8}$ by 9. Ans. $3\frac{19}{72}$.
3. Divide $14\frac{1}{2}$ by 7. Ans. $2\frac{1}{14}$.
4. Divide $13\frac{3}{8}$ by 8.
5. Divide $14\frac{3}{5}$ by 6. Ans. $2\frac{13}{30}$.
6. Divide $\$37\frac{3}{7}$ among 9 men. Ans. $\$4\frac{10}{63}$.
7. Divide $\$96\frac{2}{3}$ among 11 persons. Ans. $\$8\frac{26}{33}$.
8. What is $\frac{1}{8}$ of $167\frac{7}{11}$ cwt. of iron? Ans. $20\frac{21}{22}$ cwt.
9. Divide $\frac{7}{8}$ of a prize, valued at \$1723, equally between 12 seamen.

10. What will a barrel of flour cost, if 19 barrels can be purchased for $\$ 107\frac{3}{8}$? Ans. $\$ 5.65\frac{5}{38}$.

11. If 15 pounds of raisins can be obtained for $\$ 3\frac{2}{7}$, what will 1 pound cost? Ans. $\$ 0.21\frac{19}{21}$.

12. If 12 quarts of wine cost $\$ 3.75\frac{3}{4}$, what will a quart cost?

13. If $19 will buy $375\frac{1}{16}$ acres of land, how much can be bought for $1? Ans. $19\frac{225}{304}$ acres.

REDUCTION OF COMPLEX FRACTIONS.

241. A COMPLEX fraction is one having a fraction in its numerator or denominator, or in both. Thus, $\dfrac{3}{\frac{1}{2}}$ and $\dfrac{2\frac{1}{4}}{\frac{6}{7}}$ are complex fractions.

242. To reduce complex to simple fractions.

Ex. 1. Reduce $\dfrac{\frac{2}{9}}{\frac{5}{8}}$ to a simple fraction. Ans. $\frac{16}{45}$.

OPERATION.

$$\frac{\frac{2}{9}}{\frac{5}{8}} = \frac{2}{9} \times \frac{8}{5} = \frac{16}{45}, \text{ Ans.}$$

Since the numerator of a fraction is the dividend, and the denominator the divisor (Art. 216), we divide the numerator, $\frac{2}{9}$, by the denominator, $\frac{5}{8}$, as in *division of fractions* (Art. 239).

2. Reduce $\dfrac{7}{1\frac{2}{3}}$ to a simple fraction. Ans. $\frac{21}{5} = 4\frac{1}{5}$.

OPERATION.

$$\frac{7}{1\frac{2}{3}} = \frac{\frac{7}{1}}{\frac{5}{3}} = \frac{7}{1} \times \frac{3}{5} = \frac{21}{5} = 4\frac{1}{5}, \text{ Ans.}$$

We reduce the numerator, 7, and the denominator, $1\frac{2}{3}$, to improper fractions, and then proceed as in Ex. 1. Hence, to reduce complex to simple fractions,

Consider the denominator as a divisor, and the numerator as a dividend, and proceed as in division of fractions (Art. 239).

NOTE. — Another and often a ready method of reducing a complex fraction is to multiply both its terms by the least common multiple of their denominators.

EXAMPLES.

3. Reduce $\dfrac{\frac{1}{4}}{6\frac{1}{2}}$ to a simple fraction. Ans. $\frac{1}{26}$.

OPERATION.

$$\frac{\frac{1}{4}}{6\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{13}{2}} = \frac{1}{4} \times \frac{2}{13} = \frac{2}{52} = \frac{1}{26}, \text{ Ans.}$$

Or, multiply by the least common multiple of the denominators,

$$\frac{\frac{1}{4} \times 4}{6\frac{1}{2} \times 4} = \frac{1}{26}, \text{ Ans.}$$

4. Reduce $\frac{4\frac{3}{7}}{\frac{2}{3}}$ to a simple fraction. Ans. $6\frac{9}{14}$.

5. Reduce $\frac{\frac{3}{4}}{5\frac{2}{3}}$ to a simple fraction.

6. Reduce $\frac{7}{4\frac{2}{3}}$ to a simple fraction. Ans. $1\frac{1}{2}$.

7. Reduce $\frac{7\frac{4}{11}}{8}$ to a simple fraction. Ans. $\frac{81}{88}$.

8. Reduce $\frac{6\frac{2}{9}}{8\frac{2}{3}}$ to a simple fraction. Ans. $\frac{28}{39}$.

9. Reduce $\frac{\frac{2}{7}}{\frac{3}{5}}$ to a simple fraction.

10. Reduce $\frac{8}{\frac{1}{3}}$ to a whole number. Ans. 24.

11. Reduce $\frac{\frac{4}{7}}{2}$ to a simple fraction. Ans. $\frac{2}{7}$.

12. Reduce $\frac{5\frac{1}{3}}{\frac{3}{7}}$ to a mixed number. Ans. $12\frac{4}{9}$.

13. Reduce $\frac{\frac{1}{4}}{6\frac{1}{2}}$ to a simple fraction.

14. Reduce $\frac{3}{2\frac{1}{2}}$ to a mixed number. Ans. $1\frac{1}{5}$.

15. Reduce $\frac{3\frac{1}{4}}{9}$ to a simple fraction. Ans. $\frac{13}{36}$.

16. Reduce $\frac{11\frac{2}{3}}{12\frac{3}{5}}$ to a simple fraction. Ans. $\frac{25}{27}$.

17. If 7 were to be the denominator of the fraction whose numerator is $\frac{7\frac{7}{9}}{11\frac{4}{5}}$, what would be its value?

18. If $\frac{7}{9}$ is the numerator of the fraction whose denominator is $\frac{\frac{3}{5}}{5}$, what is its value? Ans. $6\frac{13}{27}$.

243. Complex fractions, after being reduced to simple ones, may be added, subtracted, multiplied, and divided, according to the respective rules for simple fractions.

EXAMPLES.

1. Add $\frac{1}{3}$ of $\frac{2}{7}$ of $28\frac{3\frac{6}{7}}{4\frac{2}{7}}$ to $3\frac{39\frac{1}{2}}{105}$. Ans. $6\frac{9}{70}$.

2. Add $\frac{1}{9}$, $2\frac{5}{8}$, $\frac{45}{94\frac{7}{11}}$, and $\frac{47\frac{5}{9}}{314\frac{3}{5}}$ together. Ans. $3\frac{14257081}{39299832}$.

3. What is the difference between $\frac{49\frac{5}{8}}{97}$ and $\frac{34\frac{3}{5}}{145\frac{3}{11}}$? Ans. $\frac{847651}{3100120}$.

4. What is the continued product of the following numbers: $\frac{27}{37\frac{4}{5}}$, $\frac{87\frac{2}{9}}{98\frac{1}{8}}$, $\frac{\frac{7}{8}}{2\frac{1}{3}}$, and $\frac{81\frac{5}{11}}{128}$?

5. Divide $\frac{2}{3}$ of $7\frac{3}{4}$ by $\frac{4}{5}$ of $11\frac{4}{11}$. Ans. $\frac{341}{600}$.

6. Divide $\frac{4}{9}$ of 91 by $\frac{9}{10}$ of 87. Ans. $\frac{3640}{7047}$.

MISCELLANEOUS EXAMPLES IN MULTIPLICATION AND DIVISION OF FRACTIONS.

1. At $2\frac{3}{7}$ bushels to an acre, how many bushels of wheat will be required to sow $7\frac{4}{17}$ acres? Ans. $17\frac{4}{7}$.

2. Bought $8\frac{1}{5}$ bushels of apples for \$ $4.68\frac{4}{7}$; what did they cost per bushel? Ans. \$ $0.57\frac{1}{7}$.

3. Bought a bale of cloth for \$ $96\frac{3}{8}$; I dispose of it for $\frac{5}{8}$ of the cost, and by so doing I lose \$ 2 on a yard; required the number of yards in the bale. Ans. $18\frac{9}{128}$ yd.

4. If a dividend be $18\frac{1}{3}$ times $\frac{3}{4}$ and a quotient $6\frac{1}{2}$ times $\frac{2}{3}$, what was the divisor?

5. By what number must $1\frac{\frac{1}{2}}{3}$ be multiplied, that the product shall just equal 1? Ans. $\frac{6}{7}$.

6. Bought a horse and chaise for \$ 250, and paid for the harness $\frac{7}{11}$ of what I paid for the horse. The chaise cost $\frac{11}{12}$ the value of the horse. What was the price of each?

Ans. Horse, \$ $130\frac{10}{23}$; chaise, \$ $119\frac{13}{23}$; harness, \$ $83\frac{1}{253}$.

7. S. Walker has engaged to work at yearly wages of $200 and a suit of clothes. At the end of 9 months, falling sick, and being unable to labor longer, he receives the suit of clothes and $144, as the amount justly due. What was the cost of the clothes? Ans. $24.

8. What will be the result if $\frac{1}{5}$ of $\frac{3}{4}$ of $3\frac{1}{3}$ be multiplied by $\frac{1}{2}$ of itself, and the product divided by $\frac{1}{2}$?

9. Bought $13\frac{7}{8}$ acres of land at $$25\frac{3}{4}$ per acre, and paid for it in wheat at $$2\frac{3}{5}$ per bushel. How many bushels did it require? Ans. $137\frac{173}{416}$ bushels.

10. How long will it take a man to travel 553 miles, provided he travels $3\frac{1}{2}$ miles per hour, and $9\frac{7}{8}$ hours per day?

11. If $$\frac{9}{10}$ per cord is paid E. Holmes for sawing into three pieces wood that is 4 feet long, how much more should he receive per cord for sawing into pieces of the same length wood that is 8 feet long? Ans. $0.22½.

12. A steamboat leaves New Orleans, January 1st, bound up the river to a place distant $2317\frac{1}{4}$ miles. Her forward motion is at the rate of $9\frac{4}{5}$ miles per hour for $16\frac{3}{4}$ hours each day, and she lies at anchor in the night for fear of running upon a snag. But having lost her anchor on the fifth day, she each succeeding night drifts backward, at the rate of 2 miles per hour. On what day of January will she reach her point of destination? Ans. 15th day.

A PROPOSED NUMERATOR, OR DENOMINATOR.

244. To reduce one fraction to another of equal value, having a proposed numerator, or denominator.

Ex. 1. Reduce $\frac{4}{7}$ to an equivalent fraction having 4 for a numerator. Ans. $\frac{4}{5\frac{3}{5}}$.

OPERATION.

$$\frac{\frac{4}{\cancel{5}} \times \cancel{5}}{\frac{4}{5} \times 7} = \frac{4}{5\frac{3}{5}} \text{ Ans.}$$

The proposed numerator, 4, is such a part of the given numerator as 4 divided by 5, or $\frac{4}{5}$. Now, as the numerator proposed is only $\frac{4}{5}$ as large as the given numerator, in order that the value of the two fractions be the same, the denominator of the proposed fraction should be only $\frac{4}{5}$ as large as the denominator of the given fraction. Taking $\frac{4}{5}$ of the given denominator, 7, we obtain

$5\frac{3}{5}$, which, written under the proposed numerator, gives $\frac{4}{5\frac{3}{5}}$ as the fraction required.

2. Reduce $\frac{8}{9}$ to a fraction of equal value having 12 for a denominator.

OPERATION.

$$\frac{\frac{12}{9} \times 8}{\frac{12}{\cancel{9}} \times \cancel{9}} = \frac{10\frac{2}{3}}{12} \text{ Ans.}$$

Since the proposed denominator, 12, is $\frac{12}{9}$ of the given denominator, 9, we find $\frac{12}{9}$ of the given numerator, 8, for numerator of the proposed fraction; $\frac{12}{9}$ of $8 = 10\frac{2}{3}$, which, written over the proposed denominator, gives $\frac{10\frac{2}{3}}{12}$ as the fraction required.

RULE. — *Take of both terms of the given fraction such a fractional part as the proposed numerator, or denominator, is of the given numerator, or denominator, and the result will be the required fraction.*

EXAMPLES.

3. Change $\frac{17}{20}$ to a fraction whose numerator shall be 34. Ans. $\frac{34}{40}$.
4. Change $3\frac{6}{7}$ to a fraction whose numerator shall be 9. Ans. $\frac{9}{2\frac{1}{3}}$.
5. Reduce 4 to a fraction whose numerator shall be 5.
6. Reduce $\frac{15}{16}$ to a fraction having 12 for its denominator.
7. Change $\frac{3}{5}$ to fifteenths. Ans. $\frac{9}{15}$.
8. Reduce $\frac{7}{3}$ to halves.
9. Reduce $\frac{19}{20}$ to thirty-fifths. Ans. $\frac{33\frac{1}{4}}{35}$.
10. J. Holton owns $\frac{19}{57}$ of a wood-lot, and his brother $\frac{67}{201}$ of the same lot; what fraction whose denominator shall be 12 will express the part each owns? Ans. $\frac{4}{12}$.

A COMMON NUMERATOR.

245.° A COMMON numerator of two or more fractions is a common multiple of their numerators.

246.° To reduce fractions to a common numerator.

Ex. 1. Change $\frac{3}{4}$, $\frac{4}{5}$, $\frac{6}{7}$, and $\frac{9}{10}$ to other fractions of the same value, having a common numerator. Ans. $\frac{36}{48}$, $\frac{36}{45}$, $\frac{36}{42}$, $\frac{36}{40}$.

OPERATION.

36, least common multiple of the numerators, = common numerator.

$\frac{36}{3}$ of 4 = 48, new denominator. $\frac{3}{4} = \frac{36}{48}$
$\frac{36}{4}$ of 5 = 45, new denominator. $\frac{4}{5} = \frac{36}{45}$
$\frac{36}{6}$ of 7 = 42, new denominator. $\frac{6}{7} = \frac{36}{42}$
$\frac{36}{9}$ of 10 = 40, new denominator. $\frac{9}{10} = \frac{36}{40}$ } Ans.

We find the least common multiple of all the numerators, which is 36, for the common numerator; and to obtain the several new denominators we take such a part of the given denominators, respectively, as the common numerator, 36, is of each given numerator. Thus, both terms of each fraction being proportionably increased, its value is not changed.

RULE. — *Find the least common multiple of the given numerators for a common numerator. Take, for the new denominator of each fraction, respectively, such a part of its given denominator as the common numerator is of its given numerator.*

NOTE. — Compound fractions, or whole and mixed numbers, must be reduced to simple fractions, and all to their lowest terms, before finding the common numerator.

EXAMPLES.

2. Reduce $\frac{8}{9}$, $\frac{4}{7}$, $\frac{3}{4}$, and $\frac{6}{7}$ to other fractions of equal value having a common numerator. Ans. $\frac{24}{27}$, $\frac{24}{42}$, $\frac{24}{32}$, $\frac{24}{28}$.

3. Change $\frac{5}{8}$, $2\frac{1}{2}$, and $1\frac{3}{7}$ to fractions having a common numerator.

4. A can travel round a certain island, which is 50 miles in circumference, in $4\frac{4}{15}$ days, B in $6\frac{2}{5}$ days, and C in $6\frac{2}{3}$ days. If they all set out from the same point, and travel round the island the same way, in how many days will they all meet at the point from which they started, and how many times will each have gone round the island?

Ans. They will meet in 320 days; A will have gone round the island 75 times; B, 50 times; and C, 48 times.

GREATEST COMMON DIVISOR OF FRACTIONS.

247. THE greatest common divisor of two or more fractions is the greatest number that will divide each of them, and give a whole number for the quotient.

248. To find the greatest common divisor of two or more fractions.

Ex. 1. What is the greatest common divisor of $\frac{4}{15}$, $2\frac{2}{9}$, and $5\frac{1}{3}$. Ans. $\frac{4}{45}$.

OPERATION.

$$\tfrac{4}{15},\ 2\tfrac{2}{9},\ 5\tfrac{1}{3} = \tfrac{4}{15},\ \tfrac{20}{9},\ \tfrac{16}{3} = \tfrac{12}{45},\ \tfrac{100}{45},\ \tfrac{240}{45}.$$

$$\left.\begin{array}{r}\text{Greatest common divisor of the numerators} = \ \ 4\\ \text{Least common denominator of the fractions} = \overline{45}\end{array}\right\}\ \text{Greatest common divisor required.}$$

Having reduced the fractions to equivalent fractions having the least common denominator, we find the greatest common divisor of the numerators 12, 100, and 240 to be 4. Now, since the 12, 100, and 240 represent *forty-fifths*, their greatest common divisor is not 4, a whole number, but 4 *forty-fifths;* therefore we write the 4 over the least common denominator, 45, and have $\frac{4}{45}$ as the answer.

RULE. — *Reduce the fractions, if necessary, to their least common denominator. The greatest common divisor of the numerators, written over the least common denominator, will give the greatest common divisor required.*

EXAMPLES.

2. What is the greatest common divisor of $\frac{4}{5}$, $\frac{4}{7}$, $\frac{8}{9}$, and $\frac{16}{21}$? Ans. $\frac{4}{315}$.

3. What is the greatest common divisor of $12\frac{3}{8}$, $9\frac{3}{4}$, and $8\frac{1}{4}$?

4. What is the greatest common divisor of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$? Ans. $\frac{1}{60}$.

5. What is the greatest common divisor of $3\frac{4}{5}$, $5\frac{7}{10}$, and $2\frac{8}{15}$?

6. A farmer has $33\frac{3}{4}$ bushels of corn, $67\frac{1}{2}$ bushels of rye, $70\frac{7}{8}$ bushels of wheat. He wishes to put this grain, without mixing, into the largest bags, each of which shall contain the same quantity. Required the number of bags and the quantity each will contain.

Ans. The capacity of each bag, $3\frac{3}{8}$ bushels; and the number of bags, 51.

7. I have three fields; the first contains $73\frac{7}{11}$ acres, the second $88\frac{4}{11}$ acres, the third $139\frac{10}{11}$ acres. Required the largest-sized house-lots of the same extent into which the three fields can be divided, and also the number of lots.

Ans. Size of each lot, $7\frac{4}{11}$ acres; number of lots, 41.

LEAST COMMON MULTIPLE OF FRACTIONS.

249. THE least common multiple of two or more fractions is the least number that can be divided by each of them, and give a whole number for the quotient.

250. To find the least common multiple of two or more fractions.

Ex. 1. What is the least common multiple of $\frac{3}{4}$, $\frac{6}{16}$, and $2\frac{1}{16}$? Ans. $8\frac{1}{4}$.

OPERATION.

$$\tfrac{3}{4},\ \tfrac{6}{16},\ 2\tfrac{1}{16} = \tfrac{3}{4},\ \tfrac{3}{8},\ \tfrac{33}{16}.$$

$$\left.\begin{array}{l}\text{Least common multiple of the numerators} = 33\\ \text{Greatest common divisor of denominators} = 4\end{array}\right\} \frac{33}{4} = 8\tfrac{1}{4} \left\{\begin{array}{l}\text{Least common multi-}\\ \text{ple required.}\end{array}\right.$$

Having reduced the fractions to their simplest form, we find the least common multiple of the numerators, 3, 3, and 33, to be 33. Now, since the 3, 3, and 33 are, from the nature of a fraction, dividends, of which their respective denominators, 4, 8, and 16, are the divisors (Art. 216), the least common multiple of the fractions is not 33, a whole number, but so many fractional parts of the greatest common divisor of the denominators. This common divisor we find to be 4, which, written as the denominator of the 33, gives $\frac{33}{4} = 8\frac{1}{4}$ as the least number that can be exactly divided by the given fractions.

RULE. — *Reduce the fractions, if necessary, to their lowest terms. Then find the least common multiple of the numerators, which, written over the greatest common divisor of the denominators, will give the least common multiple required.* Or,

Reduce the fractions, if necessary, to their least common denominator. Then find the least common multiple of the numerators, and write it over the least common denominator.

NOTE. — The least whole number that will contain two or more fractions an exact whole number of times, is *the least common multiple of their numerators.*

EXAMPLES.

2. What is the least common multiple of $\frac{4}{5}$, $\frac{8}{9}$, and $\frac{6}{7}$? Ans. $\frac{24}{1} = 24$.

3. Find the least number that $3\frac{15}{16}$, $7\frac{7}{8}$, and $5\frac{1}{4}$ will divide without a remainder. Ans. $15\frac{3}{4}$.

4. What is the least common multiple of $\frac{3}{8}$, $\frac{6}{7}$, and $\frac{9}{10}$?

5. What is the smallest sum of money with which I could purchase a number of sheep at \$ $2\frac{1}{4}$ each, a number of calves at \$ $4\frac{1}{2}$ each, and a number of yearlings at \$ $9\frac{3}{8}$ each? and how many of each could I purchase with this money?

Ans. \$ $112\frac{1}{2}$; 50 sheep; 25 calves; 12 yearlings.

6. There is a certain island 80 miles in circumference. A, B, and C agree to travel round it. A can walk $3\frac{1}{2}$ miles in an hour, B $4\frac{2}{3}$ miles, and C $5\frac{1}{4}$ miles. They start from the same point and travel round the same way, and continue

their travelling 8 hours a day, until they shall all meet at the point from which they started. In how many days will they all meet, and how far will each have travelled?

Ans. In $17\frac{1}{7}$ days; A 480m., B 640m., and C 720m.

7. How many times the least common multiple of $3\frac{1}{2}$, $4\frac{3}{8}$, and $5\frac{1}{4}$, is the least whole number that $3\frac{1}{2}$, $4\frac{3}{8}$, and $5\frac{1}{4}$ will exactly divide.

DENOMINATE FRACTION.

251. A DENOMINATE Fraction is one in which the unit of the fraction is a denomination of a compound number; as, $\frac{2}{3}$ of a pound, $\frac{4}{5}$ of a mile, and $\frac{7}{8}$ of a gallon.

REDUCTION OF DENOMINATE FRACTIONS.

252. REDUCTION of denominate fractions is the process of changing fractions from the unit of one denomination to that of another, without altering their value.

253. To reduce a denominate fraction from a higher denomination to a lower.

Ex. 1. Reduce $\frac{1}{640}$ of a pound to a fraction of a penny.

Ans. $\frac{3}{8}$ d.

OPERATION.

$$\frac{1}{640} \times 20 = \frac{20}{640}\text{s.}; \ \frac{20}{640} \times 12 = \frac{240}{640}\text{d.} = \frac{3}{8}\text{d. Ans.}$$

$$\text{Or, } \frac{1}{\underset{32}{\cancel{640}}} \times \cancel{20} \times \overset{3}{\cancel{12}} = \frac{3}{8}\text{d. Ans.}$$

8

Since 20s. make a pound, there will be 20 times as many shillings as pounds, or $\frac{20}{640}$s.; and since 12d. make a shilling, there will be 12 times as many pence as shillings, or $\frac{240}{640}$d. $= \frac{3}{8}$d.

RULE. — *Multiply the given fraction by the same numbers that would be employed in the reduction of whole numbers to the lower denomination required.*

EXAMPLES.

2. Reduce $\frac{1}{1200}$ of a pound to the fraction of a farthing.
3. Reduce $\frac{1}{9600}$ of a pound troy to the fraction of a grain.
4. Reduce $\frac{5}{2592}$ of a pound, apothecaries' weight, to the fraction of a scruple.
5. Reduce $\frac{3}{8800}$ of a cwt. to the fraction of an ounce.

6. Reduce $\frac{3}{8000}$ of a ton to the fraction of a pound.
7. What part of an inch is $\frac{1}{225}$ of an ell English?
8. What part of an inch is $\frac{1}{110880}$ of a mile?
9. Reduce $\frac{1}{38016}$ of a league to the fraction of an inch.
10. Reduce $\frac{3}{25090560}$ of an acre to the fraction of an inch.
11. Reduce $\frac{1}{1152}$ of a tun of wine measure to the fraction of a quart.
12. What part of a pint is $\frac{3}{320}$ of a bushel?
13. What part of a minute is $\frac{1}{4207680}$ of a year?
14. Reduce $\frac{3}{3200}$ of a hundred-weight to a fraction of an ounce.

254. To reduce a denominate fraction from a lower denomination to a higher.

Ex. 1. Reduce $\frac{3}{8}$ of a penny to a fraction of a pound.

Ans. $\frac{1}{640}$.

OPERATION.

$$\frac{3}{8} \times 12 = \frac{3}{96}\text{s.}; \ \frac{3}{96} \times 20 = \frac{3}{1920}\text{£.} = \frac{1}{640}\text{£. Ans.}$$

$$\text{Or, } \frac{\cancel{3}}{8} \times \underset{4}{\cancel{12}} \times 20 = \frac{1}{640}\text{£. Ans.}$$

Since 12 pence make a shilling, there will be $\frac{1}{12}$ as many shillings as pence, or $\frac{3}{96}$s.; and since 20s. make a pound, there will be $\frac{1}{20}$ as many pounds as shillings, or $\frac{1}{640}$ £. Ans.

RULE. — *Divide the fraction by the same numbers that would be employed in the reduction of whole numbers to a higher denomination.*

EXAMPLES.

2. What part of a pound is $\frac{4}{5}$ of a farthing?
3. What part of a pound is $\frac{3}{5}$ of a grain troy?
4. Reduce $\frac{5}{8}$ of a scruple to the fraction of a pound.
5. Reduce $\frac{6}{11}$ of an ounce to the fraction of a hundred-weight.
6. Reduce $\frac{3}{4}$ of a pound to the fraction of a ton.
7. Reduce $\frac{1}{5}$ of an inch to the fraction of an ell English.
8. Reduce $\frac{4}{7}$ of an inch to the fraction of a mile.
9. Reduce $\frac{1}{2}$ of an inch to the fraction of a league.
10. Reduce $\frac{3}{4}$ of an inch to the fraction of an acre.

11. Reduce $\frac{7}{8}$ of a quart to the fraction of a tun, wine measure.

12. Reduce $\frac{3}{5}$ of a pint to the fraction of a bushel.

13. Reduce $\frac{1}{8}$ of a minute to the fraction of a year ($365\frac{1}{4}$ days).

14. What part of a hundred-weight is $\frac{3}{2}$ of an ounce?

Ans. $\frac{3}{3200}$.

255. To find the value of a fraction in whole numbers of lower denominations.

Ex. 1. What is the value of $\frac{3}{11}$ of a £. Ans. 5s. 5d. $1\frac{9}{11}$far.

OPERATION.

$$
\begin{array}{l}
\quad\ 3\,£. \\
\quad\ \underline{2\,0} \\
11)\ 6\,0\,\text{s.}\ (5\,\text{s.} \\
\quad\ \underline{5\,5} \\
\qquad 5\,\text{s.} \\
\quad\ \underline{1\,2} \\
11)\ 6\,0\,\text{d.}\ (5\,\text{d.} \\
\quad\ \underline{5\,5} \\
\qquad 5\,\text{d.} \\
\qquad \underline{4} \\
11)\ 2\,0\,\text{far.}\ (1\frac{9}{11}\,\text{far.} \\
\quad\ \underline{1\,1} \\
\qquad \frac{9}{11}\text{far.}
\end{array}
$$

Ans. 5s. 5d. $1\frac{9}{11}$far.

Since 1£. = 20s., $\frac{3}{11}$ of a £. is $\frac{3}{11}$ of 20s. = $\frac{60}{11}$s. = $5\frac{5}{11}$s.; and since 1s. = 12d., $\frac{5}{11}$ of a shilling is $\frac{5}{11}$ of 12d. = $\frac{60}{11}$d. = $5\frac{5}{11}$d.; and, since 1d. = 4far., $\frac{5}{11}$ of a penny = $\frac{5}{11}$ of 4far. = $\frac{20}{11}$far. = $1\frac{9}{11}$far. Therefore, $\frac{3}{11}$£. = 5s. 5d. $1\frac{9}{11}$far. This is equivalent to multiplying the numerator of the fraction by the numbers required to reduce it to successive lower denominations, beginning with the highest, and dividing each product by the denominator, as in the operation.

RULE. — *Multiply the numerator of the given fraction by the number required to reduce it to the next lower denomination, and divide the product by the denominator.*

Then, if there is a remainder, proceed as before, until it is reduced to the denomination required.

EXAMPLES.

2. What is the value of $\frac{7}{24}$ of a shilling? Ans. $3\frac{1}{2}$d.

3. What is the value of $\frac{7}{9}$ of a guinea, at 28 shillings?

Ans. 21s. 9d. $1\frac{1}{3}$far.

4. What is the value of $\frac{7}{11}$ of a cwt.?

5. What is the value of $\frac{4}{9}$ of a lb. avoirdupois?

Ans. 7oz. $1\frac{7}{9}$dr.

6. What is the value of $\frac{8}{9}$ of a lb. troy?

Ans. 10oz. 13pwt. 8gr.

7. What is the value of $\frac{4}{13}$ of a ℔. apothecaries' weight? Ans. 3℥ 5ʒ 1℈ $12\frac{4}{13}$gr.

8. What is the value of $\frac{5}{8}$ of an ell English? Ans. 2qr. 3na. $0\frac{1}{4}$in.

9. What is the value of $\frac{11}{13}$ of a mile? Ans. 6fur. 30rd. 12ft. $8\frac{4}{13}$in.

10. What is the value of $\frac{8}{9}$ of a furlong? Ans. 35rd. 9ft. 2in.

11. What is the value of $\frac{7}{13}$ of an acre? Ans. 2R. 6rd. 4yd. 5ft. $127\frac{5}{13}$in.

12. What is the value of $\frac{9}{17}$ of a rod?

13. What is the value of $\frac{1}{13}$ of a cord? Ans. 9ft. $1462\frac{2}{13}$in.

14. What is the value of $\frac{2}{19}$ of a hhd. of wine? Ans. 6gal. 2qt. 1pt. $0\frac{4}{19}$gi.

15. What is the value of $\frac{2}{3}$ of a hhd. of beer? Ans. 42gal.

16. What is the value of $\frac{11}{23}$ of a year ($365\frac{1}{4}$ days)? Ans. 174d. 16h. 26m. $5\frac{5}{23}$sec.

17. What is the value of $\dfrac{7\frac{3\frac{3}{11}}{}}{4\frac{2}{5}}$ of a dollar?

256. To find the value of whole numbers in a fraction of a higher denomination.

Ex. 1. What part of a £. are 5s. 5d. $1\frac{9}{11}$far.?

OPERATION.

$$\begin{array}{ll} \text{5s. 5d. } 1\tfrac{9}{11}\text{far.} = 2880 \\ 1\text{£.} \qquad\qquad\quad = 10560 \end{array} \quad \frac{2880}{10560} = \frac{3}{11}\text{£. Ans.}$$

We reduce the 5s. 5d. $1\frac{9}{11}$far. to elevenths of a farthing, the lowest denomination in the question, for the numerator of the required fraction, and 1 £. to the same denomination for the denominator. We then have the fraction $\frac{2880}{10560}$ £. = $\frac{3}{11}$ £. as the answer.

RULE. — *Reduce the given numbers to the lowest denomination mentioned in either of them. Then write the number which is the fractional part for the numerator, and the other number for the denominator, of the required fraction.*

EXAMPLES.

2. Reduce $3\frac{1}{2}$d. to the fraction of a shilling. Ans. $\frac{7}{24}$.

3. Reduce 21s. 9d. $1\frac{1}{3}$far. to the fraction of a guinea. Ans. $\frac{7}{9}$.

4. Reduce 2qr. 13lb. 10oz. $2\frac{10}{11}$dr. to the fraction of a cwt.
Ans. $\frac{7}{11}$.

5. What part of a pound avoirdupois are 7oz. $1\frac{7}{9}$dr.?

6. What part of a pound troy are 10oz. 13pwt. 8gr.?
Ans. $\frac{8}{9}$.

7. Express 1 gallon liquid measure as a fraction of a gallon dry measure. Ans. $\frac{1344}{1155}$.

8. What part of a yard are 2qr. 0na. $1\frac{5}{13}$in.? Ans. $\frac{7}{13}$.

9. What part of an ell English are 2qr. 3na. $0\frac{1}{4}$in.?
Ans. $\frac{5}{9}$.

10. What part of a mile are 6fur. 30rd. 12ft. $8\frac{4}{13}$in.?

11. Reduce 35rd. 9ft. 2in. to the fraction of a furlong.
Ans. $\frac{8}{9}$.

12. What part of an acre are 2R. 6rd. 4yd. 5ft. $127\frac{5}{13}$in.
Ans. $\frac{7}{13}$.

13. What part of a square rod are 144ft. $19\frac{1}{17}$in.?
Ans. $\frac{9}{17}$.

14. What part of a cord are 9ft. $1462\frac{2}{13}$in.?

15. What part of a hogshead of wine are 6gal. 2qt. 1pt. $0\frac{4}{19}$gi.? Ans. $\frac{2}{19}$.

16. What part of a pound avoirdupois is 1 pound troy?
Ans. $\frac{144}{175}$.

17. What part of a year ($365\frac{1}{4}$ days) are 174d. 16h. 26m. $5\frac{5}{23}$ sec.? Ans. $\frac{11}{23}$.

ADDITION OF DENOMINATE FRACTIONS.

257. To add denominate fractions.

Ex. 1. Add $\frac{7}{13}$ of a £. to $\frac{9}{11}$ of a £.

Ans. 1£. 7s. 1d. $2\frac{54}{143}$far.

FIRST OPERATION.

$\frac{7}{13}$ of a £. = 10s. 9d. $0\frac{12}{13}$far.
$\frac{9}{11}$ of a £. = 16s. 4d. $1\frac{5}{11}$far.
Ans. 1£. 7s. 1d. $2\frac{54}{143}$far.

We find the value of each fraction separately, and add the two values together, according to the rule for adding compound numbers. (Art. 145.)

SECOND OPERATION.

$\frac{7}{13}$£. + $\frac{9}{11}$£. = $\frac{194}{143}$£. = 1£. 7s. 1d. $2\frac{54}{143}$far. Ans.

By the second operation, we first add the two fractions together, and then find the value of their sum. (Art. 255.)

Examples.

2. Add together $\frac{7}{11}$ of a ton and $\frac{17}{22}$ of a cwt.

Ans. 13cwt. 2qr.

3. Add together $\frac{3}{5}$ of a yard, $\frac{1}{7}$ of an ell English, and $\frac{6}{7}$ of a qr.

4. Add together $\frac{7}{11}$ of a mile, $\frac{4}{13}$ of a furlong, and $\frac{9}{22}$ of a yard. Ans. 5fur. 16rd. 0ft. $3\frac{93}{143}$in.

5. A has three house-lots; the first contains $\frac{5}{7}$ of an acre, the second $\frac{2}{3}$ of an acre, and the third $\frac{13}{14}$ of an acre. How many acres do they all contain?

Ans. 2A. 1R. 9p. 142ft. $87\frac{3}{7}$in.

6. A man travelled $18\frac{3}{7}$ miles the first day, $23\frac{11}{14}$ miles the second day, and $19\frac{1}{21}$ miles the third day. How far did he travel in the three days? Ans. 61m. 2fur. 3rd. 13ft. $4\frac{2}{7}$in.

7. Add $\frac{11}{12}$ of a gallon of wine to $\frac{1}{12}$ of a hhd.

8. Add $\frac{5}{16}$ of a week to $\frac{1}{5}$ of a day. Ans. 2d. 9h. 18m.

9. Add $\frac{3}{4}$ of a square foot to $\frac{1}{2}$ a foot square. Ans. 1 foot.

10. Add 6 inches to 11rd. 16ft. 5in. Ans. 12rd. 0ft. 5in.

SUBTRACTION OF DENOMINATE FRACTIONS.

258. To subtract one denominate fraction from another.

Ex. 1. From $\frac{7}{11}$ of a £. take $\frac{2}{9}$ of a £.

Ans. 8s. 3d. $1\frac{19}{33}$far.

FIRST OPERATION.

$\frac{7}{11}$ of a £. = 12s. $8\frac{8}{11}$d.
$\frac{2}{9}$ of a £. = 4s. $5\frac{1}{3}$d.
Ans. 8s. $3\frac{13}{33}$d.

We find the value of each fraction separately, and subtract one from the other, according to the rule for subtracting compound numbers. (Art. 146.)

SECOND OPERATION.

$\frac{7}{11}£. - \frac{2}{9}£. = \frac{41}{99}£.$ = 8s. 3d. $1\frac{19}{33}$far. Ans.

By the second operation, we first subtract the less fraction from the greater, and then find the value of their difference. (Art. 255.)

Examples.

2. From $\frac{7}{8}$ of an ell English take $\frac{2}{7}$ of a yard.

Ans. 3qr. 0na. $2\frac{5}{56}$in.

3. Take $\frac{7}{11}$ of a furlong from $\frac{2}{9}$ of a mile.

Ans. 1fur. 5rd. 10ft. 10in.

4. Take $\frac{3}{5}$ of a mile from $\frac{4}{7}$ of a degree.
Ans. 48m. 6fur. 17rd. 8ft. $7\frac{5}{7}$in.

5. From $\frac{4}{11}$ of an acre take $\frac{4}{5}$ of a rod.
Ans. 1R. 17p. 22yd. 2ft. 108in.

6. From $\frac{9}{10}$ of a cord take $\frac{2}{11}$ of a cord.

7. From $\frac{7}{13}$ of a hogshead of wine there leaked out $\frac{4}{5}$ of it; what remained? Ans. 6gal. 3qt. 0pt. $1\frac{7}{65}$gi.

8. From Boston to Concord, N. H., the distance is 72 miles; $\frac{4}{7}$ of this distance having been travelled, how much remains?
Ans. 30m. 6fur. 34rd. 4ft. $8\frac{4}{7}$in.

9. From $\frac{2}{7}$ of a year take $\frac{4}{5}$ of a week.
Ans. 101da. 5h. 54m. $17\frac{1}{7}$sec.

10. From $\frac{4}{11}$ of an acre take $\frac{1}{2}$ of a foot.

MISCELLANEOUS EXAMPLES IN FRACTIONS.

1. How far will a man walk in $17\frac{3}{11}$ hours, provided he goes at the rate of $4\frac{7}{9}$ miles an hour?
Ans. 82m. 4fur. 8rd. 1ft. 4in.

2. How much land is there in a field which is $29\frac{7}{13}$ rods square? Ans. 5A. 1R. 32p. 141ft. $109\frac{155}{169}$in.

3. How much wood in a pile which is $17\frac{3}{4}$ feet long, $7\frac{1}{11}$ feet high, and $4\frac{3}{5}$ feet wide? Ans. 4C. $66\frac{107}{110}$ft.

4. What is the value of $19\frac{7}{8}$ barrels of flour, at \$ $6\frac{3}{4}$ a barrel?

5. What is the value of $376\frac{11}{18}$ acres of land, at \$ $75\frac{3}{8}$ per acre? Ans. \$ $28387.06\frac{1}{4}$.

6. What cost $17\frac{19}{112}$ quintals of fish, at \$ 4.75 per quintal? Ans. \$ $81.55\frac{65}{112}$.

7. What cost $1670\frac{7}{13}$ pounds of coffee, at $12\frac{3}{4}$ cents per pound? Ans. \$ $212.99\frac{19}{52}$.

8. What cost $28\frac{4}{11}$ tons of Lackawana coal, at \$ $11\frac{3}{4}$ a ton? Ans. \$ $333.27\frac{3}{11}$.

9. Bought $37\frac{11}{63}$ hogsheads of molasses, at \$ $17.62\frac{1}{2}$ a hogshead; what was the whole cost? Ans. \$ $655.20\frac{5}{21}$.

10. What cost $\frac{7}{8}$ of a cord of wood, at \$ 5.75 a cord?

11. What are the contents of a field which is $139\frac{4}{7}$ rods long, and $38\frac{4}{5}$ rods wide? Ans. 33A. 3R. $15\frac{13}{35}$p.

12. Bought 15 loads of wood, each containing $11\frac{3}{8}$ feet, cord

measure. I divide it equally between 9 persons; what does each receive? Ans. $19\frac{1}{3}$ft.

13. If the transportation of $18\frac{3}{8}$ tons of iron costs \$48.15$\frac{3}{4}$, what is it per ton? Ans. \$2.62$\frac{4}{49}$.

14. If a hogshead of wine costs \$98$\frac{1}{8}$, what is the price of one gallon?

15. If 5 bushels of wheat cost \$8$\frac{2}{9}$, what will a bushel be worth? Ans. \$1.64$\frac{4}{9}$.

16. What will 11 hogsheads and $17\frac{1}{2}$ gallons of wine cost, at $19\frac{3}{4}$ cents a gallon? Ans. \$140.32$\frac{3}{8}$.

17. How many bottles, each containing $1\frac{3}{4}$ pints, are sufficient for bottling a hogshead of cider? Ans. 288.

18. I have a shed which is $18\frac{7}{12}$ feet long, $10\frac{5}{12}$ feet wide, and $7\frac{11}{12}$ feet high; how many cords of wood will it contain? Ans. 11C. $124\frac{829}{1728}$ft.

19. What will $6\frac{7}{8}$ pounds of tea cost, at $65\frac{3}{4}$ cents per pound? Ans. \$4.52$\frac{1}{32}$.

20. How many cubic feet does a box contain, that is $8\frac{3}{4}$ feet long, $5\frac{7}{12}$ feet wide, and 3 feet high? Ans. $146\frac{9}{16}$ft.

21. How many feet of boards will it take to cover a side of a house which is $46\frac{5}{12}$ feet long and $17\frac{1}{2}$ feet high?

22. Required the number of square feet on the surface of 7 boxes, each of which is $5\frac{1}{2}$ feet long, $2\frac{5}{12}$ feet high, and $3\frac{1}{12}$ feet wide; required also the number of cubic feet they would occupy? Ans. $527\frac{59}{72}$ft.; $286\frac{253}{288}$ cubic feet.

23. A certain room is 12 feet long, $11\frac{1}{2}$ feet wide, and $7\frac{1}{2}$ feet high; how much will it cost to plaster it, at $2\frac{3}{4}$ cents per square foot? Ans. \$13.48$\frac{7}{8}$.

24. A man has a garden that is $14\frac{1}{2}$ rods long, and $10\frac{1}{4}$ rods wide; he wishes to have a ditch dug around it, that shall be 3 feet wide and $4\frac{1}{2}$ feet deep; what will be the expense, if he give 2 cents per cubic foot? Ans. \$223.76$\frac{1}{4}$.

25. How many bushels of grain will a box contain which is $14\frac{7}{12}$ feet long, $5\frac{11}{12}$ feet deep, and $4\frac{1}{4}$ feet wide, there being $2150\frac{2}{5}$ cubic inches in a bushel? Ans. $294\frac{2429}{3584}$bu.

26. Which will contain the most, and by how much, a box that is 10 feet long, 8 feet wide, and 6 feet deep, or a cubical one, each of whose sides measures 8 feet?

Ans. The last contains 32 cubic feet the most.

27. Which will contain the most gallons, a cistern that is $7\frac{1}{2}$ feet long, 6 feet wide, and $5\frac{1}{2}$ feet deep, or one that is $9\frac{1}{2}$ feet long, $4\frac{1}{2}$ feet wide, and $5\frac{1}{2}$ feet deep?

Ans. The first cistern contains $92\frac{4}{7}$ gallons most.

28. My field has four sides. The first side is 31 rods $13\frac{3}{10}$ feet in length; the second, 41 rods $1\frac{9}{10}$ feet; the third, 38 rods $0\frac{1}{5}$ feet; and the fourth, 45 rods $12\frac{7}{10}$ feet. I wish to enclose this field with a rail-fence four rails high, using rails of equal length. Required the length of the longest rails that can be used, allowing that the rails lap by each other $\frac{7}{10}$ of a foot; also the number of rails it will take to fence it.

Ans. Length, $13\frac{1}{2}$ feet; number, 808.

29. Required the least number of yards of velvet, expressed by a whole number, that can be cut up without waste, into vest patterns of $\frac{5}{8}$, $\frac{3}{4}$, or $\frac{6}{7}$ yards each? Ans. 30 yards.

30. If a company whose capital stock is divided into 100 equal shares should conclude to divide the same stock into only 30 shares, how much larger would a share of the latter be than one of the former size?

31. D. Ripley's farm contains 31A. 3R. 6p., and J. Ford's farm contains 39A. 2R. $37\frac{1}{2}$p. What is the fraction, in its simplest form, that will express their comparative size? Ans. $\frac{4}{5}$.

32. Bought 68 barrels of flour, at $\$7\frac{11}{12}$ per barrel; what was the amount of the whole? Ans. $\$538\frac{1}{3}$.

33. What cost $8\frac{2}{3}$ acres of land, at $\$42\frac{3}{5}$ per acre?

Ans. $369.20.

34. How shall four 3's be arranged, that their value shall be nothing?

35. I have a room 20 feet long, 15 feet wide, and $8\frac{1}{2}$ feet high. This room contains 4 windows, each of which is $5\frac{1}{2}$ feet in height and $3\frac{1}{3}$ feet in width. There are two doors 7 feet high and 3 feet wide. The mop-boards are $\frac{2}{3}$ of a foot wide. A mason has agreed to plaster this room at $6\frac{1}{4}$ cents per square yard; a painter is to lay on the paper at 9 cents per square yard; the paper which I wish to have laid on is $2\frac{3}{5}$ feet wide, for which I pay 5 cents per yard. What is the amount of my bill for plastering, for papering, and for paper?

Ans. For plastering, $\$5.11\frac{29}{36}$; for papering, $4.37; for paper, $\$2.80\frac{5}{39}$.

EXAMPLES TO BE PERFORMED BY ANALYSIS.

1. If $\frac{7}{8}$ of a bushel of corn cost 63 cents, what cost a bushel? what cost 15 bushels? Ans. \$ 10.80.

ILLUSTRATION.—If 7 eighths of a bushel cost 63 cents, 1 eighth will cost 1 seventh of 63 cents = 9 cents; and 8 eighths will cost 8 times 9 cents = 72 cents, and 15 bushels will cost 15 times 72 cents = \$ 10.80.

2. If $4\frac{7}{9}$lb. of pepper cost \$ 2.15, what cost 1 pound? what cost 30lb.? Ans. \$ 13.50.

ILLUSTRATION.—In $4\frac{7}{9}$lb. there are $\frac{43}{9}$lb. Then, if 43 ninths lb. cost \$ 2.15, 1 ninth will cost 1 forty-third of \$ 2.15 = \$ 0.05, and 9 ninths or 1lb. will cost 9 times \$ 0.05 = \$ 0.45, and 30lb. will cost 30 times \$ 0.45 = \$ 13.50.

3. When \$ 1728 are paid for $30\frac{4}{15}$ tons of iron, what cost 1 ton? what cost $7\frac{17}{30}$ tons? Ans. \$ 432.

4. When \$ 432 are paid for $7\frac{17}{30}$ tons of iron, what quantity should be received for \$ 1728?

5. For $7\frac{17}{30}$ tons of iron there were paid \$ 432; what sum will it require to pay for $30\frac{4}{15}$ tons?

6. For $30\frac{4}{15}$ tons of iron \$ 1728 were paid; what quantity should be received for \$ 432?

7. Gave $7\frac{7}{10}$ bushels of rye for a barrel of flour; how much rye will it then require to purchase $6\frac{4}{9}$ barrels of flour?

Ans. $49\frac{28}{45}$ bushels.

8. Divide \$ 1728 among 17 boys and 15 girls, and give each boy $\frac{7}{11}$ as much as a girl; what sum will each receive?

Ans. Each girl, \$ $66\frac{66}{71}$; each boy \$ $42\frac{42}{71}$.

9. If $\frac{7}{9}$ of a ton of hay cost \$ 14.49, what cost $4\frac{3}{7}$ tons?

10. If $4\frac{3}{7}$ tons of hay cost \$ $82.50\frac{3}{7}$, what part of a ton will \$ 14.49 buy?

11. If \$ 14.49 will buy $\frac{7}{9}$ of a ton of hay, how much hay can be obtained for \$ $82.50\frac{3}{7}$?

12. When \$ $82.50\frac{3}{7}$ are paid for $4\frac{3}{7}$ tons of hay, what will be the cost of $\frac{7}{9}$ of a ton?

13. When $14\frac{7}{8}$ tons of copperas are sold for \$ 500, what is the value of 1 ton? what is the value of $9\frac{1}{12}$ tons?

14. When $9\frac{11}{12}$ tons of copperas are sold for \$ $333.33\frac{1}{3}$, what is the value of $14\frac{7}{8}$ tons?

15. Gave \$ $333.33\frac{1}{3}$ for $9\frac{11}{12}$ tons of copperas; what quantity of copperas should be received for \$ 500?

16. For $14\frac{7}{8}$ tons of copperas \$ 500 were paid; how much might be purchased for \$ $333.33\frac{1}{3}$?

17. Purchased $97\frac{7}{8}$ gallons of molasses for \$ 31.32; what cost 1 gallon? what cost $763\frac{5}{8}$ gallons?

18. Sold $763\frac{5}{8}$ gallons of molasses for \$ 244.36; what should I receive for $97\frac{7}{8}$ gallons?

19. If \$ 244.36 will buy $763\frac{5}{8}$ gallons of molasses, what quantity can be obtained for \$ 31.32?

20. Gave 1975lb. of flax for 40 barrels of flour; how many pounds were given for 1 barrel? how many pounds would it require to buy 144 barrels? Ans. 7110lb.

21. If 17 bushels of rye cost \$ 15.75, what cost 1 bushel? what cost $9\frac{1}{4}$ bushels? Ans. \$ $8.56\frac{67}{68}$.

22. If 9 barrels of flour cost \$ $50\frac{4}{7}$, what cost 1 barrel? what cost $87\frac{3}{5}$ barrels? Ans. \$ $492\frac{8}{35}$.

23. If 13 boarders consume a barrel of pork in 78 days, how long would it last, if 7 more boarders were added to their number? Ans. $50\frac{7}{10}$ days.

24. If a man by laboring 10 hours a day can, in 9 days, perform a certain piece of work, how many days would it require to do the same work, were he to labor 15 hours a day?

25. If a man, by laboring 15 hours a day, in 6 days can perform a certain piece of work, how many days would it require to do the same work by laboring 10 hours a day?

26. If a man, by laboring 10 hours a day, can in 9 days perform a certain piece of work, how many hours must he labor each day to perform the same work in 6 days?

27. Sold $17\frac{3}{11}$ bushels of corn for \$ $5\frac{3}{7}$; what was received for 1 bushel? what should I have charged for $97\frac{5}{9}$ bushels? Ans. \$ $30\frac{208}{315}$.

28. Bought $9\frac{3}{5}$ tons of hay at \$ $19\frac{7}{9}$ per ton; for what must it be sold per cwt. to gain \$ 7 on my bargain? Ans. \$ $1\frac{73}{2880}$.

29. If I sell hay at \$ $1\frac{3}{4}$ per cwt., what should I give for $9\frac{3}{5}$ tons, that I may make \$ 7 on my bargain? Ans. \$ 329.

30. How many bushels of corn at \$ 0.75 per bushel will

it require to purchase $47\frac{3}{11}$ bushels of wheat at \$ $2\frac{2}{3}$ per bushel? Ans. $168\frac{8}{99}$bu.

31. If 15 cords of wood cost \$ $57\frac{9}{11}$, what cost 1 cord? what cost $19\frac{7}{8}$ cords?

32. If $19\frac{7}{8}$ cords of wood cost \$ $76\frac{67}{110}$, how many cords may be obtained for \$ $57\frac{9}{11}$?

33. At $7\frac{3}{10}$ shillings per yard, what cost $47\frac{1}{3}$ yards? Ans. 17£. 5s. $6\frac{2}{5}$d.

34. When 172£. 15s. $0\frac{2}{5}$d. are paid for $47\frac{1}{3}$ yards of broadcloth, what is the value of 1 yard? Ans. 3£. 12s. $11\frac{328}{355}$d.

35. If 1lb. of sugar cost $\frac{7}{13}$ of a dollar, what is the value of $43\frac{6}{7}$lb.? Ans. \$ $23.61\frac{7}{13}$.

36. If $17\frac{3}{8}$lb. of sugar cost \$ $2\frac{7}{11}$, what cost 50lb. Ans. \$ $7.58\frac{1018}{1529}$.

37. Bought $87\frac{3}{7}$ yards of broadcloth for \$ 612; what was the value of $14\frac{7}{10}$ yards? Ans. \$ 102.90.

38. If $\frac{7}{8}$ of an acre of land cost \$ 43.75, what cost 10 acres?

39. When \$ 500 are paid for 10 acres of land, how much might be obtained for \$ 43.75?

40. If 9 hogsheads of sugar cost \$ 71.87, what cost $\frac{4}{7}$ of a hogshead?

41. Paid \$ $4.56\frac{20}{63}$ for $\frac{4}{7}$ of a hogshead of sugar; what ought to be given for 9 hogsheads?

42. If 19 men can grade a certain road in 111 days, how long would it require 47 men to perform the same labor?

43. When 47 men can grade a certain road in $44\frac{4}{47}$ days, how long would it require 19 men to perform the same labor?

44. If $\frac{4}{11}$ of a ton of hay cost \$ 9.20, what cost 17 tons?

45. When \$ 430.10 are paid for 17 tons of hay, what cost $\frac{4}{11}$ of a ton?

46. If $\frac{7}{16}$ of a tub of butter cost \$ 7.15, what cost 7 tubs?

47. When \$ 114.40 are paid for 7 tubs of butter, what cost $\frac{7}{16}$ of a tub?

48. If a horse eat $19\frac{3}{7}$ bushels of oats in $87\frac{3}{7}$ days, how many will 7 horses eat in 60 days? Ans. $93\frac{1}{3}$ bushels.

49. Henry Smith can reap a field in 10 days, by laboring 8 hours a day. His son John can reap the same field in 9 days, by laboring 12 hours a day. How long would it take both to reap the field, provided they labored 8 hours a day? Ans. $5\frac{2}{47}$ days.

DECIMAL FRACTIONS.

259. A DECIMAL FRACTION is a fraction whose denominator is some power of ten.

It, therefore, originates from dividing a *unit*, first into 10 equal parts, and then each of these parts into 10 other equal parts, and so on indefinitely, so that its fractional units are tenths, hundredths, thousandths, or some like order of parts.

260. Decimal fractions, their denominators being obvious, are commonly expressed by writing the numerator only, with the decimal point (.) before it, care being taken to put a cipher in any decimal place not requiring a digit; thus,

$\frac{9}{10}$	may be written	.9	and be read	9 tenths.
$\frac{13}{100}$	"	.13	"	13 hundredths.
$\frac{5}{1000}$	"	.005	"	5 thousandths.
$\frac{105}{10000}$	"	.0105	"	105 ten-thousandths.

261. By examining the foregoing fractions, it will be seen that, —

1. *The denominator of a decimal fraction is* 1 *with as many ciphers annexed as the numerator has places of figures.*

2. *In writing a decimal fraction without its denominator, every decimal place not having a significant figure must be filled by a cipher.*

3. *The first figure or place of a decimal fraction on the right of the decimal point is tenths; the second, hundredths; the third, thousandths; the fourth, ten-thousandths; &c.*

4. *Each figure in the expression of decimal fractions, as in whole numbers, represents value, according to its distance from the place of units.*

262. A whole number and a decimal fraction, in a single expression, constitute a mixed number. Thus, 17.63 is a mixed number, and is read seventeen, and decimal sixty-three hundredths; 150.302, read one hundred and fifty, and decimal three hundred and two thousandths.

NOTE. — For the sake of brevity, especially in reading mixed numbers, as in the instance just given, a decimal fraction is commonly called simply a decimal.

263. If ciphers are placed on the left of decimal figures, between them and the decimal point, those figures change their places, *each cipher* removing them *one* place to the right, and thus diminishing the value represented *tenfold*. Thus, .9 = $\frac{9}{10}$, but .09 = $\frac{9}{100}$, and .009 = $\frac{9}{1000}$.

264. If ciphers are placed on the right of decimal figures, or are taken away, since their places remain the same, the value represented is not changed. Thus, .7 = $\frac{7}{10}$, and .70 = $\frac{70}{100}$ = $\frac{7}{10}$ = .7.

265. By regarding the dollar as a unit, we may consider cents and mills of United States money as fractional parts of a decimal character. Thus, 3 dollars and 25 cents, is 3 dollars and 25 hundredths of a dollar, or $ 3.25; also, 10 dollars 12 cents and 5 mills, is 10 dollars and 125 thousandths of a dollar, or $ 10.125.

NOTATION AND NUMERATION OF DECIMALS.

266. The relation of decimals to whole numbers and to each other, and also the names of their different orders and places, are shown by the following

TABLE.

Millions.	Hundr. of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.	Ten-Millionths.	Hundred-Millionths.	Billionths.
7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9	2
7th order or place,	6th order or place,	5th order or place,	4th order or place,	3d order or place,	2d order or place,	1st order or place,		1st order or place,	2d order or place,	3d order or place,	4th order or place,	5th order or place,	6th order or place,	7th order or place,	8th order or place,	9th order or place,
Whole Numbers.								Decimals.								

Of the mixed number expressed in the table, the part on the left of the decimal point is the whole number, and that on the right the decimal. The decimal part is numerated from the left to the right, and the value represented is expressed in words thus: Two hundred thirty-four million five hundred sixty-seven thousand eight hundred ninety-two billionths. And the mixed number thus: Seven million six hundred fifty-four thousand three hundred twenty-one, and decimal two hundred thirty-four million five hundred sixty-seven thousand eight hundred ninety-two billionths.

267. From the table we deduce the following rules:

1. READ *a decimal as though it were a whole number, adding the name of the right-hand order.*

2. WRITE *a decimal as though it were a whole number, supplying with ciphers such places as have no significant figures.*

EXAMPLES.

Express orally, or write in words the following numbers: —

1.	.056	6.	1.631	11.	1.000007
2.	.1003	7.	48.07	12.	5.101016
3.	.2786	8.	1.315	13.	1.000327
4.	.16302	9.	5.6001	14.	0.000001
5.	.97500	10.	87.0006	15.	16.000000007

Express in the decimal form by figures: —

16.	$\frac{13}{100}$	19.	$\frac{406}{1000000}$	22.	$7\frac{17}{10000}$
17.	$\frac{6}{1000}$	20.	$\frac{1}{10000000}$	23.	$333\frac{3}{1000}$
18.	$\frac{19}{10000}$	21.	$\frac{1031}{10000000}$	24.	$1\frac{1}{10000000}$

25. Three hundred twenty-five, and seven tenths.

26. Four hundred sixty-five, and fourteen hundredths.

27. Ninety-three, and seven hundredths.

28. Twenty-four, and nine millionths.

29. Two hundred twenty-one, and nine hundred-thousandths.

30. Forty-nine thousand, and forty-nine thousandths.

31. Seventy-nine million two thousand, and one hundred five thousandths.

32. Sixty-nine thousand fifteen, and fifteen hundred-thousandths.

33. Eighty thousand, and eighty-three ten-thousandths.

34. Nine billion nineteen thousand nineteen, and nineteen hundredths.

35. Twenty-seven, and nine hundred twenty-seven thousandths.

36. Forty-nine trillion, and one trillionth.

37. Twenty-one, and one ten-thousandth.

38. Eighty-seven thousand, and eighty-seven millionths.

39. Ninety-nine thousand ninety-nine, and nine thousand nine billionths.

40. Seventeen, and one hundred seventeen ten-thousandths.

41. Thirty-three, and thirty-three hundredths.

42. Forty-seven thousand, and twenty-nine ten-millionths.

43. Fifteen, and four thousand seven hundred-thousandths.

44. Eleven thousand, and eleven hundredths.

45. Seventeen, and eighty-one quadrillionths.

46. Nine, and fifty-seven trillionths.

47. Sixty-nine thousand, and three hundred forty-nine thousandths.

268. Decimals, since they increase from right to left, and decrease from left to right, by the scale of ten, as do simple whole numbers, may be added, subtracted, multiplied, and divided in the same manner.

ADDITION OF DECIMALS.

269. Ex. 1. Add together 23.61, 161.5, 2.6789, and 61.111. Ans. 248.8999.

OPERATION.

```
  23.61
 161.5
   2.6789
  61.111
 --------
 248.8999
```

We write the numbers so that figures of the same decimal place shall stand in the same column, and then, beginning at the right hand, add them as whole numbers are added, and place the decimal point in the result directly under those above.

RULE. — *Write the numbers so that figures of the same decimal place shall stand in the same column.*

Add as in whole numbers, and point off in the sum, from the right hand, as many places for decimals as equal the greatest number of decimal places in any of the numbers added.

Proof.—The proof is the same as in addition of whole numbers.

EXAMPLES.

2. Add together the following numbers: 81.61356, 6716.31, 413.1678956, 35.14671, 3.1671, 314.6. Ans. 7564.0052656.

3. What is the sum of the following numbers: 1121.6116, 61.87, 46.67, 165.13, 676.167895? Ans. 2071.449495.

4. Add 7.61, 637.1, 6516.14, 67.1234, 6.1234 together.
Ans. 7234.0968.

5. Add 21.611, 6888.32, 3.6167 together.
Ans. 6913.5477.

6. Add together $15.06, $107.09, $1.625, and $93.765.

7. I have bought a horse for $137.50, a wagon for $55.63, a whip for $1.375, and a halter for $0.87½; what did they all cost? Ans. $195.38.

8. What is the sum of twenty-three million ten; one thousand, and five hundred-thousandths; twenty-seven, and nineteen millionths; seven, and five tenths? Ans. 23001044.500069.

9. Add the following numbers: fifty-nine, and fifty-nine thousandths; twenty-five thousand, and twenty-five ten-thousandths; five, and five millionths; two hundred five, and five hundredths. Ans. 25269.111505.

10. What is the sum of the following numbers: twenty-five, and seven millionths; one hundred forty-five, and six hundred forty-three thousandths; one hundred seventy-five, and eighty-nine hundredths; seventeen, and three hundred forty-eight hundred-thousandths? Ans. 363.536487.

11. A farmer has sold at one time 3 tons and 75 hundredths of a ton of hay, at another time 11 tons and 7 tenths of a ton, and at a third time 16 tons and 125 thousandths of a ton. How much has he sold in all? Ans. 31.575.

12. Add together 73 and 29 hundredths, 87 and 47 thousandths, 3005 and 116 ten-thousandths, 28 and 3 hundredths, 29000 and 5 thousandths.

13. Add together two hundred nine thousand, and forty-six millionths; ninety-eight thousand two hundred seven, and fifteen ten-thousandths; fifteen, and eight hundredths; and forty-nine ten-thousandths. Ans. 307222.086446.

SUBTRACTION OF DECIMALS.

270. Ex. 1. From 61.9634 take 9.182. Ans. 52.7814.

OPERATION.
6 1.9 6 3 4
9.1 8 2
5 2.7 8 1 4

Having written the less number under the greater, so that figures of the same decimal place stand in the same column, we subtract as in whole numbers, and place the decimal point in the result, as in addition of decimals.

RULE. — *Write the less number under the greater, so that figures of the same decimal place shall stand in the same column.*

Subtract as in whole numbers, and point off the remainder as in addition of decimals.

Proof. — The proof is the same as in subtraction of whole numbers.

EXAMPLES.

2.	3.	4.	5.
3 9.3	5.	6.1	4 1.7
1.6 7 8 9	1.6 7 8	1.9 9 9 9 9	2 1.9 7 6 7
3 7.6 2 1 1	3.3 2 2	4.1 0 0 0 1	1 9.7 2 3 3

6. From 29.167 take 19.66711. Ans. 9.49989.
7. From 91.61 take 2.6671. Ans. 88.9429.
8. From 96.71 take 96.709.
9. Take twenty-seven, and twenty-eight thousandths from ninety-seven, and seven tenths. Ans. 70.672.
10. Take one hundred fifteen, and seven hundredths from three hundred fifteen, and twenty-seven ten-thousandths. Ans. 199.9327.
11. From twenty-nine million four thousand and five take twenty-nine thousand, and three hundred forty-nine thousand two hundred, and twenty-four hundred-thousandths. Ans. 28625804.99976.
12. From one million take one millionth. Ans. 999999.999999.
13. From $ 19 take $ 1.375. Ans. $ 17.625.
14. A merchant bought flour to the amount of $ 316.87½, and sold it for $ 400; how much did he gain by the sale?
15. From 19 million take 19 billionths. Ans. 18999999.999999981.

16. Charles Washburne has in one farm 93.45 acres, in another 124 acres, in a third 244.285 acres, and in wood-lots 216.136 acres; how many acres more would he require to have exactly 1000 acres?

MULTIPLICATION OF DECIMALS.

271. Ex. 1. Multiply 76.81 by 3.2. Ans. 245.792.

OPERATION.

```
  7 6.8 1
      3.2
  -------
 1 5 3 6 2
2 3 0 4 3
---------
2 4 5.7 9 2
```

We multiply as in whole numbers, and point off on the right of the product as many figures for decimals as there are decimal figures in the multiplicand and multiplier counted together. The reason for pointing off the decimals in the product, as in the operation, will be seen, if we convert the multiplicand and multiplier into common fractions, and multiply them together. Thus, $76.81 = 76\frac{81}{100} = \frac{7681}{100}$; and $3.2 = 3\frac{2}{10} = \frac{32}{10}$. Then $\frac{7681}{100} \times \frac{32}{10} = \frac{245792}{1000} = 245\frac{792}{1000} =$ 245.792, Ans., the same as in the operation.

2. Multiply .1234 by .0046. Ans. .00056764.

OPERATION.

```
     .1 2 3 4
     .0 0 4 6
     --------
      7 4 0 4
     4 9 3 6
-------------
.0 0 0 5 6 7 6 4
```

Since the number of figures in the product is not equal to the number of decimals in the multiplicand and multiplier, we supply the deficiency by placing ciphers on the left hand. The reason of this process will appear, if we perform the operation thus: $.1234 = \frac{1234}{10000}$; and $.0046 = \frac{46}{10000}$. Then $\frac{1234}{10000} \times \frac{46}{10000} = \frac{56764}{100000000} = .00056764$, Ans., the same as in the operation.

RULE. — *Multiply as in whole numbers, and point off as many figures for decimals, in the product, as there are decimal figures in the multiplicand and multiplier.*

If there be not so many figures in the product as there are decimal figures in the multiplicand and multiplier, supply the deficiency by prefixing ciphers.

Proof. — The proof is the same as in multiplication of whole numbers.

EXAMPLES.

3. Multiply 61.76 by .0071. Ans. .438496.
4. Multiply .0716 by 1.326. Ans. .0949416.
5. Multiply .61001 by .061.
6. Multiply 71.61 by 365. Ans. 26137.65.
7. Multiply .1234 by 1234. Ans. 152.2756.

8. Multiply 6.711 by 6543. Ans. 43910.073.

9. Multiply .0009 by .0009. Ans. .00000081.

10. What is the product of one thousand and twenty-five, multiplied by three hundred and twenty-seven ten-thousandths? Ans. 33.5175.

11. What is the product of seventy-eight million two hundred five thousand and two, multiplied by fifty-three hundredths? Ans. 41448651.06.

12. Multiply one hundred and fifty-three thousandths by one hundred twenty-nine millionths. Ans. .000019737.

13. What will 26.7 yards of cloth cost, at $5.75 a yard? Ans. $153.525.

14. What will 14.75 bushels of wheat cost, at $1.25 a bushel? Ans. $18.4375.

15. What will 375.6 pounds of sugar cost, at $0.125 per pound?

16. What will 26.58 cords of wood cost, at $5.625 a cord? Ans. $149.512½.

17. What will 28.75 tons of potash cost, at $125.78 per ton? Ans. $3616.175.

CONTRACTIONS IN MULTIPLICATION OF DECIMALS.

272. To multiply a decimal by 10, 100, 1000, &c. *Remove the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers if required.* Thus, $1.25 \times 10 = 12.5$; and $1.6 \times 100 = 160$.

Examples.

1. Multiply 131.634 by 1000. Ans. 131634.

2. Multiply 3478.9 by 100. Ans. 347890.

3. Multiply one thousandth by one thousand.

4. What is the profit on one million yards of cotton cloth, at $0.007 per yard. Ans. $7000.

273.° When it is not necessary that all the decimal places of the product should be retained, tedious multiplications may often be obviated, by contracting the work as follows: —

Write the units' place of the multiplier under that figure of the multiplicand whose place it is proposed to retain in the product, and dispose of all the other figures of the multiplier in an order contrary to the usual one. Then, in multiplying, begin, for each partial product, with that figure of the multiplicand which stands above the multiplying figure, observing to add to the product the number nearest to that which would have been carried if the places at the right had not been rejected. Write down the several partial products, so that the right-hand figure of each shall be in the same column, and their sum will be the product required.

EXAMPLES.

1. Multiply 3.141592 by 52.7438, retaining only four places for decimals in the product. Ans. 165.6995

FIRST OPERATION.

3.1 4 1 5 9 2	=	Multiplicand
8 3 4 7.2 5	=	Multiplier reversed.
1 5 7 0 7 9 6	=	Product by 5, + 1
6 2 8 3 2	=	Product by 2, + 2
2 1 9 9 1	=	Product by 7, + 4
1 2 5 7	=	Product by 4, + 1
9 4	=	Product by 3, + 1
2 5	=	Product by 8, + 1
1 6 5.6 9 9 5	=	Product sought.

SECOND OPERATION.

3.1 4	1 5 9 2
	5 2.7 4 3 8
2 5	1 3 2 7 3 6
9 4	2 4 7 7 6
1 2 5 6	6 3 6 8
2 1 9 9 1	1 4 4
6 2 8 3 1	8 4
1 5 7 0 7 9 6	0
1 6 5.6 9 9 5	0 0 1 2 9 6

By comparison of the two methods of solution, it will be seen that the common one, as shown in the second operation, gives ten places of decimals, or six more than are required by the question, thus rendering unnecessary the several figures on the right of the vertical line. By the contrasted way, the multiplier, for convenience, has its figures reversed, or placed contrary to the usual order, so that the product of each figure by the one of the multiplicand above it, must be of the order of ten-thousandths. The first figure, at the right, of each partial product, being of the order of ten-thousandths, is written in the same column. To the product by 5 we add 1, since, if the 2 in the multiplicand had not been rejected, there would have been 1 to carry to the product of the 9 by the 5; to the product by 2 we add 2, since the product of the rejected figures, 92, by 2, approximates to 2 hundred, which would require 2 to be carried; to the product by 7 we add 4, since the product of the two rejected figures, 59, by 7, would require 4 to be carried; to the pro-

duct by 4 we add 1, since the product of the two rejected figures, 15, by 4, approximates to 1 hundred, which would require 1 to be carried; and so on, it being sufficient to increase the partial product only by such a number as approximates most nearly to that which would have been carried, provided the two rejected figures next to the figure of the multiplicand had been retained.

2. Multiply 325.701428 by .7218393, retaining only three places of decimals in the product. Ans. 235.104.

3. Multiply 56.7534916 by 5.376928, retaining only five places of decimals in the product.

4. Multiply 843.7527 by 8634.175, retaining only the integers in the product. Ans. 7285109.

DIVISION OF DECIMALS.

274. Ex. 1. Divide 1.728 by 1.2. Ans. 1.44.

OPERATION.

```
1.2 ) 1.7 2 8 ( 1.4 4 Ans.
      1 2
      ---
        5 2
        4 8
        ---
          4 8
          4 8
          ---
```

We divide as in whole numbers, and, since the divisor and quotient are the two factors, which, being multiplied together, produce the dividend, we point off two decimal figures in the quotient, to make the number in the two factors equal to the number in the product or dividend.

The reason for pointing off will also be seen by performing the example with the decimals in the form of common fractions. Thus, $1.728 = 1\frac{728}{1000} = \frac{1728}{1000}$; and $1.2 = 1\frac{2}{10} = \frac{12}{10}$. Then $\frac{1728}{1000} \div \frac{12}{10} = \frac{1728}{1000} \times \frac{10}{12} = \frac{17280}{12000} = \frac{144}{100} = 1\frac{44}{100} = 1.44$, Ans. as before.

2. Divide 36.6947 by 589. Ans. .0623.

OPERATION.

```
5 8 9 ) 3 6.6 9 4 7 ( .0 6 2 3 Ans.
        3 5 3 4
        -------
          1 3 5 4
          1 1 7 8
          -------
            1 7 6 7
            1 7 6 7
            -------
```

We divide as in whole numbers, and since we have but three figures in the quotient, we place a cipher before them, and thus make the decimal places in the divisor and quotient equal to those of the dividend.

The reason for *prefixing* the cipher will appear more obvious by solving the example in the form of common fractions. Thus, $36.6947 = 36\frac{6947}{10000} = \frac{366947}{10000}$; and $589 = \frac{589}{1}$. Then $\frac{366947}{10000} \div \frac{589}{1} = \frac{366947}{10000} \times \frac{1}{589} = \frac{366947}{5890000} = \frac{623}{10000} = .0623$, Ans. as before. Hence the following

RULE. — *Divide as in whole numbers, and point off as many figures in the quotient as the number of decimal places in the dividend exceeds the number in the divisor; but if there are not as many, supply the deficiency by prefixing ciphers.*

NOTE 1. — When the decimal places in the divisor exceed those in the dividend, make them equal by annexing ciphers to the dividend, and the quotient will be a whole number.

NOTE 2. — When there is a remainder after dividing the dividend, ciphers may be annexed, and the division continued; the ciphers thus annexed being regarded as decimals of the dividend; and to indicate in any case that the division does not terminate, the sign plus (+) can be used.

Proof. — The proof is the same as in division of whole numbers.

EXAMPLES.

3. Divide 780.516 by 2.43. Ans. 321.2.
4. Divide 7.25406 by 9.57. Ans. .758.
5. Divide .21318 by .38.
6. Divide 7.2091365 by .5201. Ans. 13.861+.
7. Divide 56.8554756 by .0759. Ans. 749.084.
8. Divide 119109094.835 by 38123.45. Ans. 3124.3.
9. Divide 1191090.94835 by 3812345.
10. Divide 11910909483.5 by 38.12345.
11. Divide 11.9109094835 by 381234.5.
12. Divide 1191.09094835 by 3.812345.
13. Divide 11910909483.5 by .3812345.
14. Divide 1.19109094835 by 3.812345.
15. Divide .119109094835 by .3812345.
16. Divide 30614.4 by .9567. Ans. 32000.
17. Divide .306144 by 9567. Ans. .000032.
18. Divide four thousand three hundred twenty-two, and four thousand five hundred seventy-three ten-thousandths by eight thousand, and nine thousandths. Ans. .5403+.
19. How many yards of calico at $ 0.0775 per yard can be purchased for $ 10.85?
20. What costs 1 acre of woodland when 19.65 acres are sold for $ 982.50? Ans. $ 50.
21. Divide three hundred twenty-three thousand seven hundred sixty-five by five millionths. Ans. 64753000000.

CONTRACTIONS IN DIVISION OF DECIMALS.

275. To divide a decimal by 10, 100, 1000, &c.

Remove the decimal point as many places to the left as there are ciphers in the divisor, and if there be not figures enough in the number, prefix ciphers. Thus, $2.15 \div 10 = .215$; and $1.9 \div 100 = .019$.

EXAMPLES.

1. Divide 31.675 by 10.
2. Divide 916.05 by 100.
3. Divide 7.0461 by 100000.
4. Divide 70.461 by 100000.
5. Divide 704.61 by 100000.
6. Divide 7046.1 by 100000.
7. Divide 70460 by 100000.
8. Divide .70460 by 100000.
9. Divide 196.5 by 1000000. Ans. .0001965.
10. If $ 3500 are paid for 1000 yards of broadcloth, what is it a yard? Ans. $ 3.50.
11. When $ 1025 are paid for 40 boxes of sugar, each containing 250 pounds, what is the cost of 1 pound?
Ans. $ 0.10¼.

276.° When the divisor contains many decimal places, and only a certain number of decimals are required to be retained in the quotient, the work may be contracted as follows:—

First consider how many figures, in all, it is necessary for the quotient to contain. Then, by using the same number of figures from the left of the divisor, find the first figure of the quotient, and, instead of bringing down a new figure from the dividend, or annexing a cipher to the remainder, reject a figure on the right of the divisor at each successive division, and make the other figures a divisor. In multiplying such a divisor by the quotient figure, observe to add to the product the number nearest to that which would have been carried if no figures had been rejected.

EXAMPLES.

1. Divide 695.57270875 by 52.35775, and retain in the quotient three places of decimals. Ans. 13.285.

FIRST OPERATION.

```
52.35775)695.57270875(13.285.
         52358  = product by 1, + 1.
         -----
         17199
         15707  = product by 3, + 2.
         -----
          1492
          1047  = product by 2, + 1.
          ----
           445
           419  = product by 8, + 3.
           ---
            26
            26  = product by 5, + 1.
            --
```

SECOND OPERATION.

```
52.35775)695.57270875(13.285.
         52357|75
         -----|--
         17199|520
         15707|325
         -----|---
          1492|1958
          1047|1550
          ----|----
           445|04087
           418|86200
           ---|-----
            26|178875
            26|178875
```

By inspection, it is evident that the first quotient figure will be of the order of tens, and therefore the quotient will contain *two* places of whole numbers; and as there are to be *three* places of decimals, it must contain *five* figures. Hence, we divide at first by five figures of the given divisor, counting them from the left toward the right, thus using the 52.357 and rejecting the figures, 75, on the right. In multiplying each contracted divisor by its quotient figure we increase the product by having regard to rejected figures, as in contracted multiplication of decimals (Art. 273).

The nature and extent of the contraction will be seen by comparison with the common method as shown in the second operation, in which the vertical line cuts off the figures not required.

NOTE. — When the given divisor does not contain as many figures as are required in the quotient, we must begin the division in the usual way, and continue till the deficiency is made up, after which begin the contraction.

2. Divide 4327.56284563 by 873.469, and retain five decimal places in the quotient. Ans. 4.95445.

3. Divide 252070.520751 by 591.57, and terminate the operation with four decimal places in the quotient. Ans. 426.1043.

4. Divide 70.23 by 7.9863, and retain in the answer four decimals.

5. Divide 12193263.1112635269 by 1234.56789, and let the quotient contain as many decimal places, plus one, as there will be integers in it. Ans. 9876.54321.

REDUCTION OF DECIMALS.

277. To reduce a decimal to a common fraction.

Ex. 1. Reduce .125 to its equivalent common fraction.

Ans. $\frac{1}{8}$.

OPERATION.

$$.125 = \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8} \text{ Ans.}$$

Erasing the decimal point and supplying the denominator, which is understood, we have $\frac{125}{1000}$, which reduced to its lowest terms equals $\frac{1}{8}$, the answer required.

RULE. — *Erase the decimal point, and write under the numerator its decimal denominator, and reduce the fraction to its lowest terms.*

EXAMPLES.

2. Reduce .875 to a common fraction. Ans. $\frac{7}{8}$.
3. Reduce .9375 to a common fraction. Ans. $\frac{15}{16}$.
4. What common fraction is equivalent to .08125? Ans. $\frac{13}{160}$.
5. Change .00075 to the form of a common fraction.
6. Express 31.75 by an integer and a common fraction. Ans. $31\frac{3}{4}$.
7. Express 96.024 by an integer and a common fraction. Ans. $96\frac{3}{125}$.
8. Express 163.04 by an integer and a common fraction.
9. Express 1001.4375 by an integer and a common fraction. Ans. $1001\frac{7}{16}$.
10. Express 1457.222 by an integer and a common fraction.
11. Express 19678.36 by an integer and a common fraction.
12. Express 9163.8755 by an integer and a common fraction. Ans. $9163\frac{1751}{2000}$.

278. To reduce a common fraction to a decimal.

Ex. 1. Reduce $\frac{3}{8}$ to a decimal. Ans. .375.

OPERATION.

$$\begin{array}{l} 8\,)\,3.0\,(\,3 \text{ tenths.} \\ \underline{\;\;2\,4\;\;} \\ \quad 8\,)\,6\,0\,(\,7 \text{ hundredths.} \\ \quad\underline{\;\;5\,6\;\;} \\ \qquad 8\,)\,4\,0\,(\,5 \text{ thousandths.} \\ \qquad\underline{\;\;4\,0\;\;} \\ \qquad\qquad \text{Ans. } .375. \end{array}$$

Or thus:

$$\begin{array}{l} 8\,)\,\underline{3.0\,0\,0} \\ \quad .3\,7\,5 \text{ Ans.} \end{array}$$

Since we cannot divide the numerator, 3, by 8, we reduce it to *tenths* by annexing a cipher, and then dividing, we obtain 3 tenths and a remainder of 6 tenths. Reducing this remainder to *hundredths* by annexing a cipher, and dividing, we obtain 7 hundredths and a remainder of 4 hundredths; which being reduced to thousandths by annexing a cipher, and then divided, gives a quotient of 5 thousandths. The sum of the several quotients, .375, is the answer.

To prove that .375 is equal to $\frac{3}{8}$, we change it to the form of a common fraction, by writing its denominator, and reducing it to its lowest terms. Thus, $.375 = \frac{375}{1000} = \frac{3}{8}$.

RULE. — *Annex ciphers to the numerator, and divide by the denominator. Point off in the quotient as many decimal places as there have been ciphers annexed.*

NOTE. — It is not usually necessary that the decimals should be carried to more than six places. When a decimal does not terminate, the sign plus (+) is generally annexed. Thus, in the expression .333+, the sign annexed indicates that the division could be carried further.

EXAMPLES.

2. Reduce $\frac{5}{8}$ to a decimal. Ans. .625.
3. Reduce $\frac{1}{2}$ to a decimal.
4. Change $\frac{3}{32}$ to a decimal. Ans. .09375.
5. Change $\frac{1}{13}$ to a decimal. Ans. .076923+.
6. Reduce $19\frac{1}{8}$ to an equivalent decimal expression.
7. Reduce \$ $315\frac{7}{8}$ to an equivalent decimal expression.
Ans. \$ 315.875.
8. Reduce \$ $1163\frac{3}{4}$ to an equivalent decimal expression.
Ans. 1163.75.

NOTE. — A decimal with a common fraction annexed constitutes what is called a *complex* decimal; as, $.87\frac{1}{2}$, $.31\frac{1}{4}$, and $.18\frac{3}{4}$. In such expressions, instead of the common fraction, its equivalent decimal, with the decimal point omitted, may be substituted. Thus, $.4\frac{1}{25} = .404$.

9. Reduce $.62\frac{1}{2}$ to a simple decimal. Ans. .625.
10. Reduce $.37\frac{1}{16}$ to a simple decimal. Ans. .370625.
11. Reduce \$ $4.31\frac{1}{4}$ to a simple decimal expression.
Ans. \$ 4.3125.

12. Reduce \$ $60.18\frac{3}{4}$ to a simple decimal expression.

Ans. \$ 60.1875.

13. What decimal expression is equivalent to $\frac{1}{2}$ of $\frac{2\frac{1}{2}}{5}$ of 2.04?

14. What decimal expression is equivalent to $2\frac{\frac{1}{2}}{8}$, + $0.37\frac{1}{2}$, + $\frac{1}{2}$ of $\frac{4}{5}$ of 4, — 1.05?

Ans. 2.9875.

279. To reduce a simple or compound number to a decimal of a higher denomination.

Ex. 1. Reduce 15s. 9d. 3far. to the decimal of a £.

Ans. .790625.

OPERATION.

4	3.0 0	far.
1 2	9.7 5 0 0	d.
2 0	1 5.8 1 2 5 0	s.
	.7 9 0 6 2 5	£.

We commence with the 3far., which we reduce to hundredths by annexing two ciphers; and then, to reduce these to the decimal of a penny, we divide by 4far., since there will be $\frac{1}{4}$ as many hundredths of a penny as of a farthing, and obtain .75d. Annexing this to the 9d., we divide by 12d., since there will be $\frac{1}{12}$ as many shillings as pence; and then, the 15s. and this quotient by 20s., since there will be $\frac{1}{20}$ as many pounds as shillings, and obtain .790625£. for the answer. Hence the following

Rule. — *Divide the lowest denomination, annexing ciphers if necessary, by that number which will reduce it to one of the next higher denomination. Then divide as before, and so continue dividing till the decimal is of the denomination required.*

Note 1. — The given number may also be first reduced to a common fraction of the given denomination (Art. 256), and then the fraction changed to a decimal. Thus, if it be required to reduce 15s. 6d. to a decimal of a £.: 15s. 6d. = 186d.; 1£. = 240d.; $\frac{186}{240}$ £. = $\frac{31}{40}$ £. = .775 £. Answer.

Note 2. — *Shillings, pence,* and farthings may be readily reduced to a decimal of three places, by inspection, thus: *Call half of the greatest even number of shillings* TENTHS, *and, if there be an odd shilling, call it* 5 HUNDREDTHS; *reduce the pence and farthings to farthings, and increase them by* 1, *if they amount to* 24 *or more, for* THOUSANDTHS. Thus, if it be required to reduce, by inspection, 19s. 10d. 2far. to the decimal of a £.; half of 18s. = 9s., which denote a value of .9£.; the 1s. denotes a value of .05£.; and 10d. 2far. = 42far., which increased by 1far. = 43far., which denote a value of .043£.; .9£. + .05£. + .043£. = .993£. Answer.

The reason for this process is, that 2s. equal a tenth of a £.; 1 shilling equals 5 hundredths of a £., and 1 farthing equals $\frac{1}{960}$£., or so nearly a thousandth of a £. that 24 farthings exactly equal 25 thousandths of a £.; and therefore farthings require to be increased only by 1 when they amount to 24 or more, to denote with sufficient accuracy their value in thousandths of a £.

Examples.

2. Reduce 9s. to the fraction of a pound. Ans. .45.
3. Reduce 15cwt. 3qr. 14lb. to the decimal of a ton.
4. Reduce 2qr. 21lb. 8oz. 12dr. to the decimal of a cwt. Ans. .71546875.
5. Reduce 1qr. 3na. to the decimal of a yard. Ans. .4375.
6. Reduce 5fur. 35rd. 2yd. 2ft. 9in. to the decimal of a mile. Ans. .73603219+.
7. Reduce 3gal. 2qt. 1pt. of wine to the decimal of a hogshead. Ans. .0575396+.
8. Reduce 1pt. to the decimal of a bushel. Ans. .015625.
9. Reduce 2R. 16p. to the decimal of an acre. Ans. .6.
10. Reduce 175 cubic feet to the decimal of a ton of timber. Ans. 4.375.
11. Reduce 3.755 pecks to the decimal of a bushel. Ans. .93875.
12. What decimal part of a degree is 25′ 34″.6?
13. Reduce 12T. 3cwt. 2qr. 20lb. to hundred-weight and the decimal of a hundred-weight. Ans. 243.7.
14. Reduce 2hhd. 30gal. 2qt. $1\frac{1}{2}$pt. to gallons and the decimal of a gallon. Ans. 156.6875.
15. Reduce to the decimal of a pound, 19s. $11\frac{3}{4}$d., 16s. $9\frac{1}{4}$d., and 17s. $5\frac{1}{2}$d., and find their sum. Ans. 2.710416+.

280. To find the value of a decimal in whole numbers of lower denominations.

Ex. 1. What is the value of .790625 £.?

Ans. 15s. 9d. 3far.

OPERATION.

```
 .7 9 0 6 2 5£.
           2 0
 ---------------
1 5.8 1 2 5 0 0s.
           1 2
 ---------------
  9.7 5 0 0 0 0d.
             4
 ---------------
  3.0 0 0 0 0 0far.
Ans. 15s. 9d. 3far.
```

There will be 20 times as many millionths of a shilling as of a pound; therefore, we multiply the decimal, .790625, by 20, and reduce the improper fraction to a mixed number by pointing off six figures on the right, which is dividing by its denominator, 1000000. The figures on the left of the point are shillings, and those on the right, the decimal of a shilling. The decimal .812500 we multiply by 12, and, pointing off as before, obtain 9d., and a decimal of a penny. The decimal

.750000 we multiply by 4, and pointing off have 3 farthings, which, taken with the other denominations obtained, gives 15s. 9d. 3far. for the answer.

RULE. — *Multiply the decimal by that number which will reduce it to the next lower denomination, and point off as in multiplication of decimals.*

Then, multiply the decimal part of the product, and point off as before. So continue till the decimal is reduced to the denominations required.

The several whole numbers of the successive products will be the answer.

NOTE. — When there is a decimal in the last product, it may be changed to a common fraction.

EXAMPLES.

2. What is the value of .625 of a shilling? Ans. $7\frac{1}{2}$d.
3. What is the value of .6725 of a cwt.? Ans. 2qr. 17lb. 4oz.
4. What is the value of .9375 of a yard?
5. What is the value of .7895 of a mile? Ans. 6fur. 12rd. 10ft. $6\frac{18}{25}$in.
6. What is the value of .9378 of an acre? Ans. 3R. 30p. 13ft. $9\frac{99}{125}$in.
7. Reduce .5615 of a hogshead of wine to its value in gallons, &c. Ans. 35gal. 1qt. 0pt. $3\frac{123}{125}$gi.
8. Reduce .367 of a year to its value in days, &c. Ans. 134d. 1h. 7m. $19\frac{1}{5}$sec.
9. What is the value of .6923828125 of a cwt.? Ans. 2qr. 19lb. 3oz. 13d.
10. What is the value of .015625 of a bushel?
11. What is the value of .55 of an ell English? Ans. 2qr. 3na.
12. What is the value of .6 of an acre? Ans. 2R. 16p.

MISCELLANEOUS EXAMPLES.

1. What is the value of 7cwt. 2qr. 18lb. of sugar, at $11.75 per cwt.? Ans. $90.24.
2. What cost 19cwt. 3qr. 14lb. of iron, at $9.25 per cwt.?

3. What cost 39A. 2R. 15p. of land, at $87.375 per acre? Ans. 3459.503\frac{29}{32}$.

4. What would be the expense of making a turnpike 87m. 3fur. 15rd., at $578.75 per mile? Ans. 50595.41\frac{1}{64}$.

5. What is the cost of a board 18ft. 9in. long, and 2ft. 3$\frac{1}{2}$in. wide, at $.053 per foot? Ans. 2.277\frac{11}{32}$.

6. Goliath of Gath was 6$\frac{1}{2}$ cubits high; what was his height in feet, the cubit being 1ft. 7.168in.? Ans. 10ft. 4.592in.

7. If a man travel 4.316 miles in an hour, how long would he be in travelling from Bradford to Boston, the distance being 29$\frac{1}{2}$ miles? Ans. 6h. 50m. 6sec. +

8. What is the cost of 5yd. 1qr. 2na. of broadcloth, at 5.62\frac{1}{2}$ per yard? Ans. 30.234\frac{3}{8}$.

9. Bought 17 bags of hops, each weighing 4cwt. 3qr. 7lb., at 5.87\frac{1}{2}$ per cwt.; what was the cost?

10. Purchased a farm, containing 176A. 3R. 25rd., at 75.37\frac{1}{2}$ per acre; what did it cost? Ans. 13334.308\frac{19}{32}$.

11. What cost 17625 feet of boards, at $12.75 per thousand? Ans. 224.718\frac{3}{4}$.

12. How many square feet in a floor 19ft. 3in. long, and 15ft. 9in. wide? Ans. 303ft. 27in.

13. How many square yards of paper will it take to cover a room 14ft. 6in. long, 12ft. 6in. wide, and 8ft. 9in. high?

14. How many solid feet in a pile of wood 10ft. 7in. long, 4ft. wide, and 5ft. 10in. high? Ans. 246$\frac{17}{18}$ft.

15. How many garments, each containing 4yd. 2qr. 3na., can be made from 112yd. 2qr. of cloth?

16. Bought 1gal. 2qt. 1pt. of wine for $1.82; what would be the price of a hogshead? Ans. $70.56.

17. Bought 125$\frac{1}{2}$yd. of lace for $15.06; what was the price of 1 yard? Ans. $0.12.

18. What cost 17cwt. 3qr. of wool, at $35.75 per hundredweight? Ans. 634.562\frac{1}{2}$.

19. What cost 7hhd. 47gal. of wine, at $87.25 per hogshead? Ans. 675.84\frac{8}{63}$.

20. How many solid feet in a stick of timber 34ft. 9in. long, 1ft. 3in. wide, and 1ft. 6in. deep? Ans. 65.15625ft.

21. If 18yd. 1qr. of cloth cost $36.50, what is the price of 1 yard? Ans. $2.00.

22. If \$477.72 be equally divided among 9 men, what will be each man's share? Ans. \$53.08.

23. A man bought a barrel of flour for \$5.375, 7gal. of molasses for \$1.78, 9gal. of vinegar for \$1.1875, 1gal. of wine for \$1.125, 14lb. of sugar for \$1.275, and 5lb. of tea for \$2.625; what did the whole amount to? Ans. \$13.367$\frac{1}{2}$.

24. A man purchased 3 loads of hay; the first contained 2$\frac{3}{8}$ tons, the second 3$\frac{7}{8}$ tons, and the third 1$\frac{1}{16}$ tons; what was the value of the whole, at \$17.625 a ton? Ans. \$128.882$\frac{3}{16}$.

25. How many hogsheads of water will it take to fill a cistern which is 15.25 feet long, 8.4 feet wide, and 10 feet deep? Ans. 152hhd. 6$\frac{6}{11}$gal.

26. At \$13.625 per cwt., what cost 3cwt. 2qr. 7lb. of sugar? Ans. \$48.641$\frac{1}{4}$.

27. At \$125.75 per acre, what cost 37A. 3R. 35rd.? Ans. \$4774.570$\frac{5}{16}$.

28. At \$11.25 per cwt., what cost 17cwt. 2qr. 21lb. of rice? Ans. \$199.237$\frac{1}{2}$.

29. What cost 7$\frac{3}{4}$ bales of cotton, each weighing 3.37cwt., at \$9.37$\frac{1}{2}$ per cwt.?

30. What cost 7hhd. 49gal. of wine, at \$97.625 per hogshead? Ans. \$759.305$\frac{35}{63}$.

31. What cost 7yd. 3qr. 3na. of cloth, at \$4.75 per yard? Ans. \$37.703$\frac{1}{8}$.

32. What cost 27T. 15cwt. 1qr. 3$\frac{1}{2}$lb. of hemp, at \$183.62 per ton? Ans. \$5098.071$\frac{117}{200}$.

33. What is the cost of constructing a railroad 17m. 3fur. 15rd., at \$1725.875 per mile? Ans. \$30067.978$\frac{33}{64}$.

34. When \$624.53125 are paid for 17A. 3R. 15p. of land, what is the cost of one acre?

35. Paid \$494.53125 for 19T. 15cwt. 2qr. 14lb. of hay; what was the cost per ton? Ans. \$24.999$\frac{172}{3297}$.

36. How much land, at \$40 per acre, can be obtained for \$1004.75? Ans. 25A. 0R. 19p.

37. How many cords of wood can be put into a space 20.5 feet long, 12.75 feet wide, and 7.6 feet high?

Ans. 15 cords 66$\frac{9}{20}$ cubic feet.

38. How many bushels of corn at \$0.62$\frac{1}{2}$ per bushel must a farmer exchange for 31 yards of sheeting at \$0.08$\frac{1}{4}$ per yard, *and* 7$\frac{1}{2}$ yards of broadcloth at \$2.75 per yard? Ans. 37$\frac{23}{250}$.

39. I have expended $ 42.875 for a quantity of grain, $\frac{3}{10}$ of it being corn, at $ 0.75 a bushel; $\frac{9}{20}$ of it wheat, at $ 2 a bushel; and the balance oats, at $ 0.40 a bushel, to the amount of $ 3.50. Required the number of bushels of each kind purchased.

40. If a mason, in constructing a drain 250.35 feet long, begin with a width of 8 inches, and increase $\frac{5}{16}$ of an inch in every foot of length, how many times the width of the beginning of the drain will its end be? Ans. 16.646875.

41. A gentleman gave $\frac{1}{4}$ of his property to his son James; $\frac{1}{3}$ of it to his son William; $\frac{1}{5}$ of the remainder to his daughter Mary; and the balance to his wife. It appeared that Mary received $ 2243.26 less than James. What was the amount divided, and how much did each receive?

Ans. Amount, $ 13459.56; James, $ 3364.89; William, $ 4486.52; Mary, 1121.63; wife, $ 4486.52.

CIRCULATING DECIMALS.

281. A CIRCULATING DECIMAL is a decimal in which one or more figures are continually repeated in the same order. Thus, in reducing $\frac{1}{3}$ to an equivalent decimal, on annexing ciphers and dividing by the denominator, the result obtained, .333+, is a circulating decimal; for, however far the division might be carried, the same figure would continue to be repeated without the decimal terminating.

Such decimals are sometimes called infinite, or repeating; and, for sake of distinguishing, those decimals that terminate are sometimes termed *finite*.

282. A *repetend* is a figure, or a series of figures, continually repeated. To mark a repetend, a point (.) is placed over a single repeating figure, or over the first and last of a series of repeating figures. Thus, in $.\dot{3}$, the point denotes that the 3 is a repetend; and in $.\dot{7}\dot{2}$, that the 72 is a repetend.

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22. If \$ 477.72 be equally divided among 9 men, what will be each man's share? Ans. \$ 53.08.

23. A man bought a barrel of flour for \$ 5.375, 7gal. of molasses for \$ 1.78, 9gal. of vinegar for \$ 1.1875, 1gal. of wine for \$ 1.125, 14lb. of sugar for \$ 1.275, and 5lb. of tea for \$ 2.625; what did the whole amount to? Ans. \$ 13.367$\frac{1}{2}$.

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25. How many hogsheads of water will it take to fill a cistern which is 15.25 feet long, 8.4 feet wide, and 10 feet deep? Ans. 152hhd. 6$\frac{6}{11}$gal.

26. At \$ 13.625 per cwt., what cost 3cwt. 2qr. 7lb. of sugar? Ans. \$ 48.641$\frac{1}{4}$.

27. At \$ 125.75 per acre, what cost 37A. 3R. 35rd.? Ans. \$ 4774.570$\frac{5}{16}$.

28. At \$ 11.25 per cwt., what cost 17cwt. 2qr. 21lb. of rice? Ans. \$ 199.237$\frac{1}{2}$.

29. What cost 7$\frac{3}{4}$ bales of cotton, each weighing 3.37cwt., at \$ 9.37$\frac{1}{2}$ per cwt.?

30. What cost 7hhd. 49gal. of wine, at \$ 97.625 per hogshead? Ans. \$ 759.305$\frac{35}{63}$.

31. What cost 7yd. 3qr. 3na. of cloth, at \$ 4.75 per yard? Ans. \$ 37.703$\frac{1}{8}$.

32. What cost 27T. 15cwt. 1qr. 3$\frac{1}{2}$lb. of hemp, at \$ 183.62 per ton? Ans. \$ 5098.071$\frac{117}{200}$.

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34. When \$ 624.53125 are paid for 17A. 3R. 15p. of land, what is the cost of one acre?

35. Paid \$ 494.53125 for 19T. 15cwt. 2qr. 14lb. of hay; what was the cost per ton? Ans. \$ 24.999$\frac{172}{3297}$.

36. How much land, at \$ 40 per acre, can be obtained for \$ 1004.75? Ans. 25A. 0R. 19p.

37. How many cords of wood can be put into a space 20.5 feet long, 12.75 feet wide, and 7.6 feet high? Ans. 15 cords 66$\frac{9}{20}$ cubic feet.

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37. How many cords of wood can be put into a space 20.5 feet long, 12.75 feet wide, and 7.6 feet high?

Ans. 15 cords 66$\frac{9}{20}$ cubic feet.

38. How many bushels of corn at \$0.62$\frac{1}{2}$ per bushel must a farmer exchange for 31 yards of sheeting at \$0.08$\frac{1}{4}$ per yard, and 7$\frac{1}{2}$ yards of broadcloth at \$2.75 per yard? Ans. 37$\frac{23}{250}$.

39. I have expended $ 42.875 for a quantity of grain, $\frac{3}{10}$ of it being corn, at $ 0.75 a bushel; $\frac{9}{20}$ of it wheat, at $ 2 a bushel; and the balance oats, at $ 0.40 a bushel, to the amount of $ 3.50. Required the number of bushels of each kind purchased.

40. If a mason, in constructing a drain 250.35 feet long, begin with a width of 8 inches, and increase $\frac{5}{10}$ of an inch in every foot of length, how many times the width of the beginning of the drain will its end be? Ans. 16.646875.

41. A gentleman gave $\frac{1}{4}$ of his property to his son James; $\frac{1}{3}$ of it to his son William; $\frac{1}{5}$ of the remainder to his daughter Mary; and the balance to his wife. It appeared that Mary received $ 2243.26 less than James. What was the amount divided, and how much did each receive?

Ans. Amount, $ 13459.56; James, $ 3364.89; William, $ 4486.52; Mary, 1121.63; wife, $ 4486.52.

CIRCULATING DECIMALS.

281. A CIRCULATING DECIMAL is a decimal in which one or more figures are continually repeated in the same order. Thus, in reducing $\frac{1}{3}$ to an equivalent decimal, on annexing ciphers and dividing by the denominator, the result obtained, .333+, is a circulating decimal; for, however far the division might be carried, the same figure would continue to be repeated without the decimal terminating.

Such decimals are sometimes called infinite, or repeating; and, for sake of distinguishing, those decimals that terminate are sometimes termed *finite*.

282. A *repetend* is a figure, or a series of figures, continually repeated. To mark a repetend, a point (.) is placed over a single repeating figure, or over the first and last of a series of repeating figures. Thus, in $.\dot{3}$, the point denotes that the 3 is a repetend; and in $.\dot{7}\dot{2}$, that the 72 is a repetend.

283. A *single repetend* is one in which only one figure is

repeated; as in .1111+ denoted by $.\dot{1}$; and 2222+, denoted by $.\dot{2}$.

284. A *compound repetend* is one in which the same set of figures is repeated; as in .135135+, denoted by $.\dot{1}3\dot{5}$, and .30363036+, denoted by $\dot{3}03\dot{6}$.

285. A *pure repetend* is one which contains only the figures of the repetend; as, $.\dot{3}$, $.\dot{0}\dot{2}$, and $.\dot{1}2\dot{3}$.

286. A *mixed repetend* is one in which a repetend is preceded in the same fraction by one or more figures. The figures preceding the repetend are called the *finite part.* Thus, $.41\dot{6}$ is a mixed repetend, of which the figure 6 is the repetend, and the figures 41 the finite part; also, $1.7\dot{2}\dot{8}$ is a mixed repetend, of which the figures 28 are the repetend, and the figures 1.7 the finite part.

287. A *perfect repetend* is a pure repetend containing the same number of figures as there are units in its denominator less one. Thus, $\frac{1}{7}$ reduced to a decimal gives $.\dot{1}4285\dot{7}$, which, as it contains as many figures as there are units in the denominator, 7, less one, is a perfect repetend.

288. *Similar repetends* are those which begin at the same distance from the decimal point; as $.\dot{3}$ and $.\dot{6}$; or $5.\dot{1}2\dot{3}$ and $3.\dot{4}7\dot{8}$.

289. *Dissimilar repetends* are those which begin at different distances from the decimal point; as $.\dot{9}8\dot{6}$ and $.4\dot{6}2\dot{5}$; or $.5\dot{9}2\dot{5}$ and $.042\dot{3}43\dot{6}$.

290. *Conterminous repetends* are those which terminate at the same distance from the decimal point; as $.6\dot{3}\dot{1}$ and $.\dot{4}6\dot{5}$, or $.0\dot{7}5\dot{3}$ and $.475\dot{2}$.

291. *Similar and conterminous repetends* are those which both begin and end at the same distance from the decimal point; as $.3\dot{5}\dot{4}$ and $.4\dot{2}\dot{5}$; or $.5\dot{7}5\dot{7}$ and $5\dot{7}2\dot{3}$.

292. *Repetends always arise from common fractions, which, when in their lowest terms, contain in their denominator other factors than* 2 *and* 5. For when a common fraction is in its lowest terms, its numerator and denominator are prime to each

other (Art. 219), and the annexing of one or more ciphers to the numerator makes the same a multiple of 10, but does not render it divisible by any factor, except 2 and 5, the factors of 10. Therefore, when the denominator of a fraction, in its lowest terms, contains other factors than those of 10, the decimal resulting from dividing the numerator with ciphers annexed, will not terminate, but will contain one or more figures that constantly repeat.

293. *A pure repetend is always equivalent to a common fraction whose numerator is the repeating figure or figures, and whose denominator as many places of nines as there are repeating figures.* For, by reducing $\frac{1}{9}$ to a decimal, we obtain as its equivalent the repetend $.\dot{1}$; and since $.\dot{1}$ is equivalent to $\frac{1}{9}$, $.\dot{2}$ will be equivalent to $\frac{2}{9}$, $.\dot{3}$ to $\frac{3}{9}$, and so on, till $.\dot{9}$ is equal to $\frac{9}{9}$ or 1. Again, $\frac{1}{99}$, and $\frac{1}{999}$, being reduced, give $.\dot{0}\dot{1}$, and $\dot{0}0\dot{1}$; that is, $\frac{1}{99} = .\dot{0}\dot{1}$, and $\frac{1}{999} = .\dot{0}0\dot{1}$; therefore, $\frac{2}{99} = .\dot{0}\dot{2}$, and $\frac{2}{999} = \dot{0}0\dot{2}$, and so on; the same principle holding true in all like cases.

294. *A mixed repetend is equivalent to a complex decimal* (Art. 278), *or to a complex fraction.* Thus, the mixed repetend $.24\dot{1}\dot{2}$ is equivalent to the mixed decimal $.24\frac{12}{99}$, which is equal to the complex fraction $\dfrac{24\frac{12}{99}}{100}$.

295. *Repetends are of the same denomination only when they are similar and conterminous.* For then alone, by having a common denominator, do they express fractional parts of the same unit.

REDUCTION OF REPETENDS.

296. To reduce a repetend to an equivalent common fraction.

Ex. 1. Reduce $.\dot{1}2\dot{3}$ to an equivalent common fraction.

Ans. $\frac{41}{333}$.

OPERATION.

$.\dot{1}2\dot{3} = \frac{123}{999} = \frac{41}{333}$ Ans.

We write the figures of the given repetend with the decimal point omitted for the numerator, and as many nines as places in the repetend for the denominator, of a common fraction (Art. 293), and obtain $\frac{123}{999}$, which, reduced to its lowest terms, $= \frac{41}{333}$, the answer required.

2. **Reduce $.13\dot{8}$ to an equivalent common fraction.**

Ans. $\frac{5}{36}$.

OPERATION.

$$.1\,3\,\dot{8} = \frac{13\frac{8}{9}}{100} = \frac{125}{900} = \frac{5}{36} \text{ Ans.}$$

The mixed repetend $.13\dot{8}$ is equivalent to the mixed decimal $.13\frac{8}{9}$ (Art. 294), which we readily change to the form of a complex fraction by erasing the decimal point and writing the denominator, 100, which is understood; and thus obtain $\frac{13\frac{8}{9}}{100}$, which, reduced to its simplest form, gives $\frac{5}{36}$, the answer required. Hence,

If the given repetend be SIMPLE, *make the repeating figure or figures the numerator, and take as many nines as the repetend has figures for the denominator.*

If the given repetend be MIXED, *change it to an equivalent complex fraction, and that fraction to its simplest form.*

NOTE. — Any circulating decimal may be transformed into another decimal, having a repetend of the same number of figures; as, $.\dot{7}\dot{8} = .7\dot{8}\dot{7}$, and $.\dot{5}3\dot{4} = .5\dot{3}4\dot{5} = .53\dot{4}5\dot{3}$. Thus, when such expressions as $1\dot{2}.\dot{5}$ or $1\dot{7}.5\dot{6}$ occur, they may be also transformed; as $1\dot{2}.\dot{5} = 12.\dot{5}\dot{2}$, and $1\dot{7}.5\dot{6} = 17.\dot{5}6\dot{7} = 17.5\dot{6}7\dot{5}$, &c.

EXAMPLES.

3. Required the common fraction equal to $.\dot{6}$. Ans. $\frac{6}{9} = \frac{2}{3}$.

4. Reduce $\dot{1}.6\dot{2}$ to its equivalent mixed number.

Ans. $1\frac{23}{37}$.

5. Change $.5\dot{3}$ to an equivalent common fraction.

6. What common fraction is equivalent to $.\dot{7}6923\dot{0}$?

Ans. $\frac{10}{13}$.

7. What common fraction is equivalent to $.5\dot{9}2\dot{5}$?

8. Change $3\dot{1}.6\dot{2}$ to an equivalent mixed number.

Ans. $31\frac{23}{37}$.

9. Reduce $.008\dot{4}9713\dot{3}$ to an equivalent common fraction.

Ans. $\frac{83}{9768}$.

297. To determine the kind of decimal to which a given common fraction can be reduced.

Ex. 1. Required to find whether the decimal equal to $\frac{210}{1120}$ be finite or circulating; and if finite, of how many places the decimal will consist. Ans. Finite, of 4 places.

OPERATION.

$$\tfrac{210}{1120} = \tfrac{3}{16} = \frac{3}{2 \times 2 \times 2 \times 2} = .1875.$$

We reduce the given fraction to its lowest terms, and then resolve the denominator, 16, of the fraction obtained, $\frac{3}{16}$, into its prime factors, which we find to be $2 \times 2 \times 2 \times 2$. Now, since the denominator contains no prime factor other than 2 or 5, it is evident that, by annexing ciphers to the numerator, 3, and dividing by the denominator, 16, the decimal arising will terminate, and thus be finite (Art. 292).

Since, in reducing a common fraction to its equivalent decimal, we annex ciphers to the numerator and divide by the denominator (Art. 278), every 10, or 2 and 5, that enter into the denominator as factors must produce one decimal place, and no more, and therefore every other factor 2 or 5 must give one, and only one, decimal place. The denominator, 16, contains only the factor 2 taken 4 times, or 2^4; and the exponent of the 2 indicates that the decimal equivalent to $\frac{3}{16}$ must contain exactly 4 decimal places, which we verify by reducing the $\frac{3}{16}$ to its equivalent decimal, .1875.

2. Find whether the decimal equal to $\frac{235}{1750}$ be finite or circulating; and if circulating, of how many places the finite part, if any, and the circulating part, will each consist.

Ans. Circulating: the finite part, 2 places; the repetend, 6 places.

OPERATION.

$$\tfrac{235}{1750} = \tfrac{47}{350} = \frac{47}{2 \times 5 \times 5 \times 7} = .13\dot{4}2857\dot{1}.$$

We reduce the given fraction to its lowest terms, and obtain $\frac{47}{350}$. The denominator, $350, = 2 \times 5 \times 5 \times 7$, contains a prime factor, 7, other than 2 and 5; therefore the decimal equivalent to $\frac{47}{350}$ will contain a repetend; and as, of the factors 2 and 5, the higher exponent of either, that of 5, is 2, the decimal will have 2 finite places before the repetend commences. This we verify by reducing $\frac{47}{350}$ to its equivalent decimal, $.13\dot{4}2857\dot{1}$. Hence, to determine whether the decimal to which a given common fraction can be reduced is finite or circulating, and the number of finite decimal places, if any,

Having reduced the given common fraction to its lowest terms, resolve the denominator into its prime factors. If these factors be not other than 2 or 5, the decimal will be FINITE; *if other prime factors occur with 2 or 5, the decimal will be a* MIXED REPETEND; *and if neither 2 nor 5 occurs as factor, the decimal will be a* PURE REPETEND.

Whichever factor 2 or 5 occurs in the denominator with the higher exponent will by its exponent denote the number of finite decimal places.

NOTE. — The number of figures of which a repetend will consist may be discovered by dividing 1 with ciphers annexed by the factors other than 2 or 5 of the denominator, until there is a remainder 1. Thus, if it be required to discover the number of figures in the repeating part of the decimal equivalent to $\frac{47}{350}$, we divide 1 with ciphers annexed by 7, the only prime factor in the denominator other than 2 or 5, until there is a remainder of 1, which occurs after the *sixth* division, thereby indicating that the repeating part will consist of *six* figures. We have seen that these must be preceded by two places of finite decimals, so that the mixed repetend equal to $\frac{47}{350}$ must consist of eight places in all.

EXAMPLES.

3. To what kind of a decimal can $\frac{1}{11}$ be reduced?

Ans. A pure repetend, of 2 places.

4. How many places of decimals, finite and repeating, will be required to express $\frac{39}{1320}$?

Ans. 5 places; 3 finite and 2 repeating.

5. To what kind of a decimal can $\frac{107}{253}$ be reduced?

6. Reduce $13\frac{17}{45}$ to a mixed repetend. Ans. $13.3\dot{7}$.

7. Change $\frac{166}{19536}$ to a mixed repetend. Ans. $.008\dot{4}9713\dot{3}$.

8. Of how many figures will the repetend consist that corresponds to $\frac{2}{29}$? Ans. 28 figures.

TRANSFORMATION OF REPETENDS.

298. Any finite decimal may be considered as a mixed repetend by making ciphers continually recur; thus, $.42 = .42\dot{0} = .42\dot{0}\dot{0} = .42\dot{0}0\dot{0}$, &c.

299. Any circulating decimal may be transformed into another having the same number of repeating figures; thus, $.1\dot{2}\dot{7} = .12\dot{7}\dot{2} = .127\dot{2}\dot{7}$, &c.

300. Any circulating decimal having as repetend any number of figures may be transformed to another having twice or thrice that number of figures, or any multiple thereof; thus, $.5\dot{9}2\dot{5}$, having a repetend of three figures, may be transformed to one having 6, 9, 12, &c. places; therefore $.5\dot{9}2\dot{5} = .5\dot{9}2592\dot{5} = 5\dot{9}2592592\dot{5} = 5\dot{9}2592592592592\dot{5}$, &c.

301. The value of a decimal is not changed by any of the above transformations, as may be seen by reducing the given

repetends to their equivalent common fractions (Art. 296) and comparing them together. Hence, they can be used in making dissimilar repetends similar and conterminous.

302.° To make any number of dissimilar repetends similar and conterminous.

Ex. 1. Make similar and conterminous $9.1\dot{6}\dot{7}$, 14.6, $3.16\dot{5}$, $12.\dot{4}3\dot{2}$, $8.1\dot{8}\dot{1}$, and $1.\dot{3}0\dot{7}$.

OPERATION.

Dissimilar.		Similar.		Similar and Conterminous.	
$9.1\dot{6}\dot{7}$	=	$9.16\dot{7}\dot{6}$	=	$9.16\dot{7}6767\dot{6}$	
14.6	=	$14.60\dot{0}$	=	$14.60\dot{0}0000\dot{0}$	
$3.16\dot{5}$	=	$3.16\dot{5}$	=	$3.16\dot{5}5555\dot{5}$	Ans.
$12.\dot{4}3\dot{2}$	=	$12.43\dot{2}4\dot{3}$	=	$12.43\dot{2}4324\dot{3}$	
$8.1\dot{8}\dot{1}$	=	$8.18\dot{1}\dot{8}$	=	$8.18\dot{1}8181\dot{8}$	
$1.\dot{3}0\dot{7}$	=	$1.30\dot{7}3\dot{0}$	=	$1.30\dot{7}3073\dot{0}$	

We make the finite mixed decimal, 14.6, a mixed repetend by annexing recurring ciphers, and make it and all the given repetends *similar*, by extending the figures to the right, so that the circulating part of each may begin at the same distance from the decimal point as does that repetend which is preceded by the most finite decimal places. Then, to make *conterminous* the repetends that have thus been rendered similar, as some of them consist of 1, some of 2, and the others of 3 places, we extend the repeating figures of each repetend till those of each occupy as many places as there are units in the least common multiple of 1, 2, and 3, which is 6. Hence, to make dissimilar repetends similar and conterminous,

Transform the given repetends so that the circulating parts shall commence at the same distance from the decimal point, and shall consist of as many circulating places as there are units in the least common multiple of the number of repeating figures found in the given decimals.

EXAMPLES.

2. Make $3.\dot{6}7\dot{1}$, $1.\dot{0}07\dot{1}$, $8.\dot{5}\dot{2}$, and $7.\dot{6}1632\dot{5}$ similar and conterminous.

3. Make $1.\dot{5}\dot{2}$, $8.\dot{7}15\dot{6}$, $3.56\dot{7}$, and $1.3\dot{7}\dot{8}$ similar and conterminous.

4. Make $.00\dot{0}\dot{7}$, $.141\dot{4}1\dot{4}$, and $887.\dot{1}$ similar and conterminous.

5. Make $.3\dot{1}2\dot{3}$, $3.\dot{2}\dot{7}$, and $5.0\dot{2}$ similar and conterminous.

6. Make $17.08\dot{8}\dot{4}$, $1563.0\dot{9}2\dot{9}$, and $15.\dot{1}234\dot{5}$ similar and conterminous.

ADDITION OF CIRCULATING DECIMALS.

303.° Ex. 1. Add $2.7\dot{6}\dot{5}$, $7.1\dot{6}67\dot{4}$, $\dot{3}.67\dot{1}$, $.\dot{7}$, and $.\dot{1}72\dot{8}$ together. Ans. $14.5\dot{5}43\dot{6}$.

OPERATION.

Dissimilar.		Similar and Conterminous.
$2.7\dot{6}\dot{5}$	=	$2.7\dot{6}56\dot{5}$
$7.1\dot{6}67\dot{4}$	=	$7.1\dot{6}67\dot{4}$
$\dot{3}.67\dot{1}$	=	$3.6\dot{7}13\dot{6}$
$.\dot{7}$	=	$.7\dot{7}77\dot{7}$
$.\dot{1}72\dot{8}$	=	$.1\dot{7}28\dot{1}$
	Ans.	$14.5\dot{5}43\dot{6}$

Having made the given repetends similar and conterminous (Art. 302), we add as in addition of whole numbers, and obtain $14.5\dot{5}43\dot{3}$. The right-hand figure of this result we increase by such a number as would have been carried, if the repetends had been continued farther to the right. In that case we should have had to carry 3 after finding the amount of the first left-hand column of the repetends continued. We therefore increase the sum as first found, and thus have the true amount as in the operation, $14.5\dot{5}43\dot{6}$.

RULE. — *Make the given repetends, when dissimilar, similar and conterminous. Add as in addition of finite decimals, observing to increase the repetend of the amount by the number, if any, to be carried from the left-hand column of the repetends.*

EXAMPLES.

2. Add $3.\dot{5}$, $7.\dot{6}5\dot{1}$, $1.\dot{7}6\dot{5}$, $6.1\dot{7}\dot{3}$, $51.\dot{7}$, 3.7, $27.\dot{6}3\dot{1}$, and $1.\dot{0}0\dot{3}$ together. Ans. $103.2\dot{5}9122\dot{7}$.

3. Reduce $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{1}{8}$ to decimals, and find their sum.

4. Find the sum of $2\dot{7}.5\dot{6}$, $5.\dot{6}3\dot{2}$, $6.\dot{7}$, $16.\dot{3}5\dot{6}$, $.7\dot{1}$, and $6.\dot{1}23\dot{4}$. Ans. $63.1\dot{6}9067086888\dot{8}$.

5. Add together $.165002$, 31.64, $1.0\dot{6}$, $.3463\dot{4}$, and 13.

6. Add together $.\dot{8}\dot{7}$, $.\dot{8}$, and $.\dot{8}7\dot{6}$. Ans. $2.\dot{6}4455\dot{3}$.

7. Required the value of $.\dot{3} + .45 + .\dot{4}\dot{5} + .\dot{3}5\dot{1} + .6468 + .646\dot{8} + .64\dot{6}\dot{8}$, and $.6\dot{4}6\dot{8}$. Ans. $4.1766\dot{3}4561\dot{8}$.

8. Find the value of $1.25 + 3.\dot{4} + .6\dot{3}\dot{7} + 7.88\dot{5} + 7.875 + 7.\dot{8}7\dot{5} + 11.\dot{1}$. Ans. $40.079\dot{3}6072\dot{4}$.

9. Add together 131.613, $15.00\dot{1}$, $67.13\dot{4}$, and $1000.6\dot{3}$.

10. Find the value of $5.\dot{1}634\dot{5} + 8.\dot{6}38\dot{1} + 3.\dot{7}\dot{5}$.
Ans. $17.\dot{5}591912084737409030\dot{2}$.

SUBTRACTION OF CIRCULATING DECIMALS.

304.° Ex. 1. From $87.16\dot{4}\dot{5}$ take $19.47\dot{9}16\dot{7}$.

Ans. $67.68\dot{5}37\dot{7}$.

OPERATION.

Dissimilar.		Similar and Conterminous.
$87.16\dot{4}\dot{5}$	=	$87.16\dot{4}54\dot{5}$
$19.47\dot{9}16\dot{7}$	=	$19.47\dot{9}16\dot{7}$
	Ans.	$67.68\dot{5}37\dot{7}$

Having made the repetends similar and conterminous, we subtract as in whole numbers, regarding, however, the right-hand figure of the subtrahend as increased by 1, since 1 would have been carried to it in subtracting, if the repetends had been continued farther to the right, as is evident from the circulating part of the subtrahend being greater than that of the minuend.

RULE. — *Make the repetends, when dissimilar, similar and conterminous. Subtract as in subtraction of finite decimals; observing to regard the repetend of the subtrahend as increased by* 1, *when it exceeds that of the minuend.*

EXAMPLES.

2. From $7.\dot{1}$ take $5.0\dot{2}$. Ans. $2.0\dot{8}$.
3. From $31\dot{5}.8\dot{7}$ take $78.\dot{0}37\dot{8}$. Ans. $237.\dot{8}3807209549\dot{7}$.
4. Subtract $\frac{1}{7}$ from $\frac{2}{9}$. Ans. $.\dot{0}7936\dot{5}$.
5. From $16.13\dot{4}\dot{7}$ take $11.08\dot{8}\dot{4}$. Ans. $5.04\dot{6}\dot{2}$.
6. From $18.1\dot{6}7\dot{8}$ take $3.\dot{2}\dot{7}$. Ans. $14.89\dot{5}\dot{1}$.
7. From $3.\dot{1}2\dot{3}$ take $0.\dot{7}\dot{1}$. Ans. $2.\dot{4}0595\dot{1}$.
8. From $\frac{3}{7}$ take $\frac{2}{11}$. Ans. $.\dot{2}4675\dot{3}$.
9. From $\frac{4}{9}$ take $\frac{2}{7}$. Ans. $.\dot{1}5873\dot{0}$.
10. From $\frac{9}{17}$ take $\frac{6}{17}$. Ans. $.\dot{1}76470588235294\dot{1}$.
11. From $5.\dot{1}234\dot{5}$ take $2.3\dot{5}2345\dot{6}$.

Ans. $2.7\dot{7}11055821666927777988888599999\dot{4}$.

MULTIPLICATION OF CIRCULATING DECIMALS.

305.° Ex. 1. Multiply $.\dot{3}\dot{6}$ by $2\dot{5}$. Ans. $0\dot{9}2\dot{9}$.

OPERATION.

$$.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11};\ .2\dot{5} = \frac{2\frac{3}{9}}{10} = \frac{23}{90}.$$

$$\frac{4}{11} \times \frac{23}{90} = \frac{92}{990} = \frac{46}{495} = .0\dot{9}2\dot{9} \text{ Ans.}$$

We change the given numbers to their equivalent common fractions, and, multiplying, obtain $\frac{92}{990} = \frac{46}{495}$, which, reduced to its equivalent decimal, gives $.0\dot{9}2\dot{5}$, the answer required.

RULE. — *Change the given numbers to their equivalent common fractions. Multiply them together, and reduce the product to its equivalent decimal.*

EXAMPLES.

2. Multiply $87.32\dot{5}8\dot{6}$ by 4.37. Ans. $381.6140\dot{3}3\dot{8}$.
3. Multiply $582.\dot{3}4\dot{7}$ by $.0\dot{3}$.
4. Multiply $3.1\dot{4}\dot{5}$ by $4.\dot{2}9\dot{7}$. Ans. $13.5\dot{1}6953\dot{3}$.
5. What is the value of $.\dot{2}8571\dot{4}$ of a guinea? Ans. 8s.
6. What is the value of $.461607\dot{1}4285\dot{7}$ of a ton?
Ans. 9cwt. 0qr. 23+lb.
7. What is the value of $.2\dot{8}493150\dot{6}$ of a year?
Ans. 104d.

DIVISION OF CIRCULATING DECIMALS.

306.° Ex. 1. Divide $.\dot{5}\dot{4}$ by $.1\dot{5}$. Ans. $3.\dot{5}0649\dot{3}$.

OPERATION.

$$.\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11};\ .1\dot{5} = \frac{1\frac{5}{9}}{10} = \frac{14}{90} = \frac{7}{45}.$$

$$\tfrac{6}{11} \div \tfrac{7}{45} = \tfrac{270}{77} = 3\tfrac{39}{77} = 3.\dot{5}0649\dot{3} \text{ Ans.}$$

We change the given numbers to their equivalent common fractions, and, dividing, obtain $\frac{270}{77}$, which, reduced to its equivalent decimal, gives $3.\dot{5}0649\dot{3}$, the answer required.

RULE. — *Change the given numbers to their equivalent common fractions. Divide, and reduce the quotient to its equivalent decimal.*

EXAMPLES.

2. Divide $345.\dot{8}$ by $.\dot{6}$. Ans. $518.8\dot{3}$.
3. Divide $234.\dot{6}$ by $.\dot{7}$.
4. Divide $13.5\dot{1}6953\dot{3}$ by $3.1\dot{4}\dot{5}$. Ans. $4.\dot{2}9\dot{7}$.
5. Divide $381.6140\dot{3}3\dot{8}$ by 4.37. Ans. $87.32\dot{5}8\dot{6}$.
6. Divide $.\dot{4}2857\dot{1}$ by $.625$.
7. Find the value of $2.3\dot{7}\dot{0} \div 4.\dot{9}2307\dot{6}$. Ans. $.\dot{4}8\dot{1}$.
8. Find the value of $.\dot{0}\dot{9} \div .\dot{2}3076\dot{9}$. Ans. $.\dot{3}\dot{9}$.
9. Find the value of $316.31015 \div .\dot{3}$.
10. Find the value of $100006 \div .\dot{6}$.
11. Divide $.\dot{3}\dot{6}$ by $.2\dot{5}$. Ans. $1.\dot{4}22924901185770750988\dot{1}$.

CONTINUED FRACTIONS.

307. A CONTINUED FRACTION is a fraction having for its numerator 1, and for its denominator a whole number plus a fraction whose numerator is 1, and whose denominator is a whole number plus a fraction, and so on. Thus,

$$\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{5+\cfrac{1}{4+\frac{1}{6}}}}}$$ is a continued fraction.

The *partial* fractions composing the parts of a continued fraction are called its *terms*. Thus, in the fraction given above, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{5}$, &c. are its terms.

308. Continued fractions are used in obtaining, in smaller numbers, the approximate values of fractions whose terms, when reduced to their simplest forms, are expressed in numbers inconveniently large.

309.° To transform a common fraction into a continued fraction, and to find, in smaller numbers, its approximate values.

Ex. 1. Transform $\frac{19}{60}$ into a continued fraction, and find its several approximate values.

OPERATION.

$$\begin{array}{l} 19)60(3 \\ \quad\ \underline{57} \\ \quad\ 3)19(6 \\ \qquad\ \underline{18} \\ \qquad\ 1)3(3 \\ \qquad\quad\ \underline{3} \\ \qquad\quad\ 0 \end{array}$$

$$\left.\begin{array}{l} \text{Hence, } \dfrac{19}{60} = \cfrac{1}{3+\cfrac{1}{6+\frac{1}{3}}} \\[2ex] \dfrac{1}{3} = \dfrac{1}{3}, \text{ 1st approx. value,} \\[2ex] \dfrac{1\times 6}{(3\times 6)+1} = \dfrac{6}{19}, \text{ 2d approx. value,} \\[2ex] \dfrac{(6\times 3)+1}{(19\times 3)+3} = \dfrac{19}{60}, \text{ the original value,} \end{array}\right\} \text{Ans.}$$

Dividing both terms of $\frac{19}{60}$ by the numerator, which operation will not change the value expressed (Art. 217), the fraction becomes $\frac{1}{3\frac{3}{19}}$; the denominator of which being between 3 and 4, the value of

the given fraction must be between $\frac{1}{3}$ and $\frac{1}{4}$; and neglecting the fraction $\frac{3}{19}$, for the present, in the denominator, we have $\frac{1}{3}$ for the *first approximate value*. This approximation, however, is greater than the true value, since the denominator, 3, is less than the true denominator $3\frac{3}{19}$. We therefore divide both terms of the $\frac{3}{19}$ by its numerator, and it becomes $\frac{1}{6\frac{1}{3}}$, which is between $\frac{1}{6}$ and $\frac{1}{7}$. By neglecting the fraction $\frac{1}{3}$ in the denominator, and taking the $\frac{1}{6}$ instead of the $\frac{3}{19}$, we have $\frac{1}{3+\frac{1}{6}} = \frac{1}{3\frac{1}{6}} = \frac{6}{19}$ for the *second approximate value* of the given fraction; which approximation is too small, since in the denominator, instead of $\frac{3}{19}$, we used $\frac{1}{6}$, which is greater than the $\frac{3}{19}$. If we now include in the calculation the remaining partial fraction $\frac{1}{3}$, we have $\frac{1}{3\frac{1}{6+\frac{1}{3}}} = \frac{1}{3\frac{3}{19}} = \frac{19}{60}$, the original fraction.

By the processes of the operation it will be seen that the *first* approximate value sought was obtained by disregarding all the partial fractions after the *first*, the *second* approximate value by disregarding all the partial fractions after the *second*, &c.

RULE. — *Divide the greater term of the given fraction by the less, and the divisor by the remainder, and so on, as in finding the greatest common divisor. The quotients thus found will be the denominators of the several terms of the continued fraction, and the numerator of each will be* 1.

For the FIRST *approximation, take the first terms of the continued fraction.*

For the SECOND *approximation, multiply the terms of the first approximate fraction by the denominator of the* SECOND *term of the continued fraction, adding* 1 *to the product of the denominators.*

For each SUCCEEDING *approximation, multiply the terms of the approximation last found by the denominator of the* NEXT *term of the continued fraction, and add the corresponding terms of the preceding approximation.*

NOTE 1. — When the fraction given is improper, *the true approximations will be the reciprocals of the fractions found by the rule.*

NOTE 2. — In a series of approximations the *first* is *larger*, the *second smaller*, and so on, every *odd* fraction being *larger*, and every *even* one *smaller*, than the given fraction. Each successive approximate fraction, however, approaches more nearly than the one preceding it to the value of the given fraction. When the continued fraction indicates many approximations, it is generally sufficient for ordinary purposes to find only from three to six of them.

NOTE 3. — A continued fraction may, for convenience, be expressed by writing its terms one directly after another, with the sign plus (+) between the denominators; thus, the continued fraction equivalent to $\frac{19}{60}$ may be written $\frac{1}{3}+\frac{1}{6}+\frac{1}{3}$.

EXAMPLES.

2. Transform $\frac{13}{30}$ into a continued fraction.

$$\text{Ans. } \cfrac{1}{2+\cfrac{1}{3+\frac{1}{4}}}$$

3. Transform $\frac{261}{82}$ into a continued fraction.

4. Find the approximate values of $\frac{29}{77}$. Ans. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{29}{77}$.

5. Find the first five approximate values of $\frac{1327}{1631}$.

6. Find the first three approximate values of $\frac{829}{347}$.

Ans. $\frac{2}{1}$, $\frac{5}{2}$, $\frac{7}{3}$, or 2, $2\frac{1}{2}$, $2\frac{1}{3}$.

7. Find the first six approximate values of $\dfrac{1}{13\frac{568}{1000}}$.

Ans. $\frac{1}{13}$, $\frac{1}{14}$, $\frac{2}{27}$, $\frac{7}{95}$, $\frac{37}{502}$, $\frac{44}{597}$.

8. What are the first four approximate values of 1.27?

Ans. $\frac{1}{1}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{14}{11}$, $\frac{33}{26}$, or 1, $1\frac{1}{3}$, $1\frac{1}{4}$, $1\frac{3}{11}$, $1\frac{7}{26}$.

RATIO.

310. RATIO is the relation, in respect to magnitude or value, which one quantity or number bears to another of the same kind.

311. The comparison by ratio is made by considering how often one number contains, or is contained in, another. Thus, the ratio of 10 to 5 is expressed by 2, the quotient arising from the division of the first number by the second, or it may be expressed by $\frac{5}{10} = \frac{1}{2}$, the quotient arising from the division of the second number by the first, as the second or the first number shall be regarded as the unit or standard of comparison. In general, of the two methods, the first is regarded as the more simple and philosophical, and therefore has the preference in this work.

NOTE. — Which of the two methods is to be preferred, is not a question of so much importance as has been by some supposed, since the connection in which ratio is used is usually such as to readily determine its interpretation.

312. The two numbers necessary to form a ratio are called

the *terms* of the ratio. The first term is called the *antecedent*, and the last, the *consequent*. The two terms taken together are called a *couplet;* and the quotient of the two terms, the *index* or *exponent* of the ratio.

313. The ratio of one number to another may be expressed either by two dots (:) between the terms; or in the form of a fraction, by making the antecedent the numerator and the consequent the denominator. Thus, the ratio 6 miles to 2 miles may be expressed as 6 : 2, or as $\frac{6}{2}$.

314. The terms of a ratio must be of the same kind, or such as may be reduced to the same denomination. Thus, cents have a ratio to cents, and cents to dollars, &c.; but cents have not a ratio to yards, nor yards to gallons.

315. A *simple* ratio is that of two whole numbers; as, 3 : 4, 8 : 16, 9 : 36, &c.

316. A *complex* ratio is that of two numbers, of which one or both are fractional; as, $6 : 4\frac{1}{2}$, $\frac{3}{5} : \frac{1\frac{1}{2}}{5}$, $4\frac{1}{2} : 2\frac{1}{4}$, &c.

317. A *compound* ratio is the product of two or more ratios. Thus, the ratio compounded of 4 : 2 and 6 : 3 is $\frac{4}{2} \times \frac{6}{3} = \frac{24}{6} = 4$, or $4 \times 6 : 2 \times 3 = 4$.

A compound ratio is generally expressed by writing the ratios composing it, in a column, with the antecedents in one vertical line, and the consequents in another; thus, $\left.\begin{array}{l} 4 : 2 \\ 6 : 3 \end{array}\right\}$ expresses a compound ratio.

NOTE. — If a ratio be compounded of two equal ratios, it is called a *duplicate* ratio; of three ratios, a *triplicate* ratio, &c.

318. A ratio is either *direct* or *inverse.* A *direct* ratio is the quotient of the antecedent by the consequent; an inverse ratio, or reciprocal ratio, as it is sometimes called, is the quotient of the consequent by the antecedent, or the reciprocal of the direct ratio. Thus the direct ratio of 6 to 2 is $\frac{6}{2}$ or 3; and the inverse or reciprocal ratio of 6 to 2 is $\frac{2}{6}$ or $\frac{1}{3}$, which is the same as the reciprocal of 3, the direct ratio of 6 to 2.

NOTE 1. — One quantity is said to *vary directly* as another, when both increase or decrease together in the same ratio; one quantity is said to *vary in-*

versely as another, when the one increases in the same ratio as the other decreases.

NOTE 2. — The word ratio, when used alone, means the direct ratio.

319. When the antecedent and consequent of a ratio are equal, the ratio equals 1, and is called that of *equality*. Thus, the ratio of $6 : 6 = \frac{6}{6} = 1$, and the ratio of $6 \times 4 : 8 \times 3 = \frac{24}{24} = 1$, are ratios of equality. But if the antecedent is larger than the consequent, the ratio is that of *greater inequality*, and if the antecedent is smaller than the consequent, the ratio is that of *less inequality*. Thus, the ratio of $15 : 5 = \frac{15}{5} = 3$, is a ratio of greater inequality; and the ratio of $7 : 14 = \frac{7}{14} = \frac{1}{2}$, is a ratio of less inequality.

320. The ratio of two fractions having a *common numerator* is the same as the inverse ratio of their *denominators*. Thus, the ratio of $\frac{3}{4} : \frac{3}{8}$ is $\frac{3}{4} \div \frac{3}{8} = 2$, which is the inverse ratio of the denominator 4 to the denominator 8.

321. The ratio of two fractions having a *common denominator* is the same as the ratio of their *numerators*. Thus, the ratio of $\frac{6}{7} : \frac{3}{7}$ is $\frac{6}{7} \div \frac{3}{7} = 2$, which is the ratio of the numerator 6 to the numerator 3.

322. The inverse or reciprocal ratio of two numbers denotes *what part* or *multiple* the consequent is of the antecedent. Thus, inquiring *what part* of 4 is 3, or *what part* 3 is of 4, is the same as inquiring the inverse or reciprocal ratio of 4 : 3. The inverse ratio of 4 : 3 is $\frac{3}{4}$, and 3 is $\frac{3}{4}$ of 4.

323. In order to compare one number with another, by ratio, it is necessary that they should not only be of the same kind, but of the same denomination. Thus, to compare 2 days with 12 hours, it is necessary that the days be reduced to hours, before we can indicate the ratio, which is 48 hours : 12 hours.

324. *If the antecedent of a ratio be multiplied, or the consequent divided,* the ratio is *multiplied.* Thus, the ratio of 6 : 3 is 2, but $6 \times 2 : 3$ is 4; or $6 : 3 \div 2$ is 4.

325. If the antecedent of a ratio be divided, or the consequent multiplied, the ratio is divided. Thus, the ratio of 18 : 6 is 3, but $18 \div 3 : 6$ is 1; or $18 : 6 \times 3 = 1$.

326. If both the antecedent and consequent of a ratio be multiplied or divided by the same number, the ratio is not altered. Thus, the ratio of $8 : 2$ is 4; of $8 \times 2 : 2 \times 2$ is 4; and of $8 \div 2 : 2 \div 2$ is 4.

REDUCTION AND COMPARISON OF RATIOS.

327. Ratios, being of the nature of fractions, may be reduced, compared, and otherwise operated upon like them.

328. To reduce a ratio to its lowest terms.

Ex. 1. Reduce $18 : 9$ to its lowest terms. Ans. $2 : 1$.

OPERATION.

$$18 : 9 = \frac{18}{9} = \frac{2}{1} = 2 : 1.$$

We cancel in the two terms the common factor 9, and obtain $\frac{2}{1} = 2 : 1$, the answer. Hence

Cancel in the given ratio all factors common to its terms.

EXAMPLES.

2. Reduce to its lowest terms $63 : 72$. Ans. $\frac{7}{8}$.
3. Reduce to its lowest terms $66 : 24$.
4. Reduce to its lowest terms $4 \times 6 \times 3 : 8 \times 9 \times 2$. Ans. $\frac{1}{2}$.
5. What are the lowest terms of $19 \times 5 \times 2 \times 3 : 15 \times 12 \times 38$?

329. To reduce a complex or a compound ratio to a simple one.

Ex. 1. Reduce $5\frac{1}{2} : \frac{3}{4}$ to a simple ratio. Ans. $22 : 3$.

OPERATION.

$$5\tfrac{1}{2} : \tfrac{3}{4} = \frac{5\frac{1}{2}}{\frac{3}{4}} = \frac{44}{6} = \frac{22}{3} = 22 : 3 \text{ Ans.}$$

We express the given ratio in the form of a complex fraction, which, changed to a simple fraction (Art. 242), and reduced to its lowest terms, gives $\frac{22}{3} = 22 : 3$, the answer required.

2. Reduce $\left.\begin{matrix} 8 : 5 \\ 7 : 24 \end{matrix}\right\}$ to a simple ratio. Ans. $7 : 15$.

OPERATION.

$$\left.\begin{matrix} 8 : 5 \\ 7 : 24 \end{matrix}\right\} = \tfrac{8}{5} \text{ of } \tfrac{7}{24} = \frac{\cancel{8}}{5} \times \frac{7}{\underset{3}{\cancel{24}}} = \tfrac{7}{15} = 7 : 15 \text{ Ans.}$$

We express the given ratio in the form of a compound fraction,

which, reduced to a simple one (Art. 329), gives $\frac{7}{15} = 7 : 15$, the answer required. Hence, to reduce a complex or a compound ratio to a simple one,

Proceed as in like operations with fractions.

EXAMPLES.

3. Reduce $\frac{5}{8} : \frac{3}{7}$ to a simple ratio. Ans. 35 : 24.
4. Reduce $13\frac{1}{2} : 27$ to a simple ratio. Ans. 1 : 2.
5. Reduce 6.25 : 3.125 to a simple ratio. Ans. 2 : 1.
6. Reduce $\left.\begin{array}{r}4 : 16\\ 25 : 10\end{array}\right\}$ to a simple ratio. Ans. 5 : 8.
7. Reduce $\left.\begin{array}{r}3 : 6\\ 9 : 27\\ 108 : 12\end{array}\right\}$ to a simple ratio. Ans. 3 : 2.
8. Reduce $\left.\begin{array}{r}12\frac{1}{2} : 6\frac{1}{4}\\ 76.5 : 25.5\end{array}\right\}$ to a simple ratio. Ans. 6 : 1.

330. To find the ratio of one number to another.

Ex. 1. Required the direct ratio of 108 to 9. Ans. 12.

OPERATION.

$108 : 9 = \frac{108}{9} = 12$ Ans.

Since 9 is the unit or standard of comparison, we make it the consequent (Art. 111) and the 108 the antecedent of the ratio, and obtain $\frac{108}{9} = 12$ Ans.

2. Required the inverse ratio of 72 to 8. Ans. $\frac{1}{9}$.

OPERATION.

$72 : 8$ inverted $= \frac{8}{72} = \frac{1}{9}$ Ans.

We divide the consequent 8 by the antecedent 72, or, which is the same thing, find the reciprocal of the direct ratio of 72 : 8 (Art. 318), by inverting its terms, and thus obtain $\frac{8}{72} = \frac{1}{9}$ Ans. Hence,

The direct ratio is found by dividing the antecedent by the consequent, and the inverse ratio by dividing the consequent by the antecedent.

NOTE 1. — Ratios expressed by fractions having different denominators must be reduced to a common denominator, in order to be compared; and then they are to each other as their numerators (Art. 328).

NOTE 2. — When a ratio is expressed in terms inconveniently large and prime to each other, we may find the approximate values of the ratio expressed in smaller numbers, as in other fractional expressions (Art. 309).

EXAMPLES.

3. What is the ratio of 39 to 13? Ans. 3.
4. What is the ratio of 2 yards 2 quarters to 9 yards?

5. What is the ratio of 21 gallons to $\frac{1}{3}$ of a hogshead?
Ans. 1.

6. What is the ratio of $\frac{1}{3}$ of $\frac{1}{6}$ of \$2 : $\frac{1}{2}$ of \$0.50?
Ans. $\frac{4}{9}$.

7. What is the inverse ratio of 24 : 6? Ans. $\frac{1}{4}$.

8. What part of 36 is 4? Ans. $\frac{1}{9}$.

9. What part of a farm of 94A. 2R. 16rd. is 11A. 3R.?

10. Which is the greater, the ratio of 17 to 9, or of 39 to 19? Ans. 39 : 19.

11. By how much does the ratio of $36 \times 4 \times 3 : 12 \times 16 \times 2$ exceed that of $60 \div (3 \times 5) : 20 \times 2 \div 8$? Ans. $\frac{13}{40}$.

12. What is the inverse ratio of $.0\dot{2} : 2.\dot{5}0\dot{3}$?

13. Which is the greater, the ratio of $\frac{1}{2}$ of $\frac{1}{5}$: $\frac{1}{3}$ of $\frac{1}{4}$, or that of 5 : 4?

14.° The height of Bunker Hill Monument is 220 feet, and that of the great pyramid, Egypt, 500 feet; what is the ratio of the height of the former to that of the latter? Ans. $\frac{11}{25}$.

15.° A certain farm contains 180 acres, and the township of which it forms a part is 36 square miles in extent. What is the ratio of the latter to the former?

16.° Find approximate values for the ratio of 4900 to 11283.
Ans. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{10}{23}$, $\frac{33}{76}$, $\frac{76}{175}$, &c.

17.° The ratio of the circumference of a circle to its diameter is 3.141592. Required approximate values for this ratio.
Ans. 3, $\frac{22}{7}$, $\frac{333}{106}$, $\frac{355}{113}$, &c., or 3, $3\frac{1}{7}$, $3\frac{15}{106}$, $3\frac{16}{113}$, &c.

ANALYSIS BY RATIO.

331. Operations by analysis may often be much abridged by ratio. Thus, frequently, it is more convenient to multiply or divide by the ratio a number bears to a unit of the same kind, than to multiply or divide by the number itself.

This form of analysis is much used by business men; and, like that by aliquot parts (Art. 114), is sometimes called *Practice.*

EXAMPLES.

1. What cost 14 tons 15cwt. 3qr. 20lb. of iron, at \$60 a ton? Ans. \$887.85.

OPERATION.

	\$ 60.00 =	cost of 1 ton.	
	14		
	\$ 840.00 =	"	14 tons.
(10cwt. : 1 ton = $\frac{1}{2}$) ; $\frac{1}{2}$ of \$ 60 =	30.00 =	"	10cwt.
(5cwt. : 10cwt. = $\frac{1}{2}$) ; $\frac{1}{2}$ of \$ 30 =	15.00 =	"	5cwt.
(2qr. : 5cwt. = $\frac{1}{10}$) ; $\frac{1}{10}$ of \$ 15 =	1.50 =	"	2qr.
(1qr. : 2cwt. = $\frac{1}{2}$) ; $\frac{1}{2}$ of \$ 1.50 =	0.75 =	"	1qr.
(15lb. : 3qr. = $\frac{1}{5}$) ; $\frac{1}{5}$ of \$ 2.25 =	0.45 =	"	15lb.
(5lb. : 15lb. = $\frac{1}{3}$) ; $\frac{1}{3}$ of \$ 0.45 =	0.15 =	"	5lb.
Ans.	\$ 887.85 =	"	15T. 3qr. 20lb.

Since 1 ton costs \$ 60, 14 tons will cost 14 times \$ 60, or \$ 840. 15cwt. = 10cwt. + 5cwt. Since the ratio of 10cwt. to 1 ton or 20cwt. = $\frac{1}{2}$, 10cwt. will cost $\frac{1}{2}$ as much as 1 ton, or \$ 30 ; and as the ratio of 5cwt. to 10cwt. = $\frac{1}{2}$, 5cwt. will cost $\frac{1}{2}$ as much as 10cwt. or \$ 15. 3qr. = 2qr. + 1qr. Since the ratio of 2qr. to 5cwt. or 20qr. = $\frac{1}{10}$, 2qr. will cost $\frac{1}{10}$ as much as 5cwt., or \$ 1.50 ; and as the ratio of 1qr. to 2qr. = $\frac{1}{2}$, 1qr. will cost $\frac{1}{2}$ as much as 2qr., or \$ 0.75. 20lb. = 15lb. + 5lb. Since the ratio of 15lb. to 3qr. or 75lb. = $\frac{1}{5}$, 15lb. will cost $\frac{1}{5}$ as much as 3qr., or \$ 0.45 ; and as the ratio of 5lb. to 15lb. = $\frac{1}{3}$, 5lb. will cost $\frac{1}{3}$ as much as 15lb., or \$ 0.15. The cost of the several parts equals the cost of the whole, or \$ 887.85, Ans.

2. What is the value of 17 acres 3 roods 35 rods of land, at \$ 80 per acre? Ans. \$ 1437.50.

3. What cost 16cwt. 3qr. 10lb. of guano, at \$ 2.50 per cwt.?

4. What cost 27cwt. 1qr. 20lb. of coffee, at \$ 14 per cwt.? Ans. \$ 384.30.

5. If 1 yard of cloth cost \$ 5.60, what will 7yd. 3qr. 2na. cost? Ans. \$ 44.10.

6. What cost 7 tons 13cwt. 2qr. of hay, at \$ 20 per ton?

7. What cost 99bu. 1pk. 4qt. of wheat, at \$ 1.92 per bushel? Ans. \$ 191.80.

OPERATION.

	\$ 1.92	= cost of 1bu.		
	100			
	\$ 192.00	=	"	100bu.
(2pk. : 1bu. = $\frac{1}{2}$) ; $\frac{1}{2}$ of \$ 1.92 =	0.96	=	"	2pk.
(4qt. : 2pk. = $\frac{1}{4}$) ; $\frac{1}{4}$ of \$ 0.96 =	0.24	=	"	4qt.
Ans.	\$ 191.80	=	"	99bu. 1pk. 4qt.

The quantity being nearly 100 bushels, we find the cost of 100 bushels by annexing two ciphers to \$ 1.92, the cost of 1 bushel, and obtain \$ 192, from which we subtract the cost of 2pk. 4qt., the difference of quantity between that given and 100 bushels; the cost of 2pk. = \$ 0.96; and that of 4qt. = \$ 0.24; \$ 192 — \$ 0.96 + \$ 0.24 = \$ 191.80 Ans.

8. What cost 19yd. 3qr. 2na. of cloth, at \$ 4.40 per yard? Ans. \$ 87.45.

9. How much must be paid for 24A. 3R. 20p. of land, at \$ 32 per acre?

10. How much must be paid for 199lb. 12oz. of butter, at \$ 0.30 per lb.? Ans. \$ 59.925.

11. What cost 714 yards of broadcloth, at 15s. 6d. per yard? Ans. 553£. 7s.

12. How much must be paid for the services of a man 2y. 9mo. 15da., at \$ 450 yer year? Ans. \$ 1256.25.

13. If 1 acre of land cost \$ 80.50, what will 25 acres 2 roods 35 rods cost? Ans. \$ 2070.35+.

14. What cost 498lb. of tea, at 2s. 6d. per lb.?

15. If 1cwt. 2qr. 12lb. of alum can be purchased for \$ 4.05, how much can be purchased for \$ 28.35?

Ans. 11cwt. 1qr. 9lb.

OPERATION.

\$ 28.35 ÷ \$ 4.05 = 7;

1cwt. 2qr. 12lb. × 7 = 11cwt. 1qr. 9lb. Ans.

Since the ratio of \$ 28.35 to \$ 4.05 = 7, \$ 28.35 will purchase 7 times as much as \$ 4.05. By multiplying what the latter will purchase by the ratio, we have the answer required.

16. If 11gal. 3qt. 1pt. of molasses cost \$ 5.83$\frac{3}{4}$, what will 35gal. 2qt. 1pt. cost? Ans. \$ 17.51$\frac{1}{4}$.

17. If 24yd. 3qr. of cloth cost \$ 49.50, what will 12yd. 1qr. 2na. cost?

18. If 17bu. 2pk. 4qt. of oats be paid for 14bu. 3pk. of salt, what quantity of oats must be paid for 73bu. 3pk. of salt?

Ans. 88bu. 0pk. 4qt.

19. If \$ 9.75 will purchase 1T. 2cwt. 2qr. 15lb. of coal, how much will \$ 3.25 purchase?

20. If a train of cars move at the average velocity of 27m. 3fur. 20rd. per 1h. 20m., how far will it move in 4h.?

Ans. 82m. 2fur. 20rd.

PROPORTION.

332. A PROPORTION is an equality of ratios. Any *four* numbers are in proportion, when the ratio of the *first* to the *second* is the same as that of the *third* to the *fourth*. Thus, the ratios 9 : 3 and 6 : 2, being equal to each other, when written, 9 : 3 = 6 : 2, or $\frac{9}{3} = \frac{6}{2}$, form a proportion.

Proportion is written with the sign of equality (=), or, as is more common, with four dots (: :), between the ratios. Thus, 9 : 3 = 6 : 2, or 9 : 3 :: 6 : 2, expresses a proportion, and is read, The ratio of 9 to 3 is equal to the ratio of 6 to 2, or 9 is to 3 as 6 is to 2.

333. The *terms* of a proportion are the four numbers which form the proportion. These numbers are also called *proportionals*. The first and third terms, or proportionals, are called *antecedents*, the second and fourth are called *consequents;* the *first* and *last* are called the *extremes*, the *second* and *third* the *means;* the first and second compose the *first couplet*, the third and fourth compose the *second;* and when the ratio of the first of three terms is to the second as the ratio of the second is to the third, the second term is called a *mean proportional* to the other two terms.

334. A *direct* proportion is an equality between two direct ratios; an *inverse* or *reciprocal* proportion is an equality between a direct and an inverse or reciprocal ratio. Thus, the numbers 4, 2, 6, 3 are, as they stand, in direct proportion, denoting 4 : 2 :: 6 : 3; but in the order 4, 2, 3, 6, are in inverse proportion, denoting that 4 : 2 :: $\frac{1}{3}$: $\frac{1}{6}$, or the direct ratio of 4 to 2 is equal to the inverse ratio of 3 to 6.

NOTE. — The term *proportion*, used alone, always means direct proportion.

335. *In any proportion, if the antecedents or consequents, or both, are divided, or multiplied, by the same number, they are still proportionals.* Thus, dividing the antecedents of the proportion 4 : 8 :: 10 : 20 by 2, we have 2 : 8 :: 5 : 20; dividing the consequents by 2, we have 4 : 4 :: 10 : 10; and dividing both the antecedents and consequents by 2, we have 2 : 4 :: 5 : 10; each of which results is a proportion, since if we divide

the second term of each by the first, and the fourth by the third, the two quotients will be equal. The effect is the same when the terms are multiplied by the same number.

336. *In every proportion the product of the two extremes is equal to the product of the two means.* Thus, the proportion 16 : 8 :: 20 : 10 may be expressed $\frac{16}{8} = \frac{20}{10}$. Now, if we reduce these fractions to a common denominator, we have $\frac{160}{80} = \frac{160}{80}$; but in this operation we multiplied together the two *extremes* of the proportion, 16 and 10, and the two *means*, 8 and 20; thus, $16 \times 10 = 8 \times 20$. Hence,

1. *If the extremes and one of the means are given, the other mean may be found by dividing the product of the extremes by the given mean;* or,

2. *If the means and one of the extremes are given, the other extreme may be found by dividing the product of the means by the given extreme.*

SIMPLE PROPORTION.

337. Simple Proportion is an equality between two simple ratios.

NOTE. — Simple Proportion is sometimes called the Rule of Three, and formerly was termed by arithmeticians the Golden Rule.

338. The object of that part of simple proportion which is usually included in arithmetics, is *to find a fourth proportional to three given numbers*, or, in other words, *to find the fourth term of a proportion, when the other three terms are given.*

Ex. 1. If a man travel 243 miles in 9 days, how far will he travel in 24 days? Ans. 648 miles.

OPERATION.

Extreme.	Mean.	Mean.	Extreme.
9 da. :	24 da. ::	243 m. :	— m.
		24	
		972	
		486	
	9)	5832	
	Ans.	648 m.	Extreme.

Since 9 days have the same ratio to 24 days as 243 miles, the distance of travel in 9 days, have to the distance of travel in 24 days, we have the first three terms of a proportion given, namely, the two means and one of the extremes, from which to find the required extreme. Now, to arrange the given numbers in the order of

a proportion, or *state the question*, we make the 243 miles the *third* term, because it is of the same kind as the required *fourth* term, and as from the nature of the question the latter must be greater than the third term, we make the greater of the other two numbers the *second* term, and the less the *first;* and, then, the product of the means divided by the given extreme gives the required extreme (Art. 336).

BY ANALYSIS. — If a man travel 243 miles in 9 days, he will in 1 day travel $\frac{1}{9}$ of 243 miles $=$ 27 miles; then, if he travel 27 miles in 1 day, in 24 days he will travel 24 times 27 miles $=$ 648 miles, the answer, as before.

BY RATIO. — $9 : 24 = \frac{9}{24} = \frac{3}{8}$; 243 miles $\div \frac{3}{8} =$ 648 miles, Ans.

2. If 15 yards of cloth cost \$48.90, what will 5 yards cost?
Ans. \$16.30.

OPERATION.

$15 : 5 :: \$48.90 : \$—$

$$\frac{\cancel{5} \times 48.90}{\underset{3}{\cancel{15}}} = 16.30.$$

Ans. \$16.30.

We state the question by making \$48.90 the *third* term, because it is of the same kind as the required term. Then, since the answer must be less than \$48.90, because 5 yards will cost less than 15 yards, we make 5 yards, the less of the two numbers, the *second* term, and 15 yards the *first;* and proceed as in the first example, except that we abridge the work by cancellation.

BY ANALYSIS. — If 15 yards cost \$48.90, 1 yard will cost $\frac{1}{15}$ of \$48.90 $=$ \$3.26; then, if 1 yard cost \$3.26, 5 yards will cost 5 times \$3.26 $=$ \$16.30.

RULE. — *Write the given number that is of the same kind as the required fourth term, or answer, for the third term of the proportion.*

Of the other two numbers write the larger for the second term, and the less for the first, when the answer should exceed the third term; but write the less for the second term, and the larger for the first, when the answer should be less than the third term.

Multiply the second and third terms together, and divide their product by the first; or divide the third term by the ratio of the first term to the second.

NOTE 1. — When the first and second terms are of different denominations, they must be reduced to the same denomination; and when the third term is a compound number, it must be reduced to the lowest denomination mentioned in it. The answer will be of the same denomination as the third term.

NOTE 2. — To shorten the operations, factors common to the dividend and divisor may be cancelled.

NOTE 3. — The pupil should perform these questions by analysis, as well as by proportion, and introduce cancellation when it will abbreviate the operation.

EXAMPLES.

3. If 16 acres of land cost $ 720, what will 197 acres cost? Ans. $ 8865.

4. If $ 8865 buy 197 acres, how many acres may be bought for $ 720?

5. What will 84hhd. of molasses cost, if 15hhd. can be purchased for $ 175.95? Ans. $ 985.32.

6. If $ 100 gain $ 6 in 12 months, how much would it gain in 40 months? Ans. $ 20.

7. If a certain vessel has provisions sufficient to last a crew of 10 men 45 days, how long would the provisions last if the vessel were to ship 5 new hands? Ans. 30 days.

8. If 7 and 9 were 12, what, on the same supposition, would 8 and 4 be?

9. If 9 men can perform a certain piece of labor in 17 days, how long would it take 3 men to do it? Ans. 51 days.

10. If 3 men can perform a piece of labor in 51 days, how many must be added to the number to perform the labor in 17 days? Ans. 6.

11. A rectangular piece of land containing an acre is $5\frac{1}{2}$ rods in breadth. What is its length? Ans. $29\frac{1}{11}$ rods.

12. If $ 100 gain $ 6 in a year, how much will $ 850 gain?

13. If $ 100 gain $ 6 in a year, how much would be sufficient to gain $ 32 in a year? Ans. $ $533.33\frac{1}{3}$.

14. If 20 gallons of water weigh 167lb., what will 180 gallons weigh? Ans. 1503lb.

15. If a staff 3 feet long cast a shadow of 2 feet, how high is that steeple whose shadow is 75 feet? Ans. $112\frac{1}{2}$ feet.

16. If $5\frac{3}{4}$cwt. be carried 36 miles for $ 4.75, how far might it be carried for $ 160? Ans. $1212\frac{12}{19}$ miles.

17. If 100 workmen can perform a piece of work in 12 days, how many men are sufficient to perform the work in 8 days? Ans. 150.

18. If $\frac{7}{12}$ of a yard cost $\frac{7}{20}$ of a dollar, what will $\frac{4}{5}$ of a yard cost? Ans. $ 0.48.

19. What must be paid for 21A. 3R. 20p. of land, if 36A. 3R. cost $ 1260? Ans. $ 750.

20. What is the value of 2000lb. of standard gold, the eagle, or $ 10 piece, weighing 10pwt. 18gr.?

21. If $4\frac{1}{2}$ yards of cloth cost $ 9.75, what will $13\frac{1}{2}$ yards cost? Ans. $ 29.25.

22. What is the length of a rectangle whose contents are 1 sq. ft. and whose breadth is $2\frac{1}{2}$ inches? Ans. $57\frac{3}{5}$ inches.

23. If $\frac{7}{16}$ of a ship cost 51£., what are $\frac{3}{32}$ of her worth? Ans. 10£. 18s. $6\frac{6}{7}$d.

24. If the moon moves 13° 10′ 35″ in one day, in what time does she perform one revolution? Ans. 27da. 7h. 43m.+

25. If 7lb. of sugar cost $\frac{3}{4}$ of a dollar, what are 12lb. worth? Ans. $ $1.28\frac{4}{7}$.

26. If $ 1.75 will buy 7lb. of loaf-sugar, how much will $ 213.50 buy? Ans. 8cwt. 2qr. 4lb.

27. If 7 ounces of gold are worth 30£., what is the value of 7lb. 11oz. Ans. 407£. 2s. $10\frac{2}{7}$d.

28. A friend borrowed of me $ 500 for 6 months; how long ought he to lend me $ 600, to requite the favor?

29. If the penny loaf weighs 7oz. when flour is $ 8 per barrel, how much should it weigh when flour is $ 7.50 per barrel? Ans. $7\frac{7}{15}$ ounces.

30. If a regiment of soldiers, consisting of 1000 men, are to be clothed, each suit to contain $3\frac{3}{4}$ yards of cloth that is $1\frac{7}{8}$ yards wide, and to be lined with flannel $1\frac{1}{4}$ yards wide, how many yards will it take to line the whole? Ans. 5625yd.

31. If by working 14 hours per day C. Simmons can plant half of a field in 9 days, in what time will he plant the remainder, working 10 hours per day at the same rate each hour? Ans. $12\frac{3}{5}$ days.

32. If 75 gallons of water fall into a cistern containing 500 gallons, and 40 gallons run out, in an hour, in what time will it be filled? Ans. 14h. 17m. $8\frac{4}{7}$sec.

33. How many dozen pairs of gloves, at $ 0.56 per pair, can be bought for $ 120.96? Ans. 18 dozen.

34. A certain cistern has three pipes; the first will empty it in 20 minutes, the second in 40 minutes, and the third in 75 minutes; in what time would they all empty it? Ans. 11m. $19\frac{13}{53}$sec.

35. A can mow a certain field in 5 days, and B can mow it

in 6 days; in what time would both of them together mow it? Ans. $2\frac{8}{11}$ days.

36. A wall, which was to be built 32 feet high, was raised 8 feet by 6 men in 12 days; how many men must be employed to finish the wall in 6 days?

37. A can build a boat in 20 days, but with the assistance of C he can do it in 12 days; in what time would C do it alone? Ans. 30 days.

38. In a fort there are 700 men provided with 184000lb. of provisions, of which each man consumes 5lb. a week; how long can they subsist? Ans. 52 weeks 4 days.

39. If 25 men have $\frac{3}{4}$ of a pound of beef each, three times in a week, how long will 3150lb. last them? Ans. 56 weeks.

40. How many tiles 8 inches square will lay a floor 20 feet long and 16 feet wide? Ans. 720.

41. How many stones 10 inches long, 9 inches broad, and 4 inches thick, would it require to build a wall 80 feet long, 20 feet high, and $2\frac{1}{4}$ feet thick? Ans. 17280 stones.

42. If there be paid for 1 ton 7cwt. 3qr. 20lb. of coal $ 9.50, what will 13 tons 5cwt. 2qr. cost?

43. If 61.3 pounds of tea cost $ 44.9942, what is the price per pound? Ans. $ 0.734.

44. What is the value of .15 of a hogshead of lime, at $ 2.39 per hogshead? Ans. $ 0.3585.

45. If .75 of a ton of hay cost $ 15, what is it per ton? Ans. $ 20.

46. How many yards of carpeting that is half a yard wide will cover a room that is 30 feet long and 18 feet wide? Ans. 120 yards.

47. If a man perform a journey in 15 days, when a day is 12 hours long, in how many days will he do it when a day is but 10 hours long?

48. If 450 men are in a garrison, and their provisions will last them but 5 months, how many must leave the garrison that the same provisions may be sufficient to supply the remaining men 9 months? Ans. 200 men.

49. The hour and minute hands of a watch are together at 12 o'clock; when will they next be together? Ans. 1h. 5m. $27\frac{3}{11}$sec.

50. A and B can perform a piece of work in $5\frac{5}{11}$ days, B and C in $6\frac{2}{3}$ days, and A and C in 6 days; in what time would each of them perform the work alone, and how long would it take them to do the work together?

Ans. A would do the work in 10 days; B, in 12 days; C, in 15 days; A, B, and C together, in 4 days.

339. To divide a number or quantity into parts, which are proportional to given numbers.

Ex. 1. Divide \$250 into two parts which shall be one to the other as 2 to 3. Ans. \$100 and \$150.

OPERATION.

$2 + 3 = 5$

$$\left.\begin{array}{l}5 : 3 :: \$250 : \$150, \text{ the greater part,} \\ 5 : 2 :: \$250 : \$100, \text{ the less part,}\end{array}\right\} \text{Ans.}$$

Since the parts are to be proportional to 2 and 3, whose sum is 5, it is evident that the sum of the two numbers, 5, will have the same ratio to the greater of them, 3, as the amount to be divided, \$250, has to the greater of the required parts; and that the sum, 5, will have the same ratio to the less number, 2, as the \$250 has to the less of the required parts; we therefore make two statements, and then find the required term of each proportion as in Art. 337. Hence,

As the sum of the given numbers is to any one of them, so is the whole quantity to be divided to the part corresponding to the number used as the second term.

NOTE. — This application of proportion is sometimes called *Distributive* or *Partitive Proportion.*

EXAMPLES.

2. A farmer divides between his three sons 246A. 1R. 32p. of land, sharing it between them as the numbers 3, 4, and 5. What were the shares?

Ans. 61A. 2R. 18p.; 82A. 0R. 24p.; 102A. 2R. 30p.

3. Divide 319 into four parts, that shall be to each other as the numbers $4\frac{1}{4}$, $6\frac{1}{2}$, $6\frac{3}{5}$, and 7.

Ans. $55\frac{330}{487}$; $85\frac{75}{487}$; $86\frac{226}{487}$; $91\frac{343}{487}$.

4. Standard gold for coinage consists of 9 parts of pure gold, and 1 part alloy. Allowing the alloy to be silver and copper in equal parts, how much pure gold, silver, and copper are contained in a double eagle, its weight being 1oz. 1pwt. 12gr.?

Ans. 19pwt. $8\frac{2}{5}$gr. gold; 1pwt. $1\frac{4}{5}$gr. silver; 1pwt. $1\frac{4}{5}$gr. copper.

5. The half-dollar of the United States coinage weighs 192 grains Troy, and consists of 9 parts pure silver and 1 part of copper. How much pure silver and how much copper in 20 half-dollars? Ans. 7oz. 4pwt. silver; 16pwt. copper.

6. Divide $ 600 between three men, so that the second man shall receive one third more than the first, and the third man shall receive two thirds more than the second.

7. A, B, and C freight a steamer; A puts on board 98 tons, B 86 tons, and C 64 tons. Owing to danger of being wrecked, there were thrown overboard while at sea 93 tons. What should be the number of tons lost by each?

Ans. A, $36\frac{3}{4}$ tons; B, $32\frac{1}{4}$ tons; and C, 24 tons.

8. A and B start together by railroad from Chicago for Galena; A travels by freight train at the rate of 15 miles per hour, and B by passenger train at the rate of 25 miles per hour. C leaves Galena for Chicago at the same time by express train, whose velocity is at the rate of 32 miles per hour. Allowing the distance between the two places to be 160 miles, how far from Chicago will A and B each be, when C passes them? Ans. A, $51\frac{3}{47}$ miles; B $70\frac{10}{57}$ miles.

COMPOUND PROPORTION.

340. A Compound Proportion is an expression of equality between a compound and a simple ratio. Thus,

$$\left.\begin{matrix} 4 : 3 \\ 5 : 7 \end{matrix}\right\} :: 60 : 63, \text{ is a compound proportion.}$$

Compound proportion is employed in the solution of such questions as would require two or more statements in Simple Proportion.

Ex. 1. If 8 men spend $ 32 in 13 weeks, what will 24 men spend in 52 weeks? Ans. $ 384.

OPERATION.

$$\begin{array}{ll} \text{Extreme.} & \text{Mean.} \\ 8 \text{ men} & : 24 \text{ men} \\ 13 \text{ weeks} & : 52 \text{ weeks} \end{array} \Bigg\} :: \begin{array}{ccc} \text{Mean.} & & \text{Extreme.} \\ \$\,32 & : & \$\,— \end{array}$$

$$\frac{\overset{3}{\cancel{24}} \times \overset{4}{\cancel{52}} \times 32}{\cancel{8} \times \cancel{13}} = 384. \quad \text{Ans. } \$\,384.$$

In stating the question, we make $ 32, which is of the same kind as the required term, the third term. Then, taking of the remaining terms two of the same

kind, 8 men and 24 men, we inquire whether the answer depending on these alone must be greater or less than the third term; and since it must be greater, because 24 men will spend more than 8 men in the same time, we make 24 men the second term, and 8 men the first term. Again, we take the two remaining terms, and make 52 weeks the second term, and 13 weeks the first, since the same number of men would spend more in 52 weeks than in 13 weeks. We then find the continued product of the second and third terms, and divide by the product of the first terms.

BY ANALYSIS. — If 8 men spend \$ 32 in 13 weeks, 1 man will spend $\$\frac{32}{8} = \$4$ in 13 weeks, and 24 men will spend $\$4 \times 24 = \$96$ in 13 weeks. If 24 men spend \$ 96 in 13 weeks, in 1 week they will spend $\$\frac{96}{13}$, and in 52 weeks, $\$\frac{96}{13} \times 52 = \$384$, Ans.

BY RATIO. — The ratio of $8 : 24 = \frac{1}{3}$; the ratio of $13 : 52 = \frac{1}{4}$; $\$32 \div \frac{1}{3} \times \frac{1}{4} = \$384$, Ans.

NOTE. — To have solved the question by simple proportion, two statements would have been required, which would have produced the following proportions: —

8 men : 24 men : : \$ 32 : \$ 96.
13 weeks : 52 weeks : : \$ 96 : \$ 384.

RULE. — *Make that number which is of the same kind as the answer required the third term of a proportion. Of the remaining numbers, take any two, that are of the same kind, and consider whether an answer depending upon these alone would be greater or less than the third term, and place them as directed in Simple Proportion.*

Then take any other two, and consider whether an answer depending only upon them would be greater or less than the third term, and arrange them accordingly; and so on until all are used.

Multiply the product of the second terms by the third, and divide the result by the product of the first terms. The quotient will be the fourth term, or answer.

EXAMPLES.

2. If a man can travel 117 miles in 30 days, travelling 9 hours a day, how far can he go in 20 days, travelling 12 hours a day? Ans. 104 miles.

3. If 6 men in 16 days of 9 hours each build a wall 20 feet long, 6 feet high, and 4 feet thick, in how many days of 8 hours each will 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick?

4. If \$ 100 gain \$ 6 in one year, how much would \$ 500 gain in four months? Ans. \$ 10.

5. If \$ 100 gain \$ 6 in one year, what must be the sum to gain \$ 10 in 4 months? Ans. \$ 500.

6. How long will it take $ 500 to gain $ 10, if $ 100 gain $ 6 in one year? Ans. 4 months.

7. If $ 500 gain $ 10 in 4 months, what is the rate per cent?

8. If 5 compositors in 16 days, 11 hours long, can compose 25 sheets of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line, in how many days, 10 hours long, can 9 compositors compose a volume (to be printed on the same kind of type), consisting of 36 sheets, 16 pages to a sheet, 50 lines to a page, and 45 letters in a line? Ans. 12 days.

9. If 12 men can build a wall 30 feet long, 6 feet high, and 3 feet thick, in 15 days, when the days are 12 hours long, in what time will 60 men build a wall 300 feet long, 8 feet high, and 6 feet thick, when they work only 8 hours a day?

Ans. 120 days.

10. If 16 horses consume 84 bushels of grain in 24 days, how many bushels will suffice 32 horses 48 days?

Ans. 336 bushels.

11. If the carriage of 5cwt. 3qr. 150 miles cost $ 24.58, what must be paid for the carriage of 7cwt. 3qr. 64 miles, at the same rate? Ans. $ 14.135+.

12. If $7\frac{1}{4}$oz. of bread be bought for $4\frac{3}{4}$d. when corn is 4s. 2d. per bushel, what weight of it may be bought for 1s. 2d. when the price per bushel is 5s. 6d.?

13. If 496 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardness, 465 feet long, $3\frac{2}{3}$ wide, $2\frac{1}{3}$ deep, in how many days of 9 hours long will 24 men dig a trench of 4 degrees of hardness, $337\frac{1}{2}$ feet long, $5\frac{2}{3}$ wide, and $3\frac{1}{2}$ deep?

Ans. 132 days.

CONJOINED PROPORTION.

341. A Conjoined Proportion is a proportion which has each antecedent of a compound ratio equal in value to its consequent.

The first term of each pair of equivalent terms is an *antecedent*, and the term following a *consequent*.

NOTE. — This kind of proportion is sometimes called the Chain Rule.

Ex. 1. If 10 barrels of apples will pay for 5 cords of wood,

and 20 cords of wood for 4 tons of hay, how many barrels of apples will it take to purchase 50 tons of hay? Ans. 500.

OPERATION.

Antecedents.		Consequents.
10 barrels	=	5 cords.
20 cords	=	4 tons.
50 tons	=	— barrels.

$$\frac{10 \times \cancel{20} \times 50}{\cancel{5} \times \cancel{4}} = 500 \text{ barrels, Ans.}$$

We arrange the antecedents at the left of the sign of equality, and the consequents at the right of the same; and, as the answer is to be of the same kind as the first term, we place the *odd term* in the column of antecedents. Then, to find the term corresponding in value with the odd term, we divide the continued product of the 10, 20, and 50 of the column of antecedents, by the product of the 5 and 4 of the column of consequents; and obtain 500 barrels, Ans.

BY ANALYSIS. — If 4 tons of hay equal in value 20 cords of wood, 1 ton equals in value 5 cords of wood. If 5 cords of wood equal 10 barrels of apples, then 1 ton of hay equals 10 barrels of apples, and 50 tons equal 50 times the value of 10 barrels of apples, or 500 barrels, answer as before.

RULE. — *Place the antecedents in one column and the consequents in another, on the right, with the sign of equality between them. Divide the continued product of the terms in the column containing the odd term by the continued product of the other column, and the quotient will be the answer.*

NOTE 1. — The odd term will always belong to that column which does not contain the other term of the same kind.

NOTE 2. — Conjoined proportion is nothing else than simple or compound proportion, exhibited in a different, and in many cases a more convenient form; and, to show the correctness of the rule, the following examples may also be performed by either simple or compound proportion.

EXAMPLES.

2. If 100 acres in Bradford be worth 120 in Haverhill, and 50 in Haverhill be worth 65 in Methuen, how many acres in Bradford are equal to 150 in Methuen? Ans. $96\frac{2}{13}$ acres.

3. If 10lb. of cheese are equal in value to 7lb. of butter, and 11lb. of butter to 2 bushels of corn, and 11 bushels of corn to 8 bushels of rye, and 4 bushels of rye to one cord of wood, how many pounds of cheese are equal in value to 10 cords of wood? Ans. $432\frac{1}{7}$lb.

4. If 12 men can do as much work as 25 women, and 5

women do as much as 6 boys, how many men would it take to do the work of 75 boys? Ans. 30 men.

5. If 6 gallons liquid measure equal 5 English imperial gallons, and if 10 English imperial gallons equal 6 velts of France, and if 26 velts of France equal 16 Russian vedros of wine, how many vedros of wine equal 63 gallons liquid measure?

6. If 7 bushels of corn in Boston be worth 8 bushels in Buffalo, and 10 bushels in Buffalo be worth 14 bushels in Chicago, and 21 bushels in Chicago be worth 25 bushels at Davenport, how many bushels in Boston are worth 1200 bushels at Davenport? Ans. 630 bushels.

7. If 24 shillings in Massachusetts are equal to 32 shillings in New York, and if 48 shillings in New York are equal to 45 shillings in Pennsylvania, and if 15 shillings in Pennsylvania are equal to 10 shillings in Canada, how many shillings in Cannada are equal to 100 shillings in Massachusetts?

Ans. $83\frac{1}{3}$ shillings.

MISCELLANEOUS EXAMPLES IN PROPORTION.

1. A sets out on a journey, and travels 27 miles a day; 7 days after, B sets out and travels the same road 36 miles a day; in how many days will B overtake A? Ans. 21 days.

2. If I sell coffee at 2s. 3d. per pound and gain 35 per cent, what did I give per pound?

3. A detachment of 2000 soldiers were supplied with bread sufficient to last them 12 weeks, allowing each man 14 ounces a day; but on examination they find 105 barrels, containing 200lb. each, wholly spoiled; how much a day may each man eat, that the remainder may supply them 12 weeks?

Ans. 12oz.

4. In consequence of having a seventh part of their bread spoiled, 2000 soldiers were put on an allowance of 12 ounces of bread per day for 12 weeks; what was the whole weight of their bread (good and bad), and how much was spoiled?

Ans. The whole weight, 147000lb.; spoiled, 21000lb.

5. Two thousand soldiers, having lost 105 barrels of bread, weighing 200lb. each, were obliged to subsist on 12 ounces a

day for 12 weeks; but had none been lost, they might have had 14 ounces a day for the same time. What was the whole weight, including what was lost, and how much had they left to subsist on?

Ans. The whole weight, 147000lb.; left to subsist on, 126000lb.

6. Bought threescore pieces of Hollands for three times as many dollars, and sold them again for four times as many dollars; but if they had cost me as much as I sold them for, for what should I have sold them to gain at the same rate?

Ans. \$320.

7. Bought 20 pounds of tea at the rate of $1\frac{1}{2}$ pounds for a dollar, and 62 pounds more at the rate of 15 pounds for \$12; and sold the whole at the rate of 3 pounds for \$4. How much did I gain, or lose?

8. Each of the four sides of a certain field appeared to be 2 furlongs 30 rods and 3 yards in length when measured by a line supposed to be 4 rods long; but the line was found to have been only 64 feet in length. Required the true distance round the field. Ans. $7074\frac{10}{11}$ feet.

9. If 5 oxen or 7 cows eat $3\frac{4}{11}$ tons of hay in 87 days, in what time will 2 oxen and 3 cows eat the same quantity of hay? Ans. 105 days.

10. If 360 men be placed in a garrison, and have provisions for 6 months, how many men must be sent away at the end of 4 months that the remaining provisions may last the garrison 8 months longer? Ans. 270 men.

11. My tailor informs me it will take $10\frac{1}{4}$ square yards of cloth to make me a full suit of clothes. The cloth I am about to purchase is $1\frac{7}{8}$ yards wide, and on sponging it will shrink $\frac{1}{20}$ in width and length. How many yards of the above cloth must I purchase for my "new suit"?

Ans. $6\frac{62}{1083}$yd.

12. If the price of a farm of 130A. 2R. 20p. be \$6537.50, what will be the price of another, containing 100A. 0R. 30p., if 4 acres of the latter be worth 5 of the former?

Ans. \$6267.71+.

13. How much each of copper, tin, zinc, and lead will be required to make a bell weighing 192 tons 17cwt. 16lb., the

weight of that at Moscow, bell-metal being composed of 800 parts copper, 101 tin, 56 zinc, and 43 lead?

Ans. Copper, $308572\frac{4}{5}$lb.; tin, $38957\frac{78}{250}$lb.; zinc, $21600\frac{12}{125}$lb.; lead, $16585\frac{127}{250}$lb.

14. The relative heating power per cord of white-oak to white-pine being as 81 to 42, and that of white-pine to pitch-pine as 7 to 8; how many cords composed of white-pine and pitch-pine in equal parts will be required to produce as much heat as 20 cords of white-oak? Ans. 36 cords or 18 of each.

15. If a cask of wine, containing 85 gallons, cost \$116.95, what would be the value of a hogshead containing 63 gallons, and composed of four parts of the same wine and one part of water? Ans. \$69.34$\frac{38}{85}$.

16. If 8 horses in 30 days consume $3\frac{1}{2}$ tons of hay, how long will $4\frac{9}{10}$ tons last 10 horses, 15 cows, and 7 sheep, each cow consuming $\frac{3}{4}$ as much as a horse, and each sheep $\frac{1}{3}$ as much as a cow?

17. The relative nutritious matter per pound of potatoes to beets being as 25 to 14, that of beets to carrots as 7 to 5, and that of carrots to turnips as 5 to 2; how many pounds of turnips are equal in nutritious matter to 2000 pounds of potatoes? and allowing 60 pounds to a bushel, how much cheaper or dearer relatively per bushel are potatoes at 80 cents, and turnips at 20 cents, than either beets at 35 cents or carrots at 25 cents.

Ans. 12500 pounds; potatoes $17\frac{1}{2}$cts. dearer; turnips 10cts. dearer.

18. A and B set out from the same place, and in the same direction. A travels uniformly 18 miles per day, and after 9 days turns and goes back as far as B has travelled during those 9 days; he then turns again, and, pursuing his journey, overtakes B $22\frac{1}{2}$ days after the time they first set out. It is required to find the rate at which B uniformly travelled.

Ans. 10 miles per day.

19. Two men in Boston hire a carriage for \$25, to go to Concord, N. H., and back, the distance being 72 miles, with the privilege of taking in three more persons. Having gone 20 miles, they take in A; at Concord, they take in B; and when within 30 miles of Boston, they take in C. How much shall each man pay?

Ans. First man, \$7.609$\frac{103}{108}$; second man, \$7.609$\frac{103}{108}$; A \$5.873$\frac{81}{108}$; B, \$2.864$\frac{7}{12}$; C, \$1.041$\frac{8}{12}$.

PERCENTAGE.

342. PERCENTAGE is an allowance for every hundred, consisting of a part of that on which it is reckoned.

Per cent., a contraction of *per centum*, signifies by the hundred. By 1 per cent. is to be understood 1 out of a hundred, or 1 *hundredth;* by 2 per cent. is to be understood 2 out of a hundred; or 2 *hundredths*, &c.

343. The *rate* per cent. is the number denoting the part allowed for every hundred, as 5 per cent., 6 per cent., &c.

344. The *basis of percentage* is the number on which percentage is reckoned.

345. The *amount* is the basis of percentage increased by the percentage; and the *remainder* is the basis decreased by the percentage.

346. The rates per cent. may be written decimally as in the following

TABLE.

1 per cent.	=	.01	9 per cent.	=	.09
2 per cent.	"	.02	10 per cent.	"	.10
3 per cent.	"	.03	15 per cent.	"	.15
4 per cent.	"	.04	20 per cent.	"	.20
5 per cent.	"	.05	25 per cent.	"	.25
6 per cent.	"	.06	63 per cent.	"	.63
7 per cent.	"	.07	100 per cent.	"	1.00
8 per cent.	"	.08	116 per cent.	"	1.16

AND

$\frac{1}{2}$ per cent. may be written .005, or $.00\frac{1}{2}$, and be read $\frac{5}{10}$ of 1 per cent., or $\frac{1}{2}$ of 1 per cent.

$\frac{1}{4}$ per cent. may be written .0025, or $.00\frac{1}{4}$, and be read $\frac{25}{100}$ of 1 per cent., or $\frac{1}{4}$ of 1 per cent.

$\frac{1}{8}$ per cent. may be written .00125, or $.00\frac{1}{8}$, and be read $\frac{125}{1000}$ of 1 per cent., or $\frac{1}{8}$ of 1 per cent.

$7\frac{1}{2}$ per cent. may be written .075, or $.07\frac{1}{2}$, and be read $7\frac{5}{10}$ per cent., or $7\frac{1}{2}$ per cent.

NOTE. — If a fraction of 1 per cent. cannot be exactly expressed in decimal figures, it may be written as a part of a mixed decimal. Thus, $4\frac{1}{3}$ per cent. may be written $.04\frac{1}{3}$; $6\frac{1}{7}$ per cent. may be written $.06\frac{1}{7}$; and $11\frac{2}{3}$ per cent. may be written $.11\frac{2}{3}$.

Exercises.

1. Express decimally 19 per cent. Ans. .19.
2. Express decimally 27 per cent.
3. Express decimally $13\frac{1}{2}$ per cent.
4. Express decimally $1\frac{3}{4}$ per cent.
5. Express decimally $7\frac{2}{5}$ per cent.
6. Express decimally $77\frac{1}{4}$ per cent.
7. Express decimally 106 per cent.
8. Express decimally 107 per cent.
9. Express decimally 305 per cent.
10. Express decimally $999\frac{12}{15}$ per cent.

347. To find the percentage any given rate per cent. is of any number or quantity.

Ex. 1. Bought $\frac{1}{2}$ of a ship for \$15650, and sold the same at a gain of 12 per cent. How much did I make by the transaction? Ans. \$1878.

OPERATION.

Basis of percentage,	\$1 5 6 5 0
Rate per cent.	.1 2
Percentage,	\$1 8 7 8.0 0 Ans.

Since 12 per cent. equals .12 of the original cost, we multiply \$15650 by the decimal expression .12.

RULE. — *Multiply the given number by the rate per cent. expressed decimally, and the product will be the percentage.* Or,

As 100 *per cent. is to the given rate per cent., so is the given basis of percentage to the percentage required.*

Examples.

2. What is 15 per cent. of 500 bushels? Ans. 75bu.
3. What is 20 per cent. of 75cwt.? Ans. 15cwt.
4. What is 30 per cent. of 150 tons? Ans. 45 tons.
5. What is 75 per cent. of \$500?
6. What is 95 per cent. of 700 chaldrons?
7. What is 2 per cent. of 40 miles? Ans. .8 mile.
8. What is 99 per cent. of \$1000? Ans. \$990.
9. What is $33\frac{1}{3}$ per cent. of 144 barrels? Ans. 48bbl.
10. What is $66\frac{2}{3}$ per cent. of 90 hogsheads?
11. What is $\frac{1}{4}$ per cent. of \$100? Ans. \$0.25.
12. What is $\frac{7}{8}$ per cent. of 1728lb.? Ans. 15.12lb.
13. A certain colonel, whose regiment consisted of 900 men,

lost 8 per cent. of them in battle, and 50 per cent. of the remainder by sickness. How many had he remaining?

Ans. 414 men.

14. A merchant, having $ 1728 in the Union Bank, wishes to withdraw 15 per cent.; how much will remain?

15. A gentleman, who had an estate of $ 25,000, in his will gave to his wife 40 per cent. of his property, and to his son Samuel 30 per cent. of the remainder. The residue he divided equally among his daughters, Marcia, Isabella, and Clara, after having deducted $ 60 as a present to his clergyman. What did each receive?

Ans. Wife, $ 10,000; son, $ 4,500; daughters, $ 3,480 each.

348. To find what rate per cent. one given number is of another.

Ex. 1. What per cent. of 50 is 12? Ans. 24 per cent.

OPERATION.

$$\tfrac{12}{50} = \tfrac{6}{25} = .24, \text{ Ans.}$$

$$\text{Or, } \frac{12 \times \cancel{1}\cancel{0}\overset{2}{\cancel{0}}}{\cancel{5}\cancel{0}} = 24 \text{ per cent.}$$

Since the percentage equals the product of the basis of percentage by the number denoting the rate per cent. (Art. 343), the quotient arising from dividing the percentage by the number denoting the basis must equal the rate per cent. We therefore divide 12 by the 50, and obtain $\frac{12}{50} = \frac{6}{25}$, which, expressed decimally, equals .24, or 24 per cent. Since the question, evidently, is the same as to find $\frac{12}{50}$ of 100 per cent., we multiply the 12 by 100, or annex two ciphers and divide by 50, and obtain the same result as before.

RULE. — *Annex two ciphers to the number denoting the percentage, and divide by the number on which the percentage is reckoned; and the quotient will be the rate per cent.* Or,

As the given basis of percentage is to the given percentage, so is 100 *per cent. to the rate per cent. required.*

EXAMPLES.

2. What per cent. of 16 is 2? Ans. $12\frac{1}{2}$ per cent.
3. What per cent. of 110 is 11? Ans. 10 per cent.
4. What per cent. of $2\frac{1}{4}$ is $\frac{1}{8}$?
5. $\frac{1}{3}$ of 18 per cent. is what per cent. of 24 per cent.?

Ans. 25 per cent.

6. What per cent. of $ 150 is 25 per cent. of $ 36?

Ans. 6 per cent.

7. 36 bushels is what per cent. of 48 bushels?

Ans. 75 per cent.

8. What per cent. of 4 years is 1 year 6 months?

9. 31 gallons 2 quarts is what per cent. of 1 hogshead?

Ans. 50 per cent.

10. Of 160 yards of cloth there have been sold 128 yards; what per cent. of the whole remains unsold?

Ans. 20 per cent.

11. What per cent. of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{1}{3}$? Ans. 20 per cent.

12. In a certain school the number of pupils studying geography is 40 per cent. more than the number studying grammar. What per cent. less is the number studying grammar than the number studying geography? Ans. $28\frac{4}{7}$ per cent.

13. If a miller takes out 4 quarts toll from every bushel he grinds, what per cent. does he take for toll?

14. If a certain coin is made of 22 parts copper and 3 parts nickel, what per cent. of it is copper, and what per cent. nickel? Ans. 88 per cent. copper; 12 per cent. nickel.

15. In a certain orchard there are 250 trees, of which 40 per cent. are apple-trees, 12 per cent. cherry-trees, 8 per cent. plum-trees, and the remainder, with the exception of 25 pear-trees, consist of peach-trees. What per cent. of the whole are the peach-trees? Ans. 30 per cent.

349. To find a number when a given number is known to be a certain per cent. of it.

Ex. 1. I have bought two house-lots; for the one I paid \$ 300, which was 60 per cent. of what I paid for the other. What did I pay for the latter? Ans. \$ 500.

OPERATION.

$$\$300 \div 60 = \$5;\ \$5 \times 100 = \$500, \text{ Ans.}$$

$$\text{Or, } \frac{\$\,\overset{5}{\cancel{300}} \times 100}{\cancel{60}} = \$500.$$

Since \$ 300 is 60 per cent. of the unknown sum, 1 per cent. of it must equal $\frac{1}{60}$ of \$ 300, or \$ 5; and 100 per cent., or the whole of it, must equal 100 times \$ 5, or \$ 500, Ans. And since the question is evidently the same as to find the value of $\frac{100}{60}$ of \$ 300, we multiply the \$ 300 by 100, and divide by 60, and obtain the same result as before.

RULE. — *Annex two ciphers to the number denoting the percentage, and divide by that denoting the rate per cent.* Or,

As the given rate per cent. is to 100 per cent., so is the given percentage to the basis of percentage required.

EXAMPLES.

2. 25 is 10 per cent. of what number? Ans. 250.
3. $16\frac{1}{4}$ is 8 per cent. of what number? Ans. $203\frac{1}{8}$.
4. 72 is 12 per cent. of what number?
5. $\frac{7}{8}$ is 40 per cent. of what number? Ans. $2\frac{3}{16}$.
6. $\$1.62\frac{1}{2}$ is $12\frac{1}{2}$ per cent. of how many dollars? Ans. \$13.
7. $\$1\frac{9}{40}$ is $\frac{7}{8}$ per cent. of what sum? Ans. \$140.

8. A flock of sheep has lost $15\frac{1}{3}$ per cent. of its number, 17 sheep having been killed by the dogs, and 6 more having been drowned. What was the original number? Ans. 150.

9. If a man owning 45 per cent. of a mill should sell $33\frac{1}{3}$ per cent. of his share for \$450, what would be the value of the whole mill?

10. $12\frac{1}{2}$ per cent. of the length of a certain railroad is equal to 3m. 1fur. 1rd. What is its entire length?

Ans. 25m. 0fur. 8rd.

11. Gave to a benevolent society 19 bushels of corn, which was $17\frac{1}{7}$ per cent. of all I raised. How many bushels had I left? Ans. $91\frac{5}{6}$ bushels.

12. Dalton says to Turner, \$36.89 is $13\frac{2}{3}$ per cent. of the sum you borrowed of me; and Turner replies, It is just $16\frac{2}{3}$ per cent. of the amount I have repaid you. How much of the money that was borrowed remains unpaid? Ans. \$57.66.

350. To find the number on which the percentage is reckoned, when the amount, or the remainder, and the rate per cent. are given.

Ex. 1. Sold a horse for \$200, which was 25 per cent. more than he cost. What did he cost? Ans. \$160.

OPERATION.

$1 + .25 = 1.25$; $\$200 \div 1.25 = \$160$, Ans.

Since the \$200 is evidently the cost and 25 per cent. of the cost, it must equal the cost taken 1.25 times. Therefore, the given amount, \$200, divided by 1 increased by .25, or the given per cent. expressed decimally, equals \$160, the basis of percentage required.

2. Sold 160 cords of wood, which was 20 per cent. less than the whole number owned. How many cords were owned?

Ans. 200 cords.

OPERATION.

$1 - .20 = .80$; $160 \div .80 = 200$ cords, Ans.

Since the 160 cords are 20 per cent. less than the whole number owned, it must equal the whole number of cords taken .80 times. Therefore the given remainder, 160 cords, divided by 1 decreased by .20, or the given per cent. expressed decimally, equals 200 cords, the basis of percentage required.

RULE. — *Divide the given amount by* 1 *increased by the given per cent., or the given remainder by* 1 *decreased by the given per cent., expressed decimally.* Or,

As 1 *increased by the given per cent., or* 1 *decreased by the given per cent., expressed decimally, is to* 1, *so is the given amount or the given remainder to the basis of percentage required.*

EXAMPLES.

3. 126 is 5 per cent. more than what number? Ans. 120.

4. $328\frac{1}{2}$ is 10 per cent. less than what number?

Ans. 365.

5. \$19.50 is 7 per cent. less than how many dollars?

6. \$34.40 is $\frac{3}{8}$ per cent. more than how many dollars?

Ans. \$$34.27\frac{119}{803}$.

7. A man expends in a week \$24, which exceeds by $33\frac{1}{3}$ per cent. his earnings in the same time. What were the earnings? Ans. \$18.

8. D. Chandler lives distant from the village 2m. 6fur. 24rd., which is $12\frac{1}{2}$ per cent. nearer than is J. Mitchel's residence. At what distance does Mitchel live from the village?

Ans. 3m. 1fur. $33\frac{1}{7}$rd.

9. Bought a carriage for \$123.16, which was 16 per cent. less than I paid for a horse. How much was paid for the horse?

10. A carpet having been cut from a piece of carpeting, there were left 6yd. $1\frac{1}{10}$qr., which was 75 per cent. less than the quantity cut off. How many yards were there in the piece at first? Ans. 31yd. 1qr. 2na.

11. A steamer with a cargo of flour, having been lightened, during a storm, of 10 per cent. of her freight, at the end of the

voyage had but 279 barrels to deliver. How many barrels were taken aboard, and how many were lost?

Ans. Taken 310 barrels; lost 31 barrels.

12. A and B each received the same sum. A spent $86\frac{1}{2}$ per cent. of his money for land, and B lost of it by gambling as much as would equal $27\frac{1}{2}$ per cent. of what both received. They then together had left just \$ $36.85\frac{1}{2}$. What was the sum received by each, and how much had each left?

Ans. Each received \$ 63; A had left \$ $8.50\frac{1}{2}$; and B had left \$ 28.35.

MISCELLANEOUS EXAMPLES.

1. What is 7 per cent. of 1672? Ans. $117\frac{1}{25}$.
2. What is $5\frac{1}{3}$ per cent. of \$ 3266? Ans. $174\frac{14}{75}$.
3. Find 312 per cent. of $\frac{1}{2}$ of $2\frac{1}{4}$.
4. Find $1\frac{1}{2}$ per cent. of 180. Ans. $2\frac{7}{10}$.
5. $9\frac{1}{2}$ is what per cent. of 3? Ans. $316\frac{2}{3}$.
6. $\frac{1}{8}$ is what per cent. of $\frac{5}{16}$? Ans. 40.
7. Find what per cent. 3.50 is of 50.
8. 13 is 16 per cent. of what number? Ans. $81\frac{1}{4}$.
9. \$ 66 is 100 per cent. of what number? Ans. \$ 66.
10. $\frac{1}{3}$ is $6\frac{1}{5}$ per cent. of what number? Ans. $5\frac{35}{93}$.
11. \$ $21.28\frac{1}{2}$ is $3\frac{1}{4}$ per cent. less than what sum? Ans. \$ 22.00.
12. 19lb. 12oz. is $16\frac{3}{4}$ per cent. of how many pounds? Ans. $117\frac{61}{67}$ pounds.

13. A grocer bought 6 boxes of eggs, each containing 30 dozen, and found that 15 per cent. of the whole were bad; how many eggs did he lose? Ans. 324 eggs.

14. There is paid for sawing a cord of wood \$ 0.69, which is 12 per cent. of the cost of the wood. What did the wood cost?

15. Three men agreed to excavate 40500 cubic feet of earth; by the first week's labor they excavate 200 cubic yards; by the second, 6000 cubic feet; and by the third, 25 per cent. of what remained at the end of the second week. They then called the work half done; but how many cubic feet did the job lack of being half completed? Ans. 1575 cubic feet.

16. 25 per cent. of $\frac{1}{2}$ of a ship is how many per cent. of $\frac{3}{4}$ of it? Ans. $16\frac{2}{3}$ per cent.

17. If molasses cost 20 per cent. less than \$ 0.50 per gallon, and it be sold at 25 per cent. more per gallon than it cost, at what price is it sold? Ans. \$ 0.50 per gal.

18. I have 20 yards of yard-wide cloth, which will shrink on sponging 4 per cent. in the length, and 5 per cent. in the width; how much less than 20 square yards will there be of it after sponging? Ans. $1\frac{19}{25}$ yards.

19. A gentleman having a large farm gave 15 per cent. of it to his oldest daughter, 10 per cent. of what remained and $\frac{1}{10}$ of an acre he gave to his oldest son, and 25 per cent. of the remainder he gave to his wife. The residue he divided equally among his other 5 children, who received each 39 acres. How many acres did his farm contain? Ans. 340 acres.

20. If the population of the United States in 1858 be 30,500,000, what will it be in 1868, allowing the increase should be at the rate of $34\frac{1}{2}$ per cent.?

21. In a certain battle in which the English, French, and Turks were allied against the Russians, there were $33\frac{1}{3}$ per cent. more French than English, and the Turks were $8\frac{1}{3}$ per cent. more than the French and 1600 more than the English. Required the whole number of the allies, and the per cent. the English, the French and the Turks each were of that number.

Ans. Whole number 13600; English, $26\frac{8}{17}$ per cent.; French, $35\frac{5}{17}$ per cent.; and Turks, $38\frac{4}{17}$ per cent.

22. Bought a cargo of flour, consisting of 560 barrels, at \$ 7.25 per barrel, less 10 per cent., and sold the same at 10 per cent. more than \$ 7.25 per barrel. At what per cent. above the cost was the flour sold? How much was made by the operation?

23. The population of a certain city, whose gain of inhabitants in 5 years has been 25 per cent., is 87500; what was it 5 years ago? Ans. 70000.

24. Bought a horse, buggy, and harness for \$ 500. The horse cost $37\frac{1}{2}$ per cent. less than the buggy, and the harness cost 70 per cent. less than the horse. What was the price of each?

Ans. Buggy \$ $275\frac{25}{29}$; horse, \$ $172\frac{12}{29}$; and harness, \$ $51\frac{21}{29}$.

INTEREST.

351. INTEREST is an allowance made for the use of money, or for value received; and it is generally reckoned as a certain rate per cent. for any given time, but usually for one year.

The *principal* is the sum lent, on which interest is computed.

The *amount* is the interest and principal added together.

Simple interest is that reckoned on the principal only; and is that meant when the term *interest* is used alone.

Legal interest is the rate per cent. established by law.

Usury is a higher rate per cent. than is allowed by law.

The legal rate per cent. varies in the different States and in different countries.

In Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, Tennessee, Kentucky, Ohio, Indiana, Illinois, Iowa, Nebraska, Missouri, Kansas, Arkansas, Mississippi, Florida, District of Columbia, and on debts or judgments in favor o' the United States, it is 6 per cent.

In New York, Mich *i* an, Wisconsin, Minnesota, Georgia, and South Carolina, it is 7 per cent.

In Alabama and 'exas, it is 8 per cent.

In California, it 10 per cent.

In Louisiana, it is 5 per cent.

In Canada, N a Scotia, and Ireland, it is 6 per cent.

In England nd France, it is 5 per cent.

NOTE. — The 'egal rate, as above, in some of the States, is only that which the law allows, when no particular rate is mentioned. By special agreement between parties, in Ohio, Indiana, Michigan, Illinois, Iowa, Nebraska, Missouri, Kansas, Arkansas, Louisiana, and Mississippi, interest can be taken as high as 10 per cent.; in Florida, as high as 8 per cent.; in Texas and Wisconsin, as high as 12 per cent.; and in California and Minnesota, any per cent. In New Jersey, by a special law, 7 per cent. may be taken in the city of Paterson, and in the cour ties of Essex and Hudson. In Vermont 7 per cent. may be taken on railroa l bonds. Banks in Illinois cannot take above 7 per cent., and in Ohio, not above 6 per cent. In Mississippi, above 6 per cent. can be taken only for oney lent.

352 When the rate of interest is 6 per cent. per annum, t erest for 1 year or 12 months will be 6 cents on every

100 cents on which it is reckoned, or $\frac{6}{100}$ of the principal. Hence, for 2 months, or $\frac{1}{6}$ of a year, it will be 1 cent on every 100 cents, or $\frac{1}{100}$ of the principal; and for 1 month, or $\frac{1}{2}$ of 2 months, it will be 5 mills, or $\frac{1}{200}$ of the principal. Now, since the interest on 100 cents for 1 month, or 30 days, is 5 mills, for 6 days, or $\frac{1}{5}$ of 30 days, it will be 1 mill, or $\frac{1}{1000}$ of the principal; and since the interest on 100 cents for 2 months is 1 cent, or $\frac{1}{100}$ of the principal, for 100 times 2 months, or 200 months, or 16 years 8 months, it will be 100 cents, or equal the whole principal; and in the same ratio for any other length of time.

The interest of \$1, at 6 per cent., and the ratio of the interest to the principal, for 200 months, and other convenient parts of time, is shown in the following

TABLE.

Interest of \$1.

For	200 mo.	= 16yr. 8mo.	is	\$1.00	equal	the whole		principal.
"	100 mo.	= 8yr. 4mo.	"	0.50	"	$\frac{1}{2}$	of the	principal.
"	$66\frac{2}{3}$mo.	= 5yr. $6\frac{2}{3}$mo.	"	$0.333\frac{1}{3}$	"	$\frac{1}{3}$	"	"
"	50 mo.	= 4yr. 2mo.	"	0.25	"	$\frac{1}{4}$	"	"
"	40 mo.	= 3yr. 4mo.	"	0.20	"	$\frac{1}{5}$	"	"
"	$33\frac{1}{3}$mo.	= 2yr. $9\frac{1}{3}$mo.	"	$0.166\frac{2}{3}$	"	$\frac{1}{6}$	"	"
"	25 mo.	= 2yr. 1mo.	"	0.125	"	$\frac{1}{8}$	"	"
"	20 mo.	= 1yr. 8mo.	"	0.10	"	$\frac{1}{10}$	"	"
"	$16\frac{2}{3}$mo.	= 1yr. $4\frac{2}{3}$mo.	"	$0.083\frac{1}{3}$	"	$\frac{1}{12}$	"	"
"	12 mo.	= 1yr. 0mo.	"	0.06	"	$\frac{6}{100}$	"	"
"	10 mo.	= $\frac{5}{6}$ of a yr.	"	0.05	"	$\frac{1}{20}$	"	"
"	8 mo.	= $\frac{2}{3}$ of a yr.	"	0.04	"	$\frac{1}{25}$	"	"
"	$6\frac{2}{3}$mo.	= $\frac{5}{9}$ of a yr.	"	$0.033\frac{1}{3}$	"	$\frac{1}{30}$	"	"
"	6 mo.	= $\frac{1}{2}$ of a yr.	"	0.03	"	$\frac{3}{100}$	"	"
"	5 mo.	= $\frac{5}{12}$ of a yr.	"	0.025	"	$\frac{1}{40}$	"	"
"	4 mo.	= $\frac{1}{3}$ of a yr.	"	0.02	"	$\frac{1}{50}$	"	"
"	2 mo.	= $\frac{1}{6}$ of a yr.	"	0.01	"	$\frac{1}{100}$	"	"
"	1 mo.	= $\frac{1}{12}$ of a yr.	"	0.005	"	$\frac{1}{200}$	"	"
"	6 d.	= $\frac{1}{5}$ of a mo.	"	0.001	"	$\frac{1}{1000}$	"	"
"	5 d.	= $\frac{1}{6}$ of a mo.	"	$0.000\frac{5}{6}$	"	$\frac{5}{6000}$	"	"
"	4 d.	= $\frac{2}{15}$ of a mo.	"	$0.000\frac{2}{3}$	"	$\frac{4}{6000}$	"	"
"	3 d.	= $\frac{1}{10}$ of a mo.	"	$0.000\frac{1}{2}$	"	$\frac{3}{6000}$	"	"
"	2 d.	= $\frac{1}{15}$ of a mo.	"	$0.000\frac{1}{3}$	"	$\frac{2}{6000}$	"	"
"	1 d.	= $\frac{1}{30}$ of a mo.	"	$0.000\frac{1}{6}$	"	$\frac{1}{6000}$	"	"

353. The principal, the rate per cent., the time, and the interest have such a relation to each other that, any three of these terms being given, the fourth can be readily found. The computations in interest, therefore, admit of the following problems among others: — I. To find the interest; II. To find the principal; III. To find the rate per cent.; IV. To find the time.

354. To find the interest of any sum for any time at 6 per cent.

Ex. 1. What is the interest of \$ 2640 for 2 years 7 months and 26 days, at 6 per cent.? Ans. \$ 420.64.

FIRST OPERATION.

Interest of \$ 1 for 2 years	=	\$ 0.1 2
Interest of \$ 1 for 7 months	=	0.0 3 5
Interest of \$ 1 for 26 days	=	0.0 0 4 $\frac{1}{3}$
Int. of \$ 1 for 2yr. 7mo. 26d.	=	\$ 0.1 5 9 $\frac{1}{3}$

$$\begin{array}{r} \text{Principal, } \$\,2\,6\,4\,0 \\ .1\,5\,9\,\tfrac{1}{3} \\ \hline 2\,3\,7\,6\,0 \\ 1\,3\,2\,0\,0 \\ 2\,6\,4\,0 \\ 8\,8\,0 \\ \hline \text{Interest, } \$\,4\,2\,0.6\,4\,0 \end{array}$$ Ans.

The interest of \$ 2640 for 2yr. 7mo. 26d. will be 2640 times as much as the interest of \$ 1 for the given time. The interest of \$ 1 for 2 years will be twice as much as for 1 year, equal 12cts.; and since the interest for 2 months is 1 cent, for 7 months it will be 3$\frac{1}{2}$ cents, or 3 cents 5 mills. And as the interest for 6 days is 1 mill, for 26 days it will be 4$\frac{1}{3}$ mills. These several sums added together give the interest of \$ 1 at 6 per cent. for the given time, equal \$ 0.159$\frac{1}{3}$, which taken 2640 times, by multiplying the given principal by it, gives \$ 420.64, the interest required.

SECOND OPERATION.

Principal,	\$ 2 6 4 0	
$\frac{1}{8}$ of the prin. =	3 3 0	Interest for 2yr. 1mo.
$\frac{1}{30}$ of the prin. =	8 8	Interest for 6mo. 20d.
$\frac{1}{1000}$ of the prin. =	2.6 4	Interest for 6d.
Ans.	\$ 4 2 0.6 4	Interest for 2yr. 7mo. 26d.

The time, 2yr. 7mo. 26d., is equal to 2yr. 1mo. + 6mo. 20d. + 6d. Now, since the interest on any sum, at 6 per cent., in 200 months equals the principal, for 2yr. 1mo., or $\frac{1}{8}$ of 200 months, it will equal $\frac{1}{8}$ of the principal. We therefore take $\frac{1}{8}$ of the principal, \$ 2640, equal \$ 330, as the interest for 2yr. 1mo. Of the balance of

time, 6mo. 20d., or $6\frac{2}{3}$mo., being $\frac{1}{30}$ of 200 months, we take $\frac{1}{30}$ of the principal, equal $ 88, as the interest for the 6mo. 20d.; and the 6d. being $\frac{1}{1000}$ of 200 months, we take $\frac{1}{1000}$ of the principal, equal $ 2.64, as the interest for the 6d. We add together the interest for the parts of the whole time, and obtain, as by the first operation, $ 420.64 as the whole interest.

RULE 1. — *Find the interest of $ 1 for the given time, by reckoning 6 cents for every* YEAR, *1 cent for every* TWO MONTHS, *and 1 mill for every 6* DAYS; *then multiply the given principal by the number denoting that interest, and the product will be the interest required.* Or,

RULE 2. — *Take such fractional part or parts of the principal as the number expressing the time is of 200 months.*

NOTE 1. — To find the amount, add the principal to the interest.

NOTE 2. — In computing interest for a fractional part of a month, the month is considered as consisting of 30 days. This has the sanction of general usage and the decisions of the courts, though not entirely accurate.

NOTE 3. — Questions in interest, like other exercises in percentage, may be solved by proportion. The foregoing example admits of a statement and solution by the rule of compound proportion (Art. 340).

NOTE 4. — It is customary among merchants to reject the mills in the results of their computations of interest, increasing, however, the number of cents by 1 when the decimal of a cent exceeds 5.

EXAMPLES.

2. What is the interest of $ 675 for 1 year? Ans. $ 40.50.
3. What is the interest of $ 3967.87 for 2 years? Ans. $ 476.144.
4. What is the interest of $ 896.28 for 3 years? Ans $ 161.33.
5. What is the amount of $ 716.57 for 4 years?
6. What is the amount of $ 76.47 for 7 years? Ans. $ 108.587.
7. What is the interest of $ 123.45 for 6 years? Ans. $ 44.442.
8. What is the interest of $ 750 for 12 years? Ans. $ 540.
9. What is the interest of $ 130 for 2 months?
10. What is the interest of $ 85 for 3 months? Ans. $ 1.275.
11. What is the interest of $ 19.62 for 7 months? Ans. $ 0.6867.
12. What is the interest of $ 637 for 10 months?

13. What is the interest of $ 1671.32 for 14 months?
Ans. $ 116.99.

14. What is the interest of $ 891.24 for 9 months?
Ans. $ 40.10.

15. What is the interest of $ 91 for 5 days? Ans. $ 0.0758.

16. What is the interest of $ 324.66 for 18 days?

17. What is the interest of $ 3246 for 27 days?
Ans. $ 14.607.

18. What is the interest $ 1364.24 for 1 day?

19. What is the interest of $ 6444 for 29 days?

20. What is the amount of $ 18.60 for 24 days?

21. What is the interest of $ 386.19 for 100 months?
Ans. $ 193.09.

22. What is the interest of $ 0.75 for 75 years?
Ans. $ 3.37½.

23. What is the interest of $ 396.15 for 1 year 1 month and 9 days? Ans. $ 26.343.

24. What is the interest of $ 36.18 for 3 months and 7 days? Ans. $ 0.584.

25. What is the interest of $ 97.15 for 2 years 11 months and 27 days?

26. What is the interest of $ 76.89½ from January 11, 1852, to July 27, 1863? Ans. $ 53.262.

27. What is the interest of $ 98.25 from July 4, 1856, to October 19, 1859? Ans. $ 19.404.

28. What is the interest of $ 22.763 from February 19, 1836, to July 18, 1860?

29. What is the interest of $ 175.07 from January 7, 1855, to October 12, 1859? Ans. $ 50.04.

30. What is the interest of $ 197.28½ from December 6, 1852, to January 11, 1854? Ans. $ 12.987.

31. What is the amount of $ 4377.15 for 3 years?

32. What is the interest of $ 444.60 for 5 years and 6 months? Ans. $ 146.718.

355. To find the interest of any sum of money at any rate per cent. for any given time.

Ex. 1. What is the interest of $ 84.50 at 7 per cent. for 2 years 5 months and 12 days? Ans. $ 14.49.

FIRST OPERATION.

Principal,	$8 4.5 0
Rate per cent.	.0 7
Interest for 1 year,	5.9 1 5 0
	2
Int. for 2 years,	1 1.8 3 0 0
Int. for 4mo., or $\frac{1}{3}$ of 1yr.	1.9 7 1 6 +
Int. for 1mo., or $\frac{1}{4}$ of 4mo.	.4 9 2 9 +
Int. for 10d., or $\frac{1}{3}$ of 1mo.	.1 6 4 3 +
Int. for 2d., or $\frac{1}{5}$ of 10d.	.0 3 2 8 +
Int. for 2yr. 5mo. 12d.	$1 4.4 9 1 6 + Ans.

Having found the interest for 1 year, and then for 2 years, the interest for 5 months is obtained by first taking $\frac{1}{3}$ of 1 year's interest, for 4 months, and then $\frac{1}{4}$ of this last interest for 1 month. And since 10 days are $\frac{1}{3}$ of 1 month, we take $\frac{1}{3}$ of 1 month's interest for the interest of 10 days; and since 2 days are $\frac{1}{5}$ of 10 days, we take $\frac{1}{5}$ of the last interest for 2 days. The interest as found for the several parts of the whole time, added together, gives the interest required.

SECOND OPERATION.

Principal,	$8 4.5 0
Int. of $1 at 6 per cent.	.1 4 7
	5 9 1 5 0
	3 3 8 0 0
	8 4 5 0
Int. at 6 per cent.	1 2.4 2 1 5 0
$\frac{1}{6}$ of int. at 6 per cent.	2.0 7 0 2 5
Int. at 7 per cent.	$1 4.4 9 1 7 5 Ans.

We first find the interest on the given sum at 6 per cent., and then add to this interest the fractional part of itself, denoted by the excess of the rate above 6 per cent. This excess is 1 per cent.; therefore we add $\frac{1}{6}$ of the interest at 6 per cent. to that interest for the answer. If the rate per cent. had been less than 6 per cent, we should have subtracted the fractional part.

RULE 1.—*First find the interest for one year by multiplying the principal by the rate per cent. expressed decimally; and for two or more years multiply this product by the number of years.*

Find the interest for months by taking the most convenient fractional part or parts of ONE *year's interest.*

Find the interest for days by taking the most convenient fractional part or parts of ONE *month's interest.* Or,

RULE 2.—*Find the interest of the given sum at* 6 *per cent., and then add to this interest, or subtract from it, such a fractional part of itself as the given rate is greater or less than* 6 *per cent.*

NOTE 1.—$\frac{1}{6}$ of the interest at 6 per cent. may be taken for that at 1 per cent.; $\frac{1}{4}$, for that at $1\frac{1}{2}$ per cent.; $\frac{1}{3}$, for that at 2 per cent.; $\frac{1}{2}$, for that at 3 per cent. From the interest at 6 per cent. may be taken $\frac{1}{3}$ of itself for that at 4 per cent.; $\frac{1}{4}$ of itself for that at $4\frac{1}{2}$ per cent.; and $\frac{1}{6}$ of itself for that at 5 per cent. To the interest at 6 per cent. may be added $\frac{1}{6}$ of itself for that at 7 per

cent.; $\frac{1}{4}$ of itself for that at $7\frac{1}{2}$ per cent.; $\frac{1}{3}$ of itself for that at 8 per cent; and $\frac{1}{2}$ of itself for that at 9 per cent. If the rate is 12 per cent., the interest at 6 per cent. may be taken twice; if 18 per cent., the interest at 6 per cent. may be taken three times, etc.

NOTE 2. — When in this book the rate of interest is not given, 6 per cent. is to be understood.

EXAMPLES.

2. What is the interest of $16.75 for 7 months and 17 days, at 7 per cent.? Ans. $0.739.

3. What is the interest of 11.10\frac{1}{2}$ from April 17, 1852, to December 7, 1852, at 7 per cent.? Ans. $0.496.

4. What is the interest of $12.69, from January 2, 1853, to August 30, 1854, at 7 per cent.? Ans. 1.47\frac{1}{2}$.

5. What is the interest of $5000 for 2 years 5 months 26 days, at 7 per cent.?

6. What is the amount of $416 for 3 years 16 days, at 7 per cent.? Ans. $504.64.

7. What is the interest of $336 for 15 days, at 5 per cent.? Ans. $0.70.

8. What is the interest of $17869.75 from February 7, 1852, to January 11, 1860, at 5 per cent.? Ans. $7083.3703.

9. What is the interest of $300.50 for 1 year 2 months and 15 days? Ans. $21.786.

10. What is the interest of $37 for 29 days, at 5 per cent.?

11. What is the interest of $35.61 from November 11, 1861, to December 15, 1863? Ans. $4.474.

12. What is the interest of $16.76 from December 17, 1841, to January 17, 1852? Ans. $10.139+

13. What is the interest of $1728.19 from May 7, 1854, to July 17, 1860, at $\frac{1}{4}$ per cent.? Ans. $26.762.

14. What is the interest of $397.16 for 1 year 6 months and 1 day, at $5\frac{1}{2}$ per cent.? Ans. $32.826.

15. What is the amount of $100.25 for 2 months and 29 days, at 4 per cent.? Ans. $101.241.

16. What is the interest of $51.17 for 9 months and 29 days, at 4 per cent.? Ans. $1.699.

17. What is the interest of $42.20 for 1 year and 16 days, at $4\frac{1}{2}$ per cent.?

18. What is the interest of $ 16.25 for 2 years, at 3 per cent.? Ans. $ 0.975.

19. What is the interest of $ 96.84 from November 27, 1849, to July 3, 1852, at 7½ per cent.? Ans. $ 18.883.

20. What is the interest of $ 786.97 from October 19, 1857, to August 17, 1861, at 7½ per cent.?

21. What is the interest of $ 71.091 from July 29, 1853, to June 19, 1857, at 12 per cent.? Ans. $ 33.175.

22. What is the amount of $ 369.29 for 2 years 3 months and 1 day, at 9 per cent.? Ans. $ 444.163.

23. What is the interest of $ 76.35 for 1 year 8 months and 18 days? Ans. $ 7.864.

24. What is the interest of $ 47.15 for 1 month and 19 days, at 13½ per cent.? Ans. $ 0.886.

25. What is the interest of $ 36.72 from May 16, 1829, to February 18, 1857, at 7 per cent.? Ans. $ 71.342.

26. What is the interest of $ 35.50 for 3 years 5 months and 20 days, at 7 per cent.? Ans. $ 8.628.

27. What is the amount of $ 496.30 for 6 months and 20 days, at 7 per cent.? Ans. $ 515.60.

28. What is the interest of $ 691.04 for 1 month 3 days, at 5 per cent.? Ans. $ 3.167.

29. What is the interest of $ 9750 for 4 months, at 2 per cent. a month? Ans. $ 780.

30. What is the interest of $ 9162 for 3 months, at 1½ per cent. a month? Ans. $ 412.29.

31. What is the interest of $ 1500 for 7 months 20 days, at 10 per cent.? Ans. $ 95.833.

32. What is the interest of $ 640.50 for 10 months and 26 days, at 10 per cent.? Ans. $ 58.00.

33. What is the interest of $ 3178 for 15 months and 15 days?

34. If a banker borrow $ 10,000 in Boston at 6 per cent., and let the same in Wisconsin at 7 per cent., how much does he make by the operation in that way in 2 years and 6⅔ months? Ans. $ 255.555.

356. To reckon interest for any number of days, when 12 months of only 30 days each, or 360 days, are considered a *year*.

Ex. 1. What is the interest of $ 460 for 93 days?

Ans. $ 7.13.

FIRST OPERATION.

Principal,	$ 4 6 0	
$\frac{1}{6}$ of 93d. = $15\frac{1}{2}$d.;	.0 1 5 $\frac{1}{2}$	
	2 3 0 0	
	4 6 0	
	2 3 0	
Interest,	$ 7.1 3 0	Ans.

We multiply the principal by $\frac{1}{6}$ of the number of days, considered as thousandths; since the interest of $ 1 at 6 per cent. for 6 days is 1 mill or 1 *thousandth* of a dollar, (Art. 352,) and for any other number of days, at the same per cent., must be one sixth as many mills or thousandths of a dollar as there are days

SECOND OPERATION.

Principal,	$ 4 6 0	
$\frac{1}{100}$ of the principal,	4.6 0	Interest for 60d.
$\frac{1}{2}$ of the interest for 60d.	2.3 0	Interest for 30d.
$\frac{1}{10}$ of the interest for 30d.	.2 3	Interest for 3d.
Ans.	$ 7.1 3	Interest for 93d.

93 days = 60 days + 30 days + 3 days. Now, since the interest of any sum at 6 per cent. for 2 months, or 60 days, equals $\frac{1}{100}$ of the principal (Art. 352), $\frac{1}{100}$ of $ 460 = $ 4.60 will be the interest for 60 days; $\frac{1}{2}$ of the interest for 60 days, or $ 2.30, will be that for 30 days; and $\frac{1}{10}$ of the interest for 30 days, or $ 0.23, will be that for 3 days. The interest for the several parts of the time added together must give the interest for the whole time, or 93 days.

RULE. — *Multiply the principal by the number denoting one sixth of the number of days expressed decimally as thousandths.* Or,

Divide the principal by 100, *and take such a part or parts of the quotient as the given number of days is of* 60 *days.*

The result will be the interest at 6 *per cent., from which it may be found for any other rate, as in Art.* 355.

NOTE 1. — One sixth of the number of days in any number of months of 30 days each is equal to 5 times the number of months. Thus, one sixth of the number of days in 7 months equals 7 × 5, or 35 days; and one sixth of the number of days in 9 months and 13 days equals (9 × 5) + (13 ÷ 6), or 45 + $2\frac{1}{6}$ = $47\frac{1}{6}$ days.

NOTE 2. — It is a common practice among mercantile men, in calculating interest, to consider a year to consist of only 360 days, but the laws of some of the States require 365 days to a year. 360 days are in fact only $\frac{360}{365} = \frac{72}{73}$ of a common year, and therefore are $\frac{1}{73}$ less than what perfect accuracy would require. Hence, *when the year is considered to be one of* 365 *days, the interest as found by the rule must be diminished by* $\frac{1}{73}$ *of itself;* Or, *we may multiply the principal by the number of days, and, if the rate be* 6 PER CENT., *divide by* $6083\frac{1}{3}$; *if the rate be* 7 PER CENT., *divide by* 5214; *and if the rate be* 5 PER CENT., *divide by* 7300.

16. 25 per cent. of $\frac{1}{2}$ of a ship is how many per cent. of $\frac{3}{4}$ of it? Ans. $16\frac{2}{3}$ per cent.

17. If molasses cost 20 per cent. less than $ 0.50 per gallon, and it be sold at 25 per cent. more per gallon than it cost, at what price is it sold? Ans. $ 0.50 per gal.

18. I have 20 yards of yard-wide cloth, which will shrink on sponging 4 per cent. in the length, and 5 per cent. in the width; how much less than 20 square yards will there be of it after sponging? Ans. $1\frac{1}{2}\frac{2}{5}$ yards.

19. A gentleman having a large farm gave 15 per cent. of it to his oldest daughter, 10 per cent. of what remained and $\frac{1}{10}$ of an acre he gave to his oldest son, and 25 per cent. of the remainder he gave to his wife. The residue he divided equally among his other 5 children, who received each 39 acres. How many acres did his farm contain? Ans. 340 acres.

20. If the population of the United States in 1858 be 30,500,000, what will it be in 1868, allowing the increase should be at the rate of $34\frac{1}{2}$ per cent.?

21. In a certain battle in which the English, French, and Turks were allied against the Russians, there were $33\frac{1}{3}$ per cent. more French than English, and the Turks were $8\frac{1}{3}$ per cent. more than the French and 1600 more than the English. Required the whole number of the allies, and the per cent. the English, the French and the Turks each were of that number.

Ans. Whole number 13600; English, $26\frac{8}{17}$ per cent.; French, $35\frac{5}{17}$ per cent.; and Turks, $38\frac{4}{17}$ per cent.

22. Bought a cargo of flour, consisting of 560 barrels, at $ 7.25 per barrel, less 10 per cent., and sold the same at 10 per cent. more than $ 7.25 per barrel. At what per cent. above the cost was the flour sold? How much was made by the operation?

23. The population of a certain city, whose gain of inhabitants in 5 years has been 25 per cent., is 87500; what was it 5 years ago? Ans. 70000.

24. Bought a horse, buggy, and harness for $ 500. The horse cost $37\frac{1}{2}$ per cent. less than the buggy, and the harness cost 70 per cent. less than the horse. What was the price of each?

Ans. Buggy $ $275\frac{5}{29}$; horse, $ $172\frac{12}{29}$; and harness, $ $51\frac{21}{29}$.

INTEREST.

351. INTEREST is an allowance made for the use of money, or for value received; and it is generally reckoned as a certain rate per cent. for any given time, but usually for one year.

The *principal* is the sum lent, on which interest is computed.

The *amount* is the interest and principal added together.

Simple interest is that reckoned on the principal only; and is that meant when the term *interest* is used alone.

Legal interest is the rate per cent. established by law.

Usury is a higher rate per cent. than is allowed by law.

The legal rate per cent. varies in the different States and in different countries.

In Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, Tennessee, Kentucky, Ohio, Indiana, Illinois, Iowa, Nebraska, Missouri, Kansas, Arkansas, Mississippi, Florida, District of Columbia, and on debts or judgments in favor of the United States, it is 6 per cent.

In New York, Michigan, Wisconsin, Minnesota, Georgia, and South Carolina, it is 7 per cent.

In Alabama and Texas, it is 8 per cent.

In California, it is 10 per cent.

In Louisiana, it is 5 per cent.

In Canada, Nova Scotia, and Ireland, it is 6 per cent.

In England and France, it is 5 per cent.

NOTE. — The legal rate, as above, in some of the States, is only that which the law allows, when no particular rate is mentioned. By special agreement between parties, in Ohio, Indiana, Michigan, Illinois, Iowa, Nebraska, Missouri, Kansas, Arkansas, Louisiana, and Mississippi, interest can be taken as high as 10 per cent.; in Florida, as high as 8 per cent.; in Texas and Wisconsin, as high as 12 per cent.; and in California and Minnesota, any per cent. In New Jersey, by a special law, 7 per cent. may be taken in the city of Paterson, and in the counties of Essex and Hudson. In Vermont 7 per cent. may be taken on railroad bonds. Banks in Illinois cannot take above 7 per cent., and in Ohio, not above 6 per cent. In Mississippi, above 6 per cent. can be taken only for money lent.

352. When the rate of interest is 6 per cent. per annum, the interest for 1 year or 12 months will be 6 cents on every

100 cents on which it is reckoned, or $\frac{6}{100}$ of the principal. Hence, for 2 months or $\frac{1}{6}$ of a year, it will be 1 cent on every 100 cents, or $\frac{1}{100}$ of the principal; and for 1 month or $\frac{1}{2}$ of 2 months it will be 5 mills, or $\frac{1}{200}$ of the principal. Now, since the interest on 100 cents for 1 month, or 30 days, is 5 mills, for 6 days, or $\frac{1}{5}$ of 30 days, it will be 1 mill, or $\frac{1}{1000}$ of the principal; and since the interest on 100 cents for 2 months is 1 cent, or $\frac{1}{100}$ of the principal, for 100 times 2 months, or 200 months, or 16 years 8 months, it will be 100 cents, or equal the whole principal; and in the same ratio for any other length of time.

The interest of $ 1, at 6 per cent., and the ratio of the interest to the principal, for 200 months, and other convenient parts of time, is shown in the following

Table.

Interest of $ 1

For	200 mo.	= 16yr. 8mo.	is	$ 1.00	equal	the whole principal.		
"	100 mo.	= 8yr. 4mo.	"	0.50	"	$\frac{1}{2}$	of the	principal.
"	$66\frac{2}{3}$mo.	= 5yr. $6\frac{2}{3}$mo.	"	$0.333\frac{1}{3}$	"	$\frac{1}{3}$	"	"
"	50 mo.	= 4yr. 2mo.	"	0.25	"	$\frac{1}{4}$	"	"
"	40 mo.	= 3yr. 4mo.	"	0 20	"	$\frac{1}{5}$	"	"
"	$33\frac{1}{3}$mo.	= 2yr. $9\frac{1}{3}$mo.	"	$0.166\frac{2}{3}$	"	$\frac{1}{6}$	"	"
"	25 mo.	= 2yr. 1mo.	"	0.125	"	$\frac{1}{8}$	"	"
"	20 mo.	= 1yr. 8mo.	"	0.10	"	$\frac{1}{10}$	"	"
"	$16\frac{2}{3}$mo.	= 1yr. $4\frac{2}{3}$mo.	"	$0.083\frac{1}{3}$	"	$\frac{1}{12}$	"	"
"	12 mo.	= 1yr. 0mo.	"	0.06	"	$\frac{6}{100}$	"	"
"	10 mo.	= $\frac{5}{6}$ of a yr.	"	0.05	"	$\frac{1}{20}$	"	"
"	8 mo.	= $\frac{2}{3}$ of a yr.	"	0.04	"	$\frac{1}{25}$	"	"
"	$6\frac{2}{3}$mo.	= $\frac{5}{9}$ of a yr.	"	$0.033\frac{1}{3}$	"	$\frac{1}{30}$	"	"
"	6 mo.	= $\frac{1}{2}$ of a yr.	"	0.03	"	$\frac{3}{100}$	"	"
"	5 mo.	= $\frac{5}{12}$ of a yr.	"	0.025	"	$\frac{1}{40}$	"	"
"	4 mo.	= $\frac{1}{3}$ of a yr.	"	0.02	"	$\frac{1}{50}$	"	"
"	2 mo.	= $\frac{1}{6}$ of a yr.	"	0.01	"	$\frac{1}{100}$	"	"
"	1 mo.	= $\frac{1}{12}$ of a yr.	"	0.005	"	$\frac{1}{200}$	"	"
"	6 d.	= $\frac{1}{5}$ of a mo.	"	0.001	"	$\frac{1}{1000}$	"	"
"	5 d.	= $\frac{1}{6}$ of a mo.	"	$0.000\frac{5}{6}$	"	$\frac{5}{6000}$	"	"
"	4 d.	= $\frac{2}{15}$ of a mo.	"	$0.000\frac{2}{3}$	"	$\frac{4}{6000}$	"	"
"	3 d.	= $\frac{1}{10}$ of a mo.	"	$0.000\frac{1}{2}$	"	$\frac{3}{6000}$	"	"
"	2 d.	= $\frac{1}{15}$ of a mo.	"	$0.000\frac{1}{3}$	"	$\frac{2}{6000}$	"	"
"	1 d.	= $\frac{1}{30}$ of a mo.	"	$0.000\frac{1}{6}$	"	$\frac{1}{6000}$	"	"

353. The principal, the rate per cent., the time, and the interest have such a relation to each other that, any three of these terms being given, the fourth can be readily found. The computations in interest, therefore, admit of the following problems among others: — I. To find the interest; II. To find the principal; III. To find the rate per cent.; IV. To find the time.

354. To find the interest of any sum for any time at 6 per cent.

Ex. 1. What is the interest of $ 2640 for 2 years 7 months and 26 days, at 6 per cent.? Ans. $ 420.64.

FIRST OPERATION.

Interest of $ 1 for 2 years	=	$ 0.12
Interest of $ 1 for 7 months	=	0.035
Interest of $ 1 for 26 days	=	0.004$\frac{1}{3}$
Int. of $ 1 for 2yr. 7mo. 26d.	=	$ 0.159$\frac{1}{3}$

Principal, $ 2640
.159$\frac{1}{3}$

23760
13200
2640
880

Interest, $ 420.640 Ans.

The interest of $ 2640 for 2yr. 7mo. 26d. will be 2640 times as much as the interest of $ 1 for the given time. The interest of $ 1 for 2 years will be twice as much as for 1 year, equal 12cts.; and since the interest for 2 months is 1 cent, for 7 months it will be 3$\frac{1}{2}$ cents, or 3 cents 5 mills. And as the interest for 6 days is 1 mill, for 26 days it will be 4$\frac{1}{3}$ mills. These several sums added together give the interest of $ 1 at 6 per cent. for the given time, equal $ 0.159$\frac{1}{3}$, which taken 2640 times, by multiplying the given principal by it, gives $ 420.64, the interest required.

SECOND OPERATION.

Principal,	$ 2640	
$\frac{1}{8}$ of the prin. =	330	Interest for 2yr. 1mo.
$\frac{1}{30}$ of the prin. =	88	Interest for 6mo. 20d.
$\frac{1}{1000}$ of the prin. =	2.64	Interest for 6d.
Ans.	$ 420.64	Interest for 2yr. 7mo. 26d.

The time, 2yr. 7mo. 26d., is equal to 2yr. 1mo + 6mo. 20d. + 6d. Now, since the interest on any sum, at 6 per cent., in 200 months equals the principal, for 2yr. 1mo., or $\frac{1}{8}$ of 200 months, it will equal $\frac{1}{8}$ of the principal. We therefore take $\frac{1}{8}$ of the principal, $ 2640, equal $ 330, as the interest for 2yr. 1mo. Of the balance of

time, 6mo. 20d., or $6\frac{2}{3}$mo., being $\frac{1}{30}$ of 200 months, we take $\frac{1}{30}$ of the principal, equal $ 88, as the interest for the 6mo. 20d.; and the 6d. being $\frac{1}{1000}$ of 200 months, we take $\frac{1}{1000}$ of the principal, equal $ 2.64, as the interest for the 6d. We add together the interest for the parts of the whole time, and obtain, as by the first operation, $ 420.64 as the whole interest.

RULE 1. — *Find the interest of $ 1 for the given time, by reckoning 6 cents for every* YEAR, 1 *cent for every* TWO MONTHS, *and* 1 *mill for every* 6 DAYS; *then multiply the given principal by the number denoting that interest, and the product will be the interest required.* Or,

RULE 2. — *Take such fractional part or parts of the principal as the number expressing the time is of* 200 *months.*

NOTE 1. — To find the amount, add the principal to the interest.

NOTE 2. — In computing interest for a fractional part of a month, the month is considered as consisting of 30 days. This has the sanction of general usage and the decisions of the courts, though not entirely accurate.

NOTE 3. — Questions in interest, like other exercises in percentage, may be solved by proportion. The foregoing example admits of a statement and solution by the rule of compound proportion (Art. 340).

NOTE 4. — It is customary among merchants to reject the mills in the results of their computations of interest, increasing, however, the number of cents by 1 when the decimal of a cent exceeds 5.

EXAMPLES.

2. What is the interest of $ 675 for 1 year? Ans. $ 40.50.
3. What is the interest of $ 3967.87 for 2 years?
Ans. $ 476.144.
4. What is the interest of $ 896.28 for 3 years?
Ans $ 161.33.
5. What is the amount of $ 716.57 for 4 years?
6. What is the amount of $ 76.47 for 7 years?
Ans. $ 108.587.
7. What is the interest of $ 123.45 for 6 years?
Ans. $ 44.442.
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Ans. $ 116.99.

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17. What is the interest of $ 3246 for 27 days?
Ans. $ 14.607.

18. What is the interest $ 1364.24 for 1 day?

19. What is the interest of $ 6444 for 29 days?

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21. What is the interest of $ 386.19 for 100 months?
Ans. $ 193.09.

22. What is the interest of $ 0.75 for 75 years?
Ans. $ 3.37½.

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27. What is the interest of $ 98.25 from July 4, 1856, to October 19, 1859? Ans. $ 19.404.

28. What is the interest of $ 22.763 from February 19, 1836, to July 18, 1860?

29. What is the interest of $ 175.07 from January 7, 1855, to October 12, 1859? Ans. $ 50.04.

30. What is the interest of $ 197.28½ from December 6, 1852, to January 11, 1854? Ans. $ 12.987.

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32. What is the interest of $ 444.60 for 5 years and 6 months? Ans. $ 146.718.

355. To find the interest of any sum of money at any rate per cent. for any given time.

Ex. 1. What is the interest of $ 84.50 at 7 per cent. for 2 years 5 months and 12 days? Ans. $ 14.49.

FIRST OPERATION.

Principal,	$84.50
Rate per cent.	.07
Interest for 1 year,	5.9150
	2
Int. for 2 years,	11.8300
Int. for 4mo., or $\frac{1}{3}$ of 1yr.	1.9716+
Int. for 1mo., or $\frac{1}{4}$ of 4mo.	.4929+
Int. for 10d., or $\frac{1}{3}$ of 1mo.	.1643+
Int. for 2d., or $\frac{1}{5}$ of 10d.	.0328+
Int. for 2yr. 5mo. 12d.	$14.4916+ Ans.

Having found the interest for 1 year, and then for 2 years, the interest for 5 months is obtained by first taking $\frac{1}{3}$ of 1 year's interest, for 4 months, and then $\frac{1}{4}$ of this last interest for 1 month. And since 10 days are $\frac{1}{3}$ of 1 month, we take $\frac{1}{3}$ of 1 month's interest for the interest of 10 days; and since 2 days are $\frac{1}{5}$ of 10 days, we take $\frac{1}{5}$ of the last interest for 2 days. The interest as found for the several parts of the whole time, added together, gives the interest required.

SECOND OPERATION.

Principal,	$84.50
Int. of $1 at 6 per cent.	.147
	59150
	33800
	8450
Int. at 6 per cent.	12.42150
$\frac{1}{6}$ of int. at 6 per cent.	2.07025
Int. at 7 per cent.	$14.49175 Ans.

We first find the interest on the given sum at 6 per cent., and then add to this interest the fractional part of itself, denoted by the excess of the rate above 6 per cent. This excess is 1 per cent.; therefore we add $\frac{1}{6}$ of the interest at 6 per cent. to that interest for the answer. If the rate per cent. had been less than 6 per cent, we should have subtracted the fractional part.

RULE 1.—*First find the interest for one year by multiplying the principal by the rate per cent. expressed decimally; and for two or more years multiply this product by the number of years.*

Find the interest for months by taking the most convenient fractional part or parts of ONE *year's interest.*

Find the interest for days by taking the most convenient fractional part or parts of ONE *month's interest.* Or,

RULE 2.—*Find the interest of the given sum at* 6 *per cent., and then add to this interest, or subtract from it, such a fractional part of itself as the given rate is greater or less than* 6 *per cent.*

NOTE 1.—$\frac{1}{6}$ of the interest at 6 per cent. may be taken for that at 1 per cent.; $\frac{1}{4}$, for that at $1\frac{1}{2}$ per cent.; $\frac{1}{3}$, for that at 2 per cent.; $\frac{1}{2}$, for that at 3 per cent. From the interest at 6 per cent. may be taken $\frac{1}{3}$ of itself for that at 4 per cent.; $\frac{1}{4}$ of itself for that at $4\frac{1}{2}$ per cent.; and $\frac{1}{6}$ of itself for that at 5 per cent. To the interest at 6 per cent. may be added $\frac{1}{6}$ of itself for that at 7 per

cent.; $\frac{1}{4}$ of itself for that at $7\frac{1}{2}$ per cent.; $\frac{1}{3}$ of itself for that at 8 per cent; and $\frac{1}{2}$ of itself for that at 9 per cent. If the rate is 12 per cent., the interest at 6 per cent. may be taken twice; if 18 per cent., the interest at 6 per cent. may be taken three times, etc.

NOTE 2. — When in this book the rate of interest is not given, 6 per cent. is to be understood.

EXAMPLES.

2. What is the interest of $ 16.75 for 7 months and 17 days, at 7 per cent.? Ans. $ 0.739.

3. What is the interest of $ 11.10½ from April 17, 1852, to December 7, 1852, at 7 per cent.? Ans. $ 0.496.

4. What is the interest of $ 12.69, from January 2, 1853, to August 30, 1854, at 7 per cent.? Ans. $ 1.47½.

5. What is the interest of $ 5000 for 2 years 5 months 26 days, at 7 per cent.?

6. What is the amount of $ 416 for 3 years 16 days, at 7 per cent.? Ans. $ 504.64.

7. What is the interest of $ 336 for 15 days, at 5 per cent.? Ans. $ 0.70.

8. What is the interest of $ 17869.75 from February 7, 1852, to January 11, 1860, at 5 per cent.? Ans. $ 7083.3703.

9. What is the interest of $ 300.50 for 1 year 2 months and 15 days? Ans. $ 21.786.

10. What is the interest of $ 37 for 29 days, at 5 per cent.?

11. What is the interest of $ 35.61 from November 11, 1861, to December 15, 1863? Ans. $ 4.474.

12. What is the interest of $ 16.76 from December 17, 1841, to January 17, 1852? Ans. $ 10.139+

13. What is the interest of $ 1728.19 from May 7, 1854, to July 17, 1860, at ¼ per cent.? Ans. $ 26.762.

14. What is the interest of $ 397.16 for 1 year 6 months and 1 day, at 5½ per cent.? Ans. $ 32.826.

15. What is the amount of $ 100.25 for 2 months and 29 days, at 4 per cent.? Ans. $ 101.241.

16. What is the interest of $ 51.17 for 9 months and 29 days, at 4 per cent.? Ans. $ 1.699.

17. What is the interest of $ 42.20 for 1 year and 16 days, at 4½ per cent.?

18. What is the interest of $16.25 for 2 years, at 3 per cent.? Ans. $0.975.

19. What is the interest of $96.84 from November 27, 1849, to July 3, 1852, at 7½ per cent.? Ans. $18.883.

20. What is the interest of $786.97 from October 19, 1857, to August 17, 1861, at 7½ per cent.?

21. What is the interest of $71.091 from July 29, 1853, to June 19, 1857, at 12 per cent.? Ans. $33.175.

22. What is the amount of $369.29 for 2 years 3 months and 1 day, at 9 per cent.? Ans. $444.163.

23. What is the interest of $76.35 for 1 year 8 months and 18 days? Ans. $7.864.

24. What is the interest of $47.15 for 1 month and 19 days, at 13½ per cent.? Ans. $0.886.

25. What is the interest of $36.72 from May 16, 1829, to February 18, 1857, at 7 per cent.? Ans. $71.342.

26. What is the interest of $35.50 for 3 years 5 months and 20 days, at 7 per cent.? Ans. $8.628.

27. What is the amount of $496.30 for 6 months and 20 days, at 7 per cent.? Ans. $515.60.

28. What is the interest of $691.04 for 1 month 3 days, at 5 per cent.? Ans. $3.167.

29. What is the interest of $9750 for 4 months, at 2 per cent. a month? Ans. $780.

30. What is the interest of $9162 for 3 months, at 1½ per cent. a month? Ans. $412.29.

31. What is the interest of $1500 for 7 months 20 days, at 10 per cent.? Ans. $95.833.

32. What is the interest of $640.50 for 10 months and 26 days, at 10 per cent.? Ans. $58.00.

33. What is the interest of $3178 for 15 months and 15 days?

34. If a banker borrow $10,000 in Boston at 6 per cent., and let the same in Wisconsin at 7 per cent., how much does he make by the operation in that way in 2 years and 6⅔ months? Ans. $255.555.

356. To reckon interest for any number of days, when 12 months of only 30 days each, or 360 days, are considered a *year.*

Ex. 1. What is the interest of $460 for 93 days?

Ans. $7.13.

FIRST OPERATION.

Principal,	$460
$\frac{1}{6}$ of 93d. = $15\frac{1}{2}$d.;	$.015\frac{1}{2}$
	2300
	460
	230
Interest,	$7.130 Ans.

We multiply the principal by $\frac{1}{6}$ of the number of days, considered as thousandths; since the interest of $1 at 6 per cent. for 6 days is 1 mill or 1 *thousandth* of a dollar, (Art. 352,) and for any other number of days, at the same per cent., must be one sixth as many mills or thousandths of a dollar as there are days

SECOND OPERATION.

Principal,	$460	
$\frac{1}{100}$ of the principal,	4.60	Interest for 60d.
$\frac{1}{2}$ of the interest for 60d.	2.30	Interest for 30d.
$\frac{1}{10}$ of the interest for 30d.	.23	Interest for 3d.
Ans.	$7.13	Interest for 93d.

93 days = 60 days + 30 days + 3 days. Now, since the interest of any sum at 6 per cent. for 2 months, or 60 days, equals $\frac{1}{100}$ of the principal (Art. 352), $\frac{1}{100}$ of $460 = $4.60 will be the interest for 60 days; $\frac{1}{2}$ of the interest for 60 days, or $2.30, will be that for 30 days; and $\frac{1}{10}$ of the interest for 30 days, or $0.23, will be that for 3 days. The interest for the several parts of the time added together must give the interest for the whole time, or 93 days.

RULE. — *Multiply the principal by the number denoting one sixth of the number of days expressed decimally as thousandths.* Or,

Divide the principal by 100, *and take such a part or parts of the quotient as the given number of days is of* 60 *days.*

The result will be the interest at 6 *per cent., from which it may be found for any other rate, as in Art.* 355.

NOTE 1. — One sixth of the number of days in any number of months of 30 days each is equal to 5 times the number of months. Thus, one sixth of the number of days in 7 months equals 7 × 5, or 35 days; and one sixth of the number of days in 9 months and 13 days equals (9 × 5) + (13 ÷ 6), or 45 + $2\frac{1}{6}$ = $47\frac{1}{6}$ days.

NOTE 2. — It is a common practice among mercantile men, in calculating interest, to consider a year to consist of only 360 days, but the laws of some of the States require 365 days to a year. 360 days are in fact only $\frac{360}{365} = \frac{72}{73}$ of a common year, and therefore are $\frac{1}{73}$ less than what perfect accuracy would require. Hence, *when the year is considered to be one of* 365 *days, the interest as found by the rule must be diminished by* $\frac{1}{73}$ *of itself;* Or, *we may multiply the principal by the number of days, and, if the rate be* 6 PER CENT., *divide by* $6083\frac{1}{3}$; *if the rate be* 7 PER CENT., *divide by* 5214; *and if the rate be* 5 PER CENT., *divide by* 7300.

EXAMPLES.

2. What is the interest of $ 96 for 33 days, at 7 per cent.? Ans. $ .616.

3. What is the interest of $ 320.40 for 63 days, at 5 per cent.? Ans. $ 2.803.

4. What is the interest of $ 131.20 for 123 days? Ans. $ 2.689.

5. What is the interest of $ 26.60 for 78 days?

6. What is the interest of $ 5780 for 153 days, at 10 per cent.? Ans. $ 245.65.

7. What is the interest of $ 105.10 for 48 days, at 12 per cent.? Ans. $ 1.681.

8. What is the interest of $ 13.62 for 93 days, at $7\frac{1}{2}$ per cent.?

9. What is the interest of $ 4580 for 253 days, at $5\frac{1}{2}$ per cent.? Ans. $ $177.029\frac{1}{8}$.

10. What is the interest of $ 3140 for 273 days, at 7 per cent.? Ans. $ $166.681\frac{2}{3}$.

11. What is the interest of $ 10550 for 243 days, at 8 per cent.? Ans. $ 569.70.

12. What is the amount of $ 33.44 for 333 days? Ans. $ 35.295.

13. What is the amount of $ 71.60 for 3 months and 18 days?

14. What is the interest of $ 92.96 for 4 months and 3 days, at 7 per cent.? Ans. $ 2.223.

15. What is the interest of $ 144.50 for 144 days, allowing 365 days to the year, at 5 per cent.? Ans. $ 2.85.

16. What is the interest of $ 761.81 for 165 days, allowing 365 days to the year? Ans. $ 20.662.

17. What is the interest of $ 560 for 183 days, allowing 365 days to a year, at 7 per cent.? Ans. $ 19.65.

18. What is the interest of $ 1960 for 93 days?

19. What is the amount of $ 1000 for 1 month and 3 days? Ans. $ 1005.50.

20. What is the amount of $ 7300 for 18 days, at 6 per cent.? Ans. $ 7321.90.

357.° To find the interest on sterling money, at any rate per cent., for any time.

Ex. 1. What is the interest of 576£. 5s. 6d. for 1 year and 10 months? Ans. 63£. 7s. 9d.

OPERATION.

576£. 5s. 6d. =	576.275£.	Principal.
$\frac{1}{10}$ of the principal,	57.6275	Int. for 1yr. 8mo.
$\frac{1}{10}$ of the int. for 1yr. 8mo.	5.76275	Int. for 2mo.
Ans.	63.39025£. =	
	63£. 7s. 9d.+	Int. for 1yr.10mo.

We reduce the 5s. 6d. to the decimal of a pound (Art. 279), and annex it to the pounds; we then find the interest as though the sum expressed dollars and cents; and, reducing the decimal in the answer to shillings and pence, we have 63£. 7s. 9d.+ as the interest required. Hence,

Reduce the shillings, pence, and farthings, if any, in the principal to the decimal of a pound. Then proceed as in United States money; and, if there be a decimal in the result, reduce it to a compound number.

EXAMPLES.

2. What is the interest of 179£. 12s. 11d. for 1 year and 7 months, at 5 per cent.? Ans. 14£. 4s. $5\frac{1}{4}$d.

3. What is the interest of 25£. for 1 year and 9 months, at 5 per cent.?

4. What is the interest of 5440£. 10s. for 3 years and 11 months, at 6 per cent.? Ans. 1278£. 10s. 4d.

5. What is the interest of 943£. 1s. 8d. from May 1, 1857, to October 21, 1857, allowing 365 days to a year, at $5\frac{1}{4}$ per cent.? Ans. 23£. 9s. 4d.

358. To find the PRINCIPAL, the interest, the time, and the rate per cent. being given.

Ex. 1. What principal will gain $120 in 4 years, at 6 per cent.? Ans. $500.

OPERATION.

$120.00 ÷ .24 = $500 Ans.

The interest for the given time and rate is 24 cents on a principal of $1; therefore, the required principal will be as many times $1 as the number denoting the given interest contains times .24.

RULE. — *Divide the given interest by the interest of $1 for the given time, at the given rate.*

EXAMPLES.

2. What principal will gain $10.08 in 1 year, at 7 per cent.? Ans. $144.

3. What principal will yield an income of $13.20 in 1 year and 4 months, at $8\frac{1}{4}$ per cent.? Ans. $120.

4. What sum should be paid for a ground rent paying yearly, at 6 per cent., $40.50?

5. If the interest on a sum borrowed at 2 per cent. a month is $24 for 90 days, what is the sum? Ans. $400.

6. What principal at $7\frac{1}{4}$ per cent. is sufficient to produce $$206.38\frac{1}{3}$ interest in 183 days? Ans. $5600.

359. To find the RATE PER CENT., the principal, the interest, and the time being given.

Ex. 1. If the interest of $500 for 4 years is $120, what is the rate per cent.?

OPERATION.

$120 \div 20 = 6$ per cent. Ans.

We find the interest on a given principal for the given term to be $20, at 1 per cent.; then the required rate will be as many per cent. as 20 is contained times in 120, or 6 per cent.

RULE. — *Divide the given interest by the interest of the given principal at* 1 *per cent. for the given time.*

EXAMPLES.

2. The interest of $144 for 1 year is $10.08; what is the rate per cent.? Ans. 7 per cent.

3. At what rate per cent. must $120 be on interest to amount to $133.20 in 1 year 4 months?

4. At what rate per cent. must $1, or any other sum, be on interest to double itself in $14\frac{2}{7}$ years? Ans. 7 per cent.

5. At what rate per cent. must any sum of money be on interest to quadruple itself in $33\frac{1}{3}$ years? Ans. 9 per cent.

6. I receive yearly $232.50 interest on $4650 loaned to the State of Massachusetts; what is the rate per cent.? Ans. 5 per cent.

7. At what rate per cent. must $7500 be loaned to gain $60.00 in 48 days?

8. At what rate per cent. must \$ 280 be on interest to amount to \$ 411.95 in 6 years and 6 months? Ans. $7\frac{1}{4}$ per cent.

9. At what rate per cent. must \$ 480 be on interest to amount to \$ 529.60 in 1 year 3 months and 15 days?

Ans. 8 per cent.

360. To find the TIME, the principal, the interest, and the rate per cent. being given.

Ex. 1. How long must \$ 500 be on interest, at 6 per cent., to gain \$ 120? Ans. 4 years.

OPERATION.

$120 \div 30 = 4$ years, Ans.

We find that the given principal at the given rate will produce \$ 30 interest in 1 year; therefore to produce \$ 120 interest there will be required as many years as 30 is contained times in 120, or 4 years.

RULE. — *Divide the given interest by the interest of the given principal for* 1 *year.*

EXAMPLES.

2. How long must \$ 120 be on interest, at $8\frac{1}{4}$ per cent., to gain \$ 13.20? Ans. 1yr. 4mo.

3. In what time will \$ 144 produce \$ 10.08 interest, at 7 per cent.? Ans. 1 year.

4. In what time will \$ 240 at interest amount to \$ 280, at 6 per cent.? Ans. 2yr. 9mo. 10d.

5. In what time will \$ 1, or any other sum, double itself, at 5 per cent. interest? Ans. 20 years.

6. In what time will any sum double itself at 10 per cent. interest? Ans. 10 years.

7. In what time will \$ 1500 amount to \$ 2250, at 5 per cent. interest?

8. In what time will \$ 480 at $4\frac{1}{2}$ per cent. amount to \$ 561.60? Ans. 3yr. 9mo. 10d.

9. In what time, at 12 per cent., will \$ 1728 amount to \$ 3853.44? Ans. 10yr. 3mo.

10. In what time, at 6 per cent., will \$ 240 amount to \$ 720? Ans. 33yr. 4mo.

11. Borrowed, May 16, 1857, the sum of \$ 400, payable as soon as the principal, increased by the interest at 6 per cent., shall equal \$ 500. At what date is it payable?

Ans. July 16, 1861.

PROMISSORY NOTES.

361. A PROMISSORY NOTE, or note of hand, is an engagement, in writing, to pay a specified sum, either to a person named in the note, or to his order, or to the bearer.

A *joint* note is one signed by two or more persons, who together are holden for its payment.

A *joint* and *several* note is one signed by two or more persons, who separately and together are holden for its payment.

A *negotiable* note is one so made that it can be sold or transferred from one person to another.

362. The *maker* or *drawer* of a note is the person who signs it.

The *payee*, *promisee*, or *holder* of a note is the person to whom it is to be paid.

The *indorser* of a note is the person who writes his name upon its back to transfer it, or as a guaranty of its payment. If the indorser, however, wishes only to transfer the note, he may write before his name the words "without recourse," and then, though by his name he guarantees the genuineness of the note, he is not liable for its payment, should the maker not pay it when due.

363. The *face* of a note is the sum for which it is given.

364. A note should contain the words "value received," and the sum for which it is given should be expressed in written words, or, as is the general custom, the dollars may be written in words and the cents, if any, be expressed as hundredths of a dollar in the form of a common fraction.

NOTE. — The laws of Pennsylvania require that the words "without defalcation" should be inserted in a promissory note; and in Indiana, notes generally contain the words "without any relief whatever from valuation or appraisement laws."

365. A note is said to be one *on time*, when it is made payable on or after a certain date, and the day on which it becomes legally due is called the *day of maturity*.

366. According to the laws of many of the States, when a particular day is specified in the note for its payment, three

days additional are allowed, called *days of grace*, within which the maker may pay the note, unless it states "without grace." Should the third day of grace, however, fall on Sunday, or some public day, the day of maturity will be a day earlier.

367. When a note is written for months, calendar months are understood. Thus, if a note be dated April 21, for one month, it will be nominally due May 21; and if dated January 29, 30, or 31, it being for one month, it will be nominally due February 28, should it be a common year, or February 29, should it be a leap year.

368. When a note is written without interest, it can only draw interest, if on time, after the time specified for its payment, and not then lawfully, in some States, till after a demand has been made; or if not on time, after payment has been demanded.

NOTE. — A note attested or witnessed, in Massachusetts and some other States, is taken out of the statute of limitation.

PARTIAL PAYMENTS ON NOTES AND BONDS.

369. PARTIAL or part payments on notes, bonds, or other obligations, being receipted for by entry on the back of the obligation, are termed *indorsements*.

UNITED STATES RULE.

370. In the United States courts, and the courts of Massachusetts, New York, and several other States, interest on notes and bonds, when partial payments have been made, is reckoned according to the following

RULE. — *Compute the interest on the principal to the time when the first payment was made, which equals or exceeds, either alone or with preceding payments, the interest then due.*

Add that interest to the principal, and from the amount subtract the payment or payments thus far made.

The remainder will form a new principal; on which compute the interest, proceeding as before.

NOTE. — This rule is on the principle that neither interest nor payment should draw interest.

Examples.

(1.) $165.18. *Boston, June* 17, 1847.

For value received, I promise to pay Nathaniel Ford, or order, on demand, one hundred and sixty-five dollars and eighteen cents, with interest.

Attest, Joseph Field. James Peterson.

On this note are the following indorsements. December 7, 1847, received eighteen dollars and thirteen cents of the within note. October 19, 1848, received twenty-eight dollars and sixteen cents. September 25, 1849, received thirty-six dollars and twelve cents. July 10, 1850, received three dollars and eighteen cents. June 6, 1851, received thirty-six dollars and twenty-eight cents. December 28, 1852, received thirty-one dollars and seventeen cents. May 5, 1853, received three dollars and eighteen cents. September 1, 1853, received twenty five dollars and eighteen cents. October 18, 1854, received ten dollars.

How much remains due September 27, 1855?

Ans. $15.417.

OPERATION.

Principal, carrying interest from June 17, 1847,		$165.180
Interest from June 17, 1847, to Dec. 7, 1847, 5mo. 20d.,		4.680
Amount,		169.860
First payment, December 7, 1847, ...		18.130
Balance for new principal, ...		151.730
Interest from Dec. 7, 1847, to Oct. 19, 1848, 10mo. 12d.,		7.889
Amount,		159.619
Second payment, October 19, 1848, . . .		28.160
Balance for new principal, ...		131.459
Interest from Oct. 19, 1848, to Sept. 25, 1849, 11mo. 6d.,		7.361
Amount,		138.820
Third payment, September 25, 1849, ...		36.120
Balance for new principal, ...		102.700
Interest from Sept. 25, 1849, to June 6, 1851, 20mo. 11d.,		10.458
Amount,		113.158
Fourth pay't, July 10, 1850, a sum less than interest,	3.18	
Fifth pay't, June 6, 1851, a sum greater than interest,	36.28	
	——	39.460
Balance for new principal, ...		73.698
Interest from June 6, 1851, to Dec. 28, 1852, 18mo. 22d.,		6.903
Amount,		80.601

Amount brought forward,	$80.601
Sixth payment, December 28, 1852, ...	31.170
Balance for new principal, ...	49.431
Interest from Dec. 28, 1852, to May 5, 1853, 4mo. 7d.,	1.046
Amount,	50.477
Seventh payment, May 5, 1853, ...	3.180
Balance for new principal, ...	47.297
Interest from May 5, 1853, to Sept. 1, 1853, 3mo. 26d.,	.914
Amount,	48.211
Eighth payment, September 1, 1853, ...	25.180
Balance for new principal, ...	23.031
Interest from Sept. 1, 1853, to Oct. 18, 1854, 13mo. 17d.,	1.562
Amount,	24.593
Ninth payment, ...	10.000
Balance for new principal, ...	14.593
Interest from Oct. 18, 1854, to Sept. 27, 1855, 11mo. 9d.,	.824
Balance due at the time of payment, ...	$15.417

(2.) $769.87. *St. Louis, June* 17, 1849.

For value received, I promise to pay L. Swan, or order, on demand, seven hundred and sixty-nine dollars and eighty-seven cents, with interest. SAMUEL Q. PETERS.

Attest, MOSES HAYNES.

Payments: March 1, 1850, seventy-five dollars and fifty cents; June 11, 1851, one hundred and sixty-five dollars; September 15, 1851, one hundred and sixty-one dollars; Jan. 21, 1852, forty-seven dollars and twenty-five cents; March 5, 1853, twelve dollars and seventeen cents; December 6, 1853, ninety-eight dollars; July 7, 1854, one hundred and sixty-nine dollars.

What remains due September 25, 1855? Ans. $226.297.

(3.) $300. *Chicago, April* 30, 1851.

For value received, I promise Kimball & Hammond to pay them, or order, on demand, three hundred dollars, with interest.

SIMPSON W. LEAVET.

Payments: June 27, 1852, one hundred and fifty dollars; December 9, 1852, one hundred and fifty dollars.

What was due October 9, 1853? Ans. $26.735.

(4.) $54.18. *San Francisco, Feb.* 11, 1852.

For value received, I promise to pay John Trow, or order, on demand, fifty-four dollars and eighteen cents, with interest.

LUKE M. SAMPSON.

Payments: July 11, 1853, twelve dollars and twenty-five cents; August 15, 1854, two dollars and ten cents; July 9, 1855, three dollars and twelve cents; August 21, 1855, thirty-seven dollars and eighteen cents.

What was due December 17, 1855? Ans. $10.222.

(5.) $1728. *Philadelphia, Jan.* 7, 1851.

For value received, we jointly and severally promise to pay Jones, Oliver, & Co., or order, one thousand seven hundred and twenty-eight dollars, on demand, with interest, without defalcation.

JOHN WICKERSHAM.

Attest, TIMOTHY TRUE. JAMES THICKSTEIN.

(6.) $1000. *New York, January* 1, 1850.

For value received, I promise to pay James Johnson, or order, on demand, one thousand dollars, with interest at seven per cent.

SAMUEL T. FORTUNE.

Indorsements: September 28, 1850, one hundred and forty-four dollars; March 1, 1851, twenty dollars; July 17, 1851, three hundred and sixty dollars; August 9, 1851, one hundred and ninety dollars; September 25, 1852, one hundred and seventy dollars; December 11, 1853, two hundred dollars; July 4, 1855, seventy-five dollars.

What was due June 1, 1857? Ans. $7.61.

CONNECTICUT RULE.

371. The Supreme Court of the State of Connecticut has adopted the following

RULE. — *Compute the interest to the time of the first payment, if that be one year or more from the time the interest commenced; add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, till all the payments are absorbed; provided the time between one payment and another be one year or more.*

If any payments be made before one year's interest has accrued, then compute the interest on the principal sum due on the obligation for ONE YEAR, *add it to the principal, and compute the interest on the sum paid from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above.*

If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.

NOTE. — If a year extends beyond the time of settlement, find the amount of the remaining principal to the time of *settlement;* find also the amount of the indorsement or indorsements, if any, from the time they were paid to the *time of settlement*, and subtract their sum from the amount of the principal.

EXAMPLE.

(1.) $ 900. *New Haven, June* 1, 1858.

For value received, I promise to pay J. Downs, or order, nine hundred dollars, on demand, with interest.

JAMES L. EMERSON.

Indorsements: June 16, 1859, two hundred dollars; August 1, 1860, one hundred and sixty dollars; November 16, 1860, seventy-five dollars; February 1, 1862, two hundred and twenty dollars.

What was due August 1, 1862? Ans. $ 417.822.

OPERATION.

Principal, ...	$ 900.00
Interest from June 1, 1858, to June 16, 1859, 12½ months,	56.25
	956.25
First payment, ...	200.00
	756.25
Interest from June 16, 1859, to August 1, 1860, 13½ months,	51.046
	807.296
Second payment, ...	160.000
	647.296
Interest for 1 year, ...	38.837
	686.133
Am't of 3d pay't, from Nov 16, 1860, to Aug. 1, 1861, 8½mo.,	78.187
	607.946
Interest from Aug. 1, 1861, to Aug. 1, 1862, 12 months,	36.476
	644.422
Am't of 4th pay't, from Feb. 1, 1862, to Aug. 1, 1862, 6mo.,	226.600
Balance due August 1, 1862, ...	$ 417.822

MERCHANTS' RULE.

372. It is customary with merchants and others, when partial payments are made of notes or other debts, when the note or debt is settled within a year after becoming due, to adopt the following

RULE. — *Find the amount of the principal from the time it became due until the time of settlement. Then find the amount of each indorsement from the time it was paid until settlement, and subtract their sum from the amount of the principal.*

EXAMPLES.

(1.) $1728. *Baltimore, January* 1, 1853.

For value received, I promise to pay Riggs, Peabody, & Co., or order, on demand, one thousand seven hundred and twenty-eight dollars, with interest. JOHN PAYWELL, JR.

Indorsements: March 1, 1853, three hundred dollars; May 16, 1853, one hundred and fifty dollars; September 1, 1853, two hundred and seventy dollars; December 11, 1853, one hundred and thirty-five dollars.

What was due at the time of payment, which was December 16, 1853? Ans. $948.03

OPERATION.

Principal,		$1728.00
Interest for 11 months and 15 days,		99.36
		$1827.36
First payment,	$300.00	
Interest for 9 months and 15 days,	14.25	
Second payment,	150.00	
Interest for 7 months,	5.25	
Third payment,	270.00	
Interest for 3 months and 15 days,	4.72	
Fourth payment,	135.00	
Interest for 5 days,	.11	
		879.33
Balance remaining due, December 16, 1853,		$948.03

(2.) $700. *Montpelier, February* 4, 1854.

For value received, we jointly and severally promise to pay James Thomas, or order, on demand, seven hundred dollars, with interest. SAMPSON PHILLIPS,
RICHARD FLETCHER.

Payments: March 18, 1854, one hundred and sixty dollars; June 24, 1854, two hundred dollars; September 11, 1854, one hundred and twenty dollars; October 5, 1854, sixty dollars.

What was due on this note Nov. 28, 1854? Ans. $180.43.

(3.) $500. *Detroit, January* 1, 1857.

For value received, three months after date I promise to pay to the order of James Francis five hundred dollars.

WILLIAM AMSDEN.

Indorsement: July 1, 1857, two hundred dollars.

What was due April 1, 1858, the rate of interest being 7 per *cent.?* Ans. $324.50.

COMPOUND INTEREST.

373. COMPOUND INTEREST is interest on the original principal with its interest added when remaining unpaid after becoming due.

374. When the interest is added to the principal at the end of every year, and a new principal is thus formed yearly, it is said to compound annually; when the interest is added to the principal so as to form a new principal half-yearly, it is said to compound semiannually.

375. Compound interest is based upon the principle, that, if the borrower does not pay the interest as it becomes due at stated times, it is no more than just for him to pay interest for the use of it, so long as he shall have it in his possession.

NOTE. — Compound interest is not favored by the laws, though it is not usurious. A contract or promise to pay money with compound interest cannot generally be enforced, being only valid for the principal and *legal* interest.

376. To find the compound interest of any sum of money at any rate per cent. for any time.

Ex. 1. What is the compound interest of $ 300 for 3 years?

FIRST OPERATION.

Principal for 1st year,	$ 3 0 0
Interest of $ 1 for 1 year,	.0 6
Interest for 1st year,	1 8.0 0
	3 0 0.0 0
Principal for 2d year,	3 1 8.0 0
	.0 6
Interest for 2d year,	1 9.0 8 0 0
	3 1 8.0 0
Principal for 3d year,	3 3 7.0 8
	.0 6
Interest for 3d year,	2 0.2 2 4 8
	3 3 7 0 8
Amount for 3 years,	3 5 7.3 0 4 8
First principal,	3 0 0.0 0
Comp. int. for 3 years,	$ 5 7.3 0 4 8, Ans.

We first multiply the given principal by the number denoting the interest of $ 1 for one year, and add the interest thus found to the principal for the amount; on which as a new principal we find the interest for the second year, and proceed as before; and so also with the third year. From the amount of the last year we subtract the first principal, and obtain the compound interest for 3 years.

SECOND OPERATION.

Principal for 1st year,	\$ 3 0 0.0 0	
$\frac{1}{20}$ of the principal,	1 5.0 0	
$\frac{1}{5}$ of the interest at 5 per cent.,	3.0 0	
Principal for 2d year,	3 1 8.0 0	Amount for 1st year at
$\frac{1}{20}$ of the principal,	1 5.9 0	[6 per cent.
$\frac{1}{5}$ of the interest at 5 per cent.,	3.1 8	
Principal for 3d year,	3 3 7.0 8	Amount for 2d year at
$\frac{1}{20}$ of the principal,	1 6.8 4 5	[6 per cent.
$\frac{1}{5}$ of the interest at 5 per cent.,	3.3 7 0	
	3 5 7.3 0 4	Amount for 3d year
First principal,	3 0 0.0 0	[at 6 per cent.
Compound interest for 4 years,	\$ 5 7.3 0 4	Ans.

In the second operation the work is somewhat abridged, by finding the interest for each year at 6 per cent. by taking $\frac{1}{20}$ of the principal for the interest at 5 per cent., and $\frac{1}{5}$ of that for interest at 1 per cent.

RULE. — *Find the interest of the given sum to the time the interest becomes due, and add it to the principal. Then, find the interest on this amount as a new principal, and add the interest to it, as before. Proceed in the same manner for each successive period when the interest becomes due until the time of settlement.*

Subtract the principal from the last amount, and the remainder will be the compound interest.

NOTE. — When partial payments have been made on notes at compound interest, it is customary to *find the amount of the given principal, and from it to subtract the sum of the several amounts of the indorsements.*

EXAMPLES.

2. What is the amount of \$ 500 for 3 years at compound interest? Ans. \$ 595.508.

3. What is the compound interest of \$ 970 for 2 years 9 months and 24 days? Ans. \$ 173.295.

4. What is the compound interest of \$ 300 for 4 years 6 months, at 7 per cent.? Ans. \$ 107.001.

5. What is the compound interest of \$ 316 for 3 years 4 months and 18 days? Ans. \$ 69.017.

377. The computation of compound interest is rendered more expeditious by means of the following

TABLE,

SHOWING THE AMOUNT OF ONE DOLLAR AT COMPOUND INTEREST FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY.

No.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.030 000	1.035 000	1.040 000	1.050 000	1.060 000	1.070 000
2	1.060 900	1.071 225	1.081 600	1.102 500	1.123 600	1.144 900
3	1.092 727	1.108 718	1.124 864	1.157 625	1.191 016	1.225 043
4	1.125 509	1.147 523	1.169 859	1.215 506	1.262 477	1.310 796
5	1.159 274	1.187 686	1.216 653	1.276 282	1.338 226	1.402 552
6	1.194 052	1.229 255	1.265 319	1.340 096	1.418 519	1.500 730
7	1.229 874	1.272 279	1.315 932	1.407 100	1.503 630	1.605 781
8	1.266 770	1.316 809	1.368 569	1.477 455	1.593 848	1.718 186
9	1.304 773	1.362 897	1.423 312	1.551 328	1.689 479	1.838 459
10	1.343 916	1.410 599	1.480 244	1.628 895	1.790 848	1 967 151
11	1.384 234	1.459 970	1.539 454	1.710·339	1.898 299	2.104 852
12	1.425 761	1.511 069	1.601 032	1.795 856	2.012 196	2.252 192
13	1.468 534	1.563 956	1.665 074	1.885 649	2.132 928	2.409 845
14	1.512 590	1.618 694	1.731 676	1.979 932	2.260 904	2.578 534
15	1.557 967	1.675 349	1.800 944	2.078 928	2.396 558	2.759 032
16	1.604 706	1.733 986	1.872 981	2.182 875	2.540 352	2.952 164
17	1.652 848	1.794 675	1.947 901	2.292 018	2.692 773	3.158 815
18	1.702 433	1.857 489	2.025 817	2.406 619	2.854 339	3.379 932
19	1.753 506	1.922 501	2.106 849	2.526 950	3.025 600	3.616 526
20	1.806 111	1.989 789	2.191 123	2.653 298	3.207 135	3.869 684
21	1.860 295	2.059 431	2.278 768	2.785 963	3.399 564	4.140 562
22	1.916 103	2.131 512	2.369 919	2.925 261	3.603 537	4.430 402
23	1.973 587	2.206 114	2.464 716	3.071 524	3.819 750	4.740 530
24	2.032 794	2.283 328	2.563 304	3.225 100	4.048 935	5.072 367
25	2.093 778	2.363 245	2.665 836	3.386 355	4.291 871	5.427 433
26	2.156 591	2.445 959	2.772 470	3.555 673	4.549 383	5.807 353
27	2.221 289	2.531 567	2.883 369	3.733 456	4.822 346	6.213 868
28	2.287 928	2.620 177	2.998 703	3.920 129	5.111 687	6.648 838
29	2.356 566	2.711 878	3.118 651	4.116 136	5.418 388	7.114 257
30	2.427 262	2.806 794	3.243 398	4.321 942	5.743 491	7.612 255
31	2.500 080	2.905 031	3.373 133	4.538 039	6.088 101	8.145 113
32	2.575 083	3.006 708	3.508 059	4.764 941	6.453 387	8.715 271
33	2.652 335	3.111 942	3.648 381	5.003 189	6.840 590	9.325 340
34	2.731 905	3.220 860	3.794 316	5.253 348	7.251 025	9.978 114
35	2.813 862	3.333 590	3.946 089	5.516 015	7.686 087	10.676 581
36	2.890 278	3.450 266	4.103 933	5.791 816	8.147 252	11.423 942
37	2.985 227	3.571 025	4.268 090	6.081 407	8.636 087	12.223 618
38	3.074 783	3.696 011	4.438 813	6.385 477	9.154 252	13.079 271
39	3.167 027	3.825 372	4.616 366	6.704 751	9.703 507	13.994 820
40	3.262 038	3.959 260	4.801 021	7.039 989	10.285 718	14.974 458
41	3.359 899	4.097 834	4.993 061	7.391 988	10.902 861	16.022 670
42	3.460 696	4.241 258	5.192 784	7.761 588	11.557 033	17.144 257
43	3.564 517	4.389 702	5.400 495	8.149 667	12.250 455	18.344 355
44	3.671 452	4.543 342	5.616 515	8.557 150	12.985 482	19.628 460
45	3.781 596	4.702 358	5.841 176	8.985 008	13.764 611	21.002 452
46	3.895 044	4.866 941	6.074 823	9.434 258	14.590 487	22.472 623
47	4.011 895	5.037 284	6.317 816	9.905 971	15.465 917	24.045 707
48	4.132 252	5.213 589	6.570 528	10.401 270	16.393 872	25.728 907
49	4.256 219	5.396 065	6.833 849	10.921 333	17.377 504	27.529 930
50	4.383 906	5.584 927	7.106 683	11.467 400	18.420 154	29.457 025

NOTE.—If each of the numbers in the table be diminished by 1, the remainder will denote the interest of $ 1, instead of its amount.

Ex. 1. What is the compound interest of $ 360 for 5 years 6 months and 24 days? Ans. $ 138.14.

OPERATION.

Amount of $ 1 for 5 years,	$ 1.338226
Principal,	360
	80293560
	40146780
Amount of $ 360 for 5 years,	481.761360
Amount of $ 1 for 6mo. 24d.,	1.034
	1927045440
	1445284080
	481761360
Amount of $ 360 for 5y. 6mo. 24d.,	498.141246240
Principal,	360.
Comp. int. of $ 360 for 5y. 6mo. 24d.,	$ 138.14 Ans.

We find the amount of $ 1 for 5 years in the table, and, multiplying it and the number denoting the given principal together, obtain the amount of the $ 360 for 5 years. On this amount as a new principal we find the amount for the remaining 6 months and 24 days, by multiplying by the number denoting the amount of $ 1 for the same time. From the last amount subtracting the original principal, we have left the compound interest required. Hence,

Multiply the amount of $ 1 for the given time and rate, as found in the table, by the number denoting the given principal. The product will be the required amount, from which subtract the given principal, and the remainder will be the COMPOUND INTEREST.

NOTE. — When the given time includes not only the regular periods at which interest becomes due, but also a partial period, as a succession of periods of a year each, followed by one containing months or days, or both, after finding the amount for the regular periods, multiply that amount by the amount of $ 1 for the remaining time or partial period, and the product will be the required amount for the given time. In like manner, when the number of successive periods exceeds the limits of the table, make the computations for a convenient length of time by means of the table, and on the amount thus found make another computation by means of the table, and so on.

In making computation for a succession of periods shorter or longer than one year each, use the numbers in the table the same as if the periods were those of one year each.

EXAMPLES.

2. What is the compound interest of $ 1200 for 11 years at 7 per cent.? Ans. $ 1325.822.

3. What is the compound interest of $ 300 for 10 years 7 months and 15 days? Ans. $ 257.401.

4. What is the compound interest of $ 5 for 50 years at 7 per cent.? Ans. $ 142.285.

5. What is the amount of $ 480 for 40 years, at compound interest? Ans. $ 4937.144.

6. What is the compound interest of $ 40 for 4 years, at 7 per cent.? Ans. $ 12.431.

7. What is the compound interest of $ 100 for 100 years? Ans. $ 33830.20.

8. What is the difference between the simple and the compound interest of $ 1000 for 33 years and 4 months?

9. To what sum will $ 50, deposited in a savings bank, amount, at compound interest for 21 years, at 3 per cent., payable semiannually? Ans. $ 173.034.

(10.) $ 100. *Boston, September* 25, 1853.

For value received, I promise to pay J. D. Forster, or order, on demand, one hundred dollars, with interest, after six months.

ALLEN T. DAWES.

On this note are the following indorsements: — June 11, 1854, received fifty dollars; September 25, 1854, received fifty dollars.

What was due, reckoning at compound interest, August 25, 1855? Ans. $ 2.247.

(11.) $ 1000. *St. Paul, January* 1, 1850.

For value received, I promise to pay Stephen Howe, or bearer, on demand, one thousand dollars, with interest at 7 *per cent.*

WILSON GOODHUE.

Indorsements: — June 10, 1850, seventy dollars; September 25, 1851, eighty dollars; July 4, 1852, one hundred dollars; November 11, 1853, thirty dollars; June 5, 1854, fifty dollars.

At 7 per cent. compound interest, what remains due April 1, 1855? Ans. $ 1022.34.

378. To find the PRINCIPAL, the compound interest, the time, and the rate being given.

Ex. 1. What principal at 6 per cent. compound interest will produce $ 2370 in 10 years? Ans. $ 3000.

OPERATION.

$2370 ÷ .790 = $3000 Ans.

We find the compound interest of $1 for the given time, and at the given rate; and proceed as in like cases in simple interest (Art. 359).

RULE. — *Divide the given compound interest by the compound interest of* $1 *for the given time at the given rate.*

EXAMPLES.

2. What principal, at 7 per cent. compound interest, will produce $205.90 in 6 years and 6 months? Ans. $372.16.

3. What sum of money, at compound interest, will produce $1026.54 in 3 years 2 months and 12 days? Ans. $5000.

4. What sum of money must be invested at compound interest at a semiannual rate of $3\frac{1}{2}$ per cent. to produce $857.25 in $15\frac{1}{2}$ years? Ans. $450.

379.° To find the RATE PER CENT., the principal, the interest, or the amount, and the time being given.

Ex. 1. At what rate per cent. must $500 be at compound interest to become $703.55 in 7 years? Ans. 5 per cent.

OPERATION.

$703.55 ÷ 500 = $1.4071, which for 7 years, in the table, denotes a rate of 5 per cent.

Since $500 becomes $703.55 in 7 years at the required rate, $1 in the same time at the same rate will amount to $\frac{1}{500}$ as much, or $1.4071. Corresponding to this amount of $1 for the given time, we find in the table (Art. 377) 5 per cent., the rate required.

RULE. — *Divide the amount by the principal, and the quotient will be the amount of* $1 *for the given time and the required rate; and in the table, over this amount, may be found the rate per cent. required.*

NOTE. — If the given time contains a part over an exact number of periods, look in the table, against the number denoting the whole periods or years in the given time, for that amount of $1 which comes the nearest to the one found by dividing. Then see if the approximate amount, increased by its rate of interest for the fractional period, will equal the other amount; if so, the rate corresponding to the approximate amount will be the rate per cent. required; if not, the rate of the approximate amount will be as much greater or smaller than the required rate, as the interest added to the approximate amount is greater or smaller than that required to produce the amount found by dividing. The rule can only be well applied when the rate per cent. sought is within the limits of the table.

EXAMPLES.

2. At what rate per cent. will $ 400 amount to $ 640.405, at compound interest, in 12 years? Ans. 4 per cent.

3. At what rate per cent. must $ 2500 be loaned, to produce $ 2096.147 compound interest, in 9 years? Ans. 7 per cent.

4. At what rate per cent. will any sum of money double itself at compound interest in $11\frac{89}{100}$ years? Ans. 6 per cent.

5. At what rate per cent. will $ 10,000 amount to $ 31479.70 in 19 years and $8\frac{3}{4}$ months? Ans. 6 per cent.

380.° To find the TIME, the principal, the compound interest, and the rate per cent. being given.

Ex. 1. In what time will $ 500, at 7 per cent. compound interest, amount to $ 655.398? Ans. 4 years.

OPERATION.

$ 655.398 ÷ 500 = $ 1.310796, which, at the given rate in the table, denotes 4 years' time.

Since $ 500 amounts to $ 655.398 at 7 per cent. in the required time, $ 1 at the same rate, in the same time, will amount to $\frac{1}{500}$ as much, or $ 1.310796. Corresponding to this amount of $ 1 at the given rate, we find in the table (Art. 377) 4 years, the time required.

RULE. — *Divide the amount by the principal, and the quotient will be the amount of $ 1 at the given rate for the required time; and in the table, against this amount, may be found the time required.*

NOTE. — If the required time cannot be found exactly in the table, the number against that amount of $ 1 which under the given rate is next less than the amount found by dividing, will denote the whole periods or years. Then, find the fractional period or part of a year, by dividing 1 whole period or year by the ratio of the difference between the amount corresponding to the whole periods or years and that found by dividing to the difference between the former of the amounts and that next larger in the table; and the value of the fraction obtained as the result may be expressed in months or days, or both.

EXAMPLES.

2. In what time will $ 400 amount to $ 640.405 at 4 per cent. compound interest? Ans. 12 years.

3. In what time will $ 6000 amount to $ 9021.78 at 7 per cent. compound interest?

4. In what time will any sum double itself at 5 per cent. compound interest? Ans. 14y. 2mo. 13d.

5. In what time will any sum double itself at 6 per cent. compound interest? Ans. 11y. 10mo. 20+d.

6. A gentleman has deposited \$450, for the benefit of his son, in a savings bank, at compound interest at a semiannual rate of $3\frac{1}{2}$ per cent. He is to receive the amount as soon as it becomes \$$1781.66\frac{1}{2}$. Allowing that the deposit was made when the son was 1 year old, what will be his age when he can come in possession of the money? Ans. 21 years.

DISCOUNT AND PRESENT WORTH.

381. DISCOUNT is an allowance or deduction made for paying money before it is due.

382. The PRESENT WORTH is the amount of ready money that will satisfy a debt before it is due. It is equivalent to a principal which, being put at interest, will amount to the debt at the time of its becoming payable. Thus, \$100 is the present worth of \$106 due one year hence at 6 per cent.; for \$100 at 6 per cent. will amount to \$106 in that time; and \$6 is the discount.

383. In discount, the rate per cent., the time, and the sum on which the discount is made, are given, to find the *present worth*, which corresponds precisely to the rate per cent., the time and the amount being given, in either simple or compound interest, to find the *principal*.

384. The interest or percentage of any sum cannot properly be taken for the *true* discount; for we have seen (Art. 382) that the *interest* for one year is the fractional part of the sum at interest, denoted by the rate for the numerator, and 100 for the denominator; and the *discount* for one year is the fractional part of the sum on which discount is to be made, denoted by the rate for the numerator, and 100 plus the rate for the de-

nominator. Thus, if the rate of interest is 6 per cent., the *interest* for one year is $\frac{6}{100}$ of the sum at interest; but if the rate per cent. of discount is 6, the *discount* for one year is $\frac{6}{106}$ of the sum on which discount is made.

385. Business men, however, often deduct, or "take off," from the face of a bill or note due at some future time, a greater percentage than the interest would be for the given time at the given rate. Therefore, the *true* present worth and discount are not obtained by that method, but only a NOMINAL PRESENT WORTH and a NOMINAL DISCOUNT. The true discount is equal to the interest on the true present worth of the debt, while the nominal discount is equal to the interest on the *face* of the debt.

386. To find the true present worth of any sum, and the discount, for any time, at any rate per cent.

Ex. 1. What is the present worth of \$12.72, due one year hence, discounting at 6 per cent.? What is the discount?

Ans. \$12 present worth; \$0.72 discount.

OPERATION.

Amount of \$1,	1.06) \$12.72 (	\$12, Present worth.	
	106		
	212	\$12.72, Given sum.	
	212	12.00, Present worth.	
		\$0.72, Discount.	

Since \$1 is the present worth of \$1.06 due one year hence, at 6 per cent., it is evident that the present worth of \$12.72 must be as many dollars as \$1.06 is contained times in \$12.72. We thus find the present worth to be \$12, which, subtracted from the given sum, gives \$0.72 as the discount.

RULE. — *Divide the given sum by the amount of* \$1 *for the given time and rate, and the quotient will be the* PRESENT WORTH.

From the given sum subtract the present worth, and the remainder will be the DISCOUNT.

EXAMPLES.

2. What is the discount on \$802.50, at 7 per cent., due one year hence? Ans. \$52.50.

3. What is the present worth of \$117.60, due one year hence, at 12 per cent.?

4. What is the present worth of $ 769.60, due 3 years and 5 months hence? Ans. $ 638.672.

5. What is the present worth of $ 678.75, due 3 years 7 months hence, at $7\frac{1}{2}$ per cent.? Ans. $ 534.975.

6. What is the discount on $ 600, due 5 years hence, at 5 per cent.?

7. A merchant has given two notes; the first for $ 79.87, to be paid January 21, 1856; the second for $ 87.75, to be paid December 17, 1856. How much ready money will discharge both notes February 10, 1855? Ans. $ 154.545.

8. C. Gardner owes Samuel Hall as follows: $ 365.87, to be paid December 19, 1855; $ 161.15, to be paid July 16, 1856; $ 112.50, to be paid June 23, 1854; $ 96.81, to be paid April 19, 1858. What should Hall receive as an equivalent, January 1, 1854? Ans. $ 653.40.

9. What is the present worth of $ 67.25 due 3 years hence?

10. What is the present worth of $ 80.095, due 3 years hence at compound interest? Ans. $ 67.25.

11. What is the discount on $ 110.364 due 5 years hence, at 7 per cent. compound interest? Ans. $ 31.677.

387. To find a nominal present worth of any sum due at some future time, and the discount on the same at a given rate, *reckon the interest on the face of the debt for the given time and rate, and the same will be the nominal discount; and this discount subtracted from the face of the note will give the nominal present worth.*

EXAMPLES.

1. I have bought of Paine and Woodard a bill of goods amounting to $ 960, on six months, but for ready money they take off from the face of the bill, for the time, 5 per cent. What was the amount paid? Ans. $ 912.

2. How much more is the nominal than the true discount on $ 5000 due one year hence, at 7 per cent.?

3. When money is worth 6 per cent. a year, how much may be gained by hiring money to pay $ 4440 due 6 months hence, allowing the present worth of this debt to be reckoned by *deducting* the nominal discount? Ans. $ 3.996.

BANKING.

388. BANKING is the general business transacted at banks.

A bank is a joint-stock company, established for the purpose of receiving deposits, loaning money, dealing in exchange, or issuing bank-notes or bills, as a circulating medium, redeemable in specie at its place of business.

The *capital* of a bank is the money paid in by its stockholders, as the basis of business.

The affairs of a bank are usually managed by a *board of directors* chosen by the stockholders, and the *principal officers* are a *president*, a *cashier*, and one or more *tellers*.

NOTE. — The president and cashier sign the bills issued; the cashier superintends the bank accounts; and the tellers receive and pay out money. A *bank check* is an order drawn on the cashier for money.

389. Bank discount is the simple interest of a note, draft, or bill of exchange, deducted from it in advance, or before it becomes due.

The interest is computed, not only for the specified time, but for three days additional, called *days of grace*.

The legal rate of discount is usually the same as the legal rate of interest; and the difference between *bank discount* and *true discount* is the same as the difference between interest and true discount (Art. 384).

390. A note is said to be *discounted* at a bank, when it is received as security for the money that is paid for it, after deducting the interest for the time until it becomes due.

The money paid for a note is called its *avails*, *proceeds*, or *present worth*.

391. A note, though nominally due at the time specified in it, is not legally due till the days of grace have been counted.

The time a note has to run is counted in days from after the day of its being discounted to the day of its becoming legally due.

392. To find the bank discount and the avails or proceeds of a note or bill for any time or rate per cent.

Ex. 1. What is the discount on a note for $1000 having 63 days to run, discounted at a bank at 6 per cent.? How much are the proceeds?

Ans. Discount, $10.50; proceeds, $989.50.

OPERATION.

Sum discounted,	$1000.00
$\frac{1}{100}$ of sum = int. for 60d.	10.00
$\frac{1}{20}$ of int. for 60d. = int. for 3d.	.50
Bank discount,	$10.50
Proceeds, or present worth,	$989.50

We find the interest on the sum discounted as in Art. 354, and this interest is the bank discount (Art. 389), which subtracted from the sum discounted gives the proceeds or present worth. Hence,

Find the interest on the note, or sum discounted, for the given rate and time, including THREE *days of grace, and this interest will be the discount.*

Subtract the discount from the face of the note, or sum discounted, and the remainder will be the proceeds, or present worth.

Examples.

2. What is the bank discount on a note for $7800 on 90 days' time? Ans. $120.90.

3. What is the bank discount on a note for $1200, payable in 60 days, at 7 per cent.? Ans. $14.70.

4. What is the bank discount on $8000, payable in 60 days? What are the proceeds?

Ans. Discount, $84; proceeds, $7916.

5. How much money should be received on a note for $760, payable in 5 months, discounted at a bank?

6. A merchant sold a cargo of hemp for $7860, for which he received a note payable in 6 months. How much money will he receive at a bank for this note? Ans. $7620.27.

7. If the following note was discounted April 3, 1857, how long had it to run, and what were the proceeds?

160\frac{40}{100}$. *Boston, December* 3, 1856.

Six months after date, for value received, we promise to pay Robert S. Davis & Co., or order, one hundred and sixty $\frac{40}{100}$ *dollars, at the Merchants' Bank.*

HALLETT, OSGOOD, & CO.

Ans. 63 days; proceeds, $158.72.

8. Required the time when the following note will become legally due, and the bank discount, provided it was discounted May 16, 1856, the rate being 7 per cent.

$ 890$\frac{50}{100}$. *Chicago, April* 16, 1856.

One hundred and twenty days after date, I promise to pay to the order of Keen and Lee eight hundred and ninety $\frac{50}{100}$ dollars, at the Marine Bank. THOMAS L. COOK.

Ans. August 17, 1856; discount, $ 16.103.

9. Required the legal time of maturity of the following note, and the proceeds, it having been discounted June 11, 1856.

$ 1340. *Philadelphia, May* 1, 1856.

For value received, ninety days after date, we promise to pay J. B. Lippincott & Co., or order, one thousand three hundred and forty dollars, at the Girard Bank, without defalcation.

JOHNSON & POLLOCK.

10. The following note was discounted, at 2 per cent. a month, July 5, 1857; how long had it to run, and what were the proceeds?

$ 9000. *New York, June* 19, 1857.

Two months after date, for value received, I promise to pay to the order of Joseph Appleton, nine thousand dollars, at the Manhattan Bank. L. T. ROBERTS.

Ans. 48 days; proceeds, $ 8712.00.

393. To find the amount for which a note must be given that the proceeds shall be a specified sum.

Ex. 1. For what amount, payable in 60 days, must a note be given to a bank discounting at 6 per cent., to obtain $ 989.50?

Ans. $ 1000.

OPERATION.

	$ 1.0000
Interest of $ 1 for 63 days,	.0105
Proceeds of $ 1,	$ 0.9895

$ 989.50 ÷ .9895 = $ 1000.

Since $ 0.9895 of proceeds requires $ 1 to be discounted, $ 989.50 will require as many dollars as $ 0.9895 is contained times in $ 989.50, or $ 1000. Hence,

Divide the given sum by the proceeds of $ 1 for the given time and rate of bank discount, including THREE *days of grace, and the quotient will be the required amount.*

EXAMPLES.

2. What sum, payable in 90 days, if discounted at 7 per cent. bank discount, will produce $ 680? Ans. $ 692.52.

3. What must be the face of a note, which, when discounted at a bank for 120 days, shall give as its proceeds $ 540.50?

Ans. $ 551.81.

4. For what amount must a note be given, in order that the proceeds of the note discounted, when having 6 months to run, shall be $ 1938?

5. The avails of a note, having 4 months to run, discounted at a bank, were $ 1631.60; what was the face of the note?

Ans. $ 1665.74.

6. If a gentleman wishes to obtain $ 1500, for what sum must he give his note payable in 30 days, allowing it is to be discounted at 1 per cent. a month? Ans. $ 1516.68.

394. To find the rate of interest corresponding to a given rate of bank discount.

Ex. 1. What rate of interest is paid when a note payable in 60 days is discounted at 6 per cent.?

Ans. $6\frac{126}{1979}$ per cent.

OPERATION.

$.06 \div .9895 = .06\frac{126}{1979}$

Every $ 1 discounted for the given time and rate yields as its proceeds $ 0.9895. Then, if $ 1 in the given time yield a certain interest at 6 per cent., $ 0.9895 in the same time will yield the same interest, at as many per cent. as the given rate, .06, contains times .9895.

RULE. — *Divide the given rate per cent., expressed decimally, by the number denoting the proceeds of $ 1 for the given time and rate. The quotient will be the rate of interest required.*

EXAMPLES.

2. What rate of interest is paid when a note payable in 30 days is discounted at 6 per cent.? Ans $6\frac{66}{1989}$ per cent.

3. What rate of interest is paid when a note payable in 90 days is discounted at 6 per cent.? Ans. $6\frac{186}{1969}$ per cent.

4. A note payable in 4 months is discounted at 2 per cent. a month; what rate of interest is paid?

5. When a note, payable in 6 months, is discounted at 7 per

cent., to what rate of interest does the bank discount correspond?

6. When a note payable in one year, without grace, is discounted, to what rate per cent. of interest does the bank discount correspond? Ans. $6\frac{18}{47}$ per cent.

395. To find the rate of bank discount corresponding to a given rate of interest.

Ex. 1. At what rate of bank discount must a note, payable in 60 days, be discounted, to produce 6 per cent. interest?

Ans. $5\frac{1895}{2021}$ per cent.

OPERATION.

$.06 \div 1.0105 = .05\frac{1895}{2021}$.

Since bank discount is reckoned on an amount which equals the proceeds plus the interest on the proceeds for the given time and rate (Art. 389), the bank discount for every \$ 1 of proceeds is equivalent to the interest on \$ 1 + .0105 = \$ 1.0105. Then, if \$ 1 yield a certain interest in a given time at 6 per cent. per annum, \$ 1.0105 must yield the same interest in the same time, at as many per cent. per annum as the given rate, .06, contains times 1.0105.

RULE. — *Divide the given rate per cent., expressed decimally, by the number denoting the amount of \$ 1 for the given time and rate. The quotient will be the rate of bank discount required.*

EXAMPLES.

2. At what rate must a note, payable in 30 days, be discounted, to produce 6 per cent. interest?

Ans. $5\frac{1945}{2011}$ per cent.

3. At what rate must a note, running 60 days, be discounted, to yield 2 per cent. a month interest? Ans. $23\frac{17}{521}$ per cent.

4. At what rate must a note, running 90 days, be discounted, to produce 6 per cent. interest?

5. At what rate must a note, running 120 days, be discounted, to produce 8 per cent. interest?

6. At what rate must a note, running 6 months, be discounted, to produce 7 per cent. interest?

7. At what rate must a note, payable 1 year hence, without grace, be discounted, to produce 6 per cent. interest?

8. What rate of bank discount on a note, payable 8 years and 4 months hence, without grace, corresponds to 5 per cent. interest? Ans. $3\frac{9}{17}$ per cent.

MISCELLANEOUS EXAMPLES.

1. The gold coinage of the United States contains 9 parts of pure gold to 1 part of alloy, and the alloy is 1 part copper to 1 part silver. What per cent. of the whole is each metal?

Ans. Gold 90 per cent.; silver 5 per cent.; copper 5 per cent.

2. How much grain must be sent to the miller that a bushel of meal may be returned, the miller taking $\frac{1}{16}$ part for toll? Ans. $34\frac{2}{15}$qts.

3. If a gentleman, possessing $ 25000, has a net yearly income of 4 per cent. of that sum, how much may he spend each year in order that his expenses shall just equal his income?

4. The half-dollar of coinage previous to 1853 contains $206\frac{1}{4}$ grains of standard silver, and that of present coinage contains 192 grains; what per cent. more of standard silver does the one contain than the other? Ans. $7\frac{27}{64}$.

5. My horse is worth 50 per cent. more than my buggy; how many per cent. is the buggy worth less than the horse?

Ans. $33\frac{1}{3}$.

6. How many years longer will it take $ 10 to become $ 20 at 5 per cent. than at 6 per cent. simple interest?

Ans. $3\frac{1}{3}$ years.

7. What is the present worth of $ 500 due 4 years hence, at 5 per cent. compound interest? Ans. $ 411.351.

8. Bought cloth at $ 5.00 per yard. What must be "the asking price," in order to *fall* on it 10 per cent., and still make 10 per cent. on the purchase? Ans. $ $6.11\frac{1}{9}$.

9. A merchant sold a cargo of hemp for $ 7860, for which he received a note payable in 6 months. How much money will he receive for the note at a bank? Ans. $ 7620.27.

10. What is the difference between the true discount and that taken by banks on $ 1500 due one year hence without grace? Ans. $ $5.09\frac{23}{53}$.

11. When a note payable in 60 days is discounted at the rate of 2 per cent. a month, what rate of interest is bank discount equal to? Ans. $25\frac{25}{479}$ per cent.

12. A 45 day note discounted at $1\frac{1}{2}$ per cent. a month yielded a bank discount of $ 36.40. What was the face of the note?

Ans. $ $1617.77\frac{7}{9}$.

13. At what rate per cent. of bank discount should a 30 day note be discounted, that interest may be received at the rate of 12 per cent.? Ans. $11\frac{283}{337}$.

14. How much more can a bank make in 693 days with $ 50000 by discounting notes on 60 days' time, than by discounting those on 30 days, the rate of discount being 6 per cent., and the profits in both cases to be retained in the bank till the end of the time?

15. A merchant bought 450 quintals of fish at $ 3.50 cash, and sold them immediately for $ 4.00 on 6 months' credit, for which he received a note. If he should get this note discounted at a bank, what will he gain on the fish? Ans. $ 170.10.

16. A bank by discounting a note at 6 per cent. receives for its money a discount equivalent to $6\frac{1}{2}$ per cent. interest. How long must the note have been discounted before it was due?

Ans. 1yr. 3mo. 12d.

STOCKS.

396. STOCKS is a general name given to government bonds, and to money capital invested in corporations.

The capital of banks, and of insurance, railroad, manufacturing, mining, and like companies, is usually divided into equal *shares*, the market value of which is often variable.

397. Stocks are said to be *at par* when they sell for their original value; *above par*, or at a *premium*, when for more than their original value; *below par*, or at a *discount*, when for less than their original value.

398. The premium and discount on stocks are generally computed at a certain per cent. on the original or nominal value of the shares.

NOTE. — The original value of a share of bank, insurance, railroad, or like stock, is usually $ 100, but sometimes $ 50, and rarely any other sum than one of these.

399. A *dividend* is the interest or profit on stocks, dis-

tributed to the shareholders, and is reckoned on the par or nominal value of the shares.

400. To find the *market value* of stocks when they are at a premium, or at a discount.

Ex. 1. What is the value of 20 shares of bank stock, at 9 per cent. premium, their nominal or par value being \$100 each?

OPERATION.

$$\$100 \times 20 = \$2000; \ \$2000 \times 1.09 = \$2180.$$

Since the par value of 1 share is \$100, that of 20 shares is \$2000; then, as \$1 at 9 per cent. premium equals \$1.09, \$2000 will equal 2000 times as much, or \$2180.

RULE.—*Multiply the par value of the given stock by* 1 *increased by the rate per cent. premium, or by* 1 *decreased by the rate per cent. discount, expressed decimally, and the product will be the value required.*

NOTE.—The difference between the par and market value gives the per cent. of premium or discount.

EXAMPLES.

2. What is the value of \$24360 of stock, at 35 per cent. premium. Ans. \$32886.

3. Sold 15 shares of the Camden and Amboy Railroad, the par value being \$100 per share, at 13 per cent. advance. To what did they amount? Ans. \$1695.

4. What must be paid for 10 shares of the Old Colony and Fall River Railroad, at 85 per cent., the original value being \$100 each?

5. Sold 30 shares, \$100 each, in the Boston Bank, at $8\frac{3}{4}$ per cent. advance. To what did they amount, and how much was the premium?

Ans. Amount, \$3262.50; premium, \$262.50.

6. What must be given for 25 shares of insurance stock, par value being \$50, at 3 per cent. discount? Ans. \$1212.50.

7. What must be paid for 22 shares of the Iron City Manufacturing Stock, par value being \$250, at 95 per cent., and how much is the discount? Ans. \$5225; discount, \$275.

8. What will be the cost of \$50000 of United States government stock, at 17 per cent. advance? Ans. \$58500.

9. Bought $ 19500 of State stocks at 93 per cent., and sold the same at 103 per cent.; how much was gained by the operation? Ans. $ 1950.

401. To find the *par value* of stocks, when they are at a premium, or at a discount.

Ex. 1. Bought Ocean Insurance Company stock, at 7 per cent. premium, for $ 535; what is its par value? Ans. $ 500.

OPERATION.

$ 535 ÷ 1.07 = $ 500, Ans.

Since $ 1 at 7 per cent. premium equals $ 1.07, the par value of the stock must be as many dollars as 535 contains times 1.07, or $ 500.

RULE. — *Divide the market value of the given stock by* 1 *increased by the rate per cent. premium, or by* 1 *decreased by the rate per cent. discount, expressed decimally, and the quotient will be the value required.*

EXAMPLES.

2. Bought Massachusetts State stock, at $3\frac{1}{2}$ per cent. premium, for $ 6210; what is its par value? Ans. $ 6000.

3. Sold 11 shares of Reading Bank, at 5 per cent. premium, for $ 1155; what is the par value of its shares? Ans. $ 100.

4. Bought 41 shares of canal stock at 40 per cent. below par for $ 1230; what is the par value of its shares?

5. Bought 19 shares of bank stock, at a premium of 8 per cent., for $ 2052; what was the amount of premium paid?

6. When government stocks are at 5 per cent. discount, how much par value will $ 16245 purchase, and what is amount of discount? Ans. $ 17100; discount, $ 855.

7. When railroad stock at 15 per cent. advance is selling at $ 57.50 per share, how many shares may be bought for $ 862.50, and what will be the amount of premium?

Ans. 15 shares; $ 112.50 premium.

8. How many State bonds of $ 1000 each, at 12 per cent. discount, can be purchased for $ 7920, and how much may be gained by the operation, if the selling price should afterwards advance to par? Ans. 9 bonds; $ 1080 may be gained.

402. To find the rate of interest to which a dividend on any stock bought at a premium or discount corresponds.

Ex. 1. Received $12\frac{1}{2}$ per cent. dividend on an investment in stocks at 25 per cent. above par; to what rate per cent. interest did it correspond? Ans. 10 per cent.

OPERATION.

$.125 \div 1.25 = .10$, Ans.

Since the stock was bought at 25 per cent. above par, every \$1.25 of investment must represent only \$1 of par value. Then, since every \$1 of par value pays a dividend corresponding to $12\frac{1}{2}$ per cent. interest, every \$1.25 of investment pays as many per cent. expressed decimally, as 1.25 is contained times in .125, or 10 per cent.

RULE. — *Divide the rate per cent. of dividend, expressed decimally, by 1 increased by the rate per cent. premium, or by 1 decreased by the rate per cent. discount, expressed decimally, and the quotient will denote the rate of interest required.*

NOTE. — If it be required to find at what price a stock, paying a certain rate per cent. dividend, should be bought in order that the investment shall pay a given rate of interest, *divide the rate per cent. of dividend, expressed decimally, by the given rate of interest, expressed decimally, and the quotient will be the price required of each \$1 of the given stock.*

EXAMPLES.

2. Received 6 per cent. dividend on factory stock, purchased at 25 per cent. below par. What rate per cent. interest did the investment pay? Ans. 8 per cent.

3. When railroad stock paying 11 per cent. dividend is worth \$110 per share, or \$10 per share above par, to what rate of interest would the income from an investment in its shares correspond? Ans. 10 per cent.

4. How much advance must be paid for stocks paying 12 per cent. dividends, in order that the investment shall pay exactly 8 per cent. interest? Ans. 50 per cent.

5. Which is the better investment, the buying of 9 per cent. stocks at 25 per cent. advance, or 6 per cent. stocks at 25 per cent. discount?

6. At what per cent. discount must government 5 per cent. stock be bought that the investment may yield 7 per cent.? Ans. $28\frac{4}{7}$ per cent.

7. How much more income yearly may be derived from \$20000 invested in 5 per cent. stock bought at 20 per cent. discount, than by letting the same sum at 6 per cent. interest? Ans. \$50.

BROKERAGE AND COMMISSION.

403. Brokerage is the percentage paid to a dealer in money and stocks, called a *broker*, for making exchanges of money, negotiating different kinds of bills of credit, or transacting other like business.

404. Commission is the percentage paid an *agent*, *factor*, or *commission merchant* for buying or selling goods, making collections, or transacting other business.

405. When the person transacting the commission business lives in a foreign country, he is frequently called a *correspondent* or *consignee*.

The goods shipped or forwarded to a consignee to be sold on commission are termed a *consignment*, and the person sending or consigning the same is called the *consignor*.

406. The rate per cent. of brokerage or commission is not regulated by law, but varies in different places, and with the nature of the business transacted.

Brokerage and commission are computed in the same manner.

407. To find the brokerage or commission on any given sum.

Ex. 1. Paid a broker, for exchanging \$ 896 uncurrent bills for par funds, 2 per cent. brokerage. How much was the brokerage? Ans. \$ 17.92.

Since brokerage is a percentage on the given sum, the brokerage on \$ 896 at 2 per cent. will be $\$ 896 \times .02 = \$ 17.92$.

Rule. — *Find the percentage on the given sum at the given rate per cent., and the result will be the brokerage or commission.*

Note. — When the brokerage or commission, and the sum on which it is reckoned, are given, the rate per cent. may be found as in Art. 348.

Examples.

2. My agent in New Orleans has purchased cotton, on my account, to the amount of \$ 18768. What is his commission at $1\frac{3}{4}$ per cent.? Ans. \$ 328.44.

3. I have engaged a broker to purchase for me 12 shares in the New York Central Railroad, at $ 112.25 per share; what is his commission at $\frac{1}{4}$ per cent.? Ans. $ 3.36$\frac{3}{4}$.

4. My agents, Hilton and Marcy of Cincinnati, advise me that they have purchased on my account a cargo of pork, consisting of 700 barrels, at $ 12.25 per barrel; what is their commission at 1$\frac{3}{4}$ per cent.? Ans. $ 150.06$\frac{1}{4}$.

5. What rate per cent. of brokerage does a broker charge who takes $ 50 for investing $ 10000?

6. What is the commission on the sale of 173cwt. of sugar, at $ 8.95 per cwt., at 1$\frac{7}{8}$ per cent.? Ans. $ 29.03$\frac{5}{32}$.

7. My factor at Mobile advises me that he has purchased on my account 37 bales of cotton, at $ 107.75 per bale; what is his commission at $\frac{3}{8}$ per cent.? Ans. $ 14.95$\frac{1}{32}$.

8. A consignee in London writes that he has purchased for his employer goods to the amount of 395£. 15s. 5d.; what is his commission at 2$\frac{1}{4}$ per cent.? Ans. 8£. 18s. 1$\frac{65}{400}$d.

9. Paid G. Willis $ 5.46 for exchanging $ 364 of depreciated currency; what was the rate of brokerage?

Ans. 1$\frac{1}{2}$ per cent.

408. When the given amount includes both the brokerage or commission, and the sum to be invested.

Ex. 1. A gentleman intrusts $ 20050 to a broker in New York City, with instruction, after deducting his brokerage of $\frac{1}{4}$ per cent., to invest the balance in government bonds. What will be the sum invested, and how much will be the brokerage?

OPERATION.

$ 20050 ÷ $ 1.0025 = $ 20000, Investment.
$ 20050 — $ 20000 = $ 50, Brokerage.

Since the broker is entitled to $\frac{1}{4}$ per cent. of the sum he invests, it is evident he requires $ 1.0025 in order to invest $ 1. Hence, the investment he can make will be as many dollars as $ 20050 contains times 1.0025, or $ 20000; which, being subtracted from the amount forwarded him, leaves as his brokerage $ 50.

Rule. — *Divide the given amount by* 1 *increased by the rate per cent. of brokerage or commission, expressed decimally, and the quotient will be the investment.*

Subtract the investment from the given amount, and the remainder will be the brokerage or commission.

1976.00 1.04 1900 1976
104 936 $1900 1976
936

NOTE. — When the brokerage or commission and the rate per cent. of the same are given, the sum on which it is reckoned may be found as in Art. 349.

EXAMPLES.

2. An agent receives $ 1976, which includes the sum he is to lay out in goods and also his commission at 4 per cent. How much of the amount is to be expended for the goods, and how much is his commission?

Ans. $ 1900 for the goods; $ 76 commission.

3. A broker receives $ 8341.50, which includes a sum to be invested in railroad shares at $ 83 each, and his brokerage at $\frac{1}{2}$ per cent. How many shares can he purchase, and how much is his brokerage? Ans. 100 shares; $ 41.50 brokerage.

4. Sent to my agent, John Crowell, Rochester, $ 8960, to purchase a quantity of flour; his commission is 2 per cent. on the purchase, which he is to deduct from the money; what is his commission? Ans. $ 175.68$\frac{32}{51}$.

5. A town has levied a tax of $ 5150, which sum includes the amount voted for the repairs of a bridge and the collector's commission of 3 per cent. How much was voted for the bridge, and how much does the collector receive for his commission? Ans. $ 5000 for the bridge; $ 150 commission.

6. What amount of money has been invested when the broker's charges, at 1$\frac{1}{2}$ per cent. for making the investment, amount to $ 285?

7. A commission merchant purchases for me in New Orleans 34 boxes of sugar, pays for cartage and freight $ 7.50, and his commission is 1$\frac{1}{2}$ per cent. on the amount of purchase, making the whole bill $ 740.83$\frac{3}{4}$. How much was his commission, and, allowing 250 pounds to a box, how much a pound did he pay for the sugar? Ans. $ 10.83$\frac{3}{4}$ commission; $ 0.08$\frac{1}{2}$ per lb.

8. Sent a cargo of flour to Liverpool, which my factor sold for 987£. 18s. 6d. He invested this sum in broadcloths, at 1£. 3s. 8d. per yard. His commission for selling the flour is 2$\frac{1}{4}$ per cent., and for purchasing the broadcloth 1$\frac{7}{8}$ per cent., and he is to receive his commissions, for selling and buying, out of the proceeds of the flour. Required the number of yards of broadcloth that I should receive. Ans. 801$\frac{3576}{57865}$yd.

ACCOUNT OF SALES.

409. AN Account of Sales is an account of goods sold, which a commission merchant or consignee, &c. furnishes to his employer. It contains the quantity and price of the goods disposed of, charges attending the sales, and the net proceeds, or the sum to which the owner, or consignee, is entitled after all charges are deducted.

The names of purchasers are often included in the account of sales, as in Example 1, and sometimes the mark of the box, bale, &c., as in Example 2.

410. The *gross amount* of sales is the sum of all the quantities at the prices given; and *the net proceeds are found by deducting the commission at the given rate, and the other charges, from the gross amount.*

Find the commission and net proceeds required in the following

EXAMPLES.

1. Account of sales of flour received by the steamer Calvert, from Baltimore, sold on account of James Taylor, Ellicott's Mills.

Date.	To whom sold.	Description.	Barrels.	Price.	$
1857.					
Jan. 1.	William Hooper,	Superfine,	100	8.00	
"	Sidney Pope,	Extra Eagle,	50	7.50	
"	Albert Rollo,	" "	25	7.50	
Jan. 3.	S. S. Coe,	Baltimore,	30	7.00	
Feb. 1.	J. C. Hill & Co.,	"	60	7.00	
" 5.	G. T. Sampson,	Extra Eagle,	25	7.50	
					$ 2180.00

CHARGES.

Commission on $ 2180.00, at 2¼ per cent.,	$ 49.05	
Freight and drayage,	41.63	
Advertising,	4.50	
		$ 95.18
Net proceeds to credit of James Taylor,		$ 2084.82

Errors excepted.

Boston, May 15, 1857. NILES, MARVIN, & Co.

2. Sales of goods, made by Haskell, Fargo, & Co., on account of Jones, Boker, & Co., New York.

Date.	Marks.	Description.	Gals.	Price.	$
1857.					
Feb. 7.	A.	4 casks Sperm Oil, Fall,	208	1.50	
"	B. 3.	13 " " " Winter,	650	1.40	
"	Z.	23 " Whale Oil, Crude,	874	.70	
Mar. 3.	X.	20 bbls. Olive Oil,	320	1.20	
"	19.	10 " Turpentine, Spirits,	315	.60	
Apr. 11.	Y. Z.	10 " Varnish,	315	.20	
" 12.	L. L.	5 " Olive Oil,	157½	1.20	
" 12.	4. 10.	2 " Varnish,	63	.20	
					$

CHARGES.

Feb. 19.	Freight, 40 casks @ $ 2.50,	$
Mar. 20.	" 80 bbls. " 2.00,	
Apr. 29.	" 17 " " 1.50,	
		$
	Commission, 3 per cent., on $	
	Cartage, cooperage, &c.,	26.00
	Storage and insurance,	63.24
	Net proceeds due J., B., & Co.,	$

Errors and omissions excepted.

Chicago, August 7, 1857. HASKELL, FARGO, & Co.

PROFIT AND LOSS.

411. PROFIT AND LOSS is the process by which merchants and others estimate their gains or losses in business transactions.

Gains and losses are usually reckoned on the prime or first cost of articles.

412. To find the selling price when the cost and the gain or loss per cent. are given.

Ex. 1. If I buy cloth at $ 4 per yard, for how much per yard must I sell it to gain 25 per cent.? Ans. $ 5.

OPERATION.

$\$ 4 \times 1.25 = \$ 5$, Ans.

Since the selling price is to be a gain on the cost of 25 per cent., evidently it must be 125 per cent. of the cost, or $\$ 4 \times 1.25$.

RULE. — *Multiply the cost price by* 1 *increased by the gain per cent., or by* 1 *decreased by the loss per cent., expressed decimally, and the product will be the selling price required.*

EXAMPLES.

2. If I buy cloth at \$ 5 per yard, for what must I dispose of it per yard to lose 20 per cent.? Ans. \$ 4.

3. Bargained for cheese at \$ 8.50 per cwt. How must it be sold to gain 10 per cent.? Ans. \$ 9.35 per cwt.

4. Molasses having been bought at 42 cents a gallon, and not proving so good as expected, was sold at a loss of 5 per cent. For what was it sold a gallon?

5. Bought a house and lot for \$ 2500; at what price must it be sold to gain 20 per cent.? Ans. \$ 3000.

413. To find the cost when the selling price and the gain or loss per cent. are given.

Ex. 1. If I sell cloth at \$ 5 per yard, and thereby make 25 per cent., what was its first cost? Ans. \$ 4 per yard.

OPERATION.

$\$ 5.00 \div 1.25 = \$ 4$, Ans.

Since the gain is 25 per cent. of the cost, the selling price, \$ 5, is equal to the cost increased by 25 per cent. of the cost, or 1.25 of the cost. Therefore, the cost must be as many dollars as 5 contains times 1.25.

RULE. — *Divide the selling price by* 1 *increased by the gain per cent., or by* 1 *decreased by the loss per cent., expressed decimally, and the quotient will be the cost.*

EXAMPLES.

2. If I dispose of cloth at \$ 4 per yard, and by so doing lose 20 per cent., required the prime cost of the goods? Ans. \$ 5.

3. Sold 10 barrels of flour for \$ 96, and made 20 per cent. What was the prime cost a barrel? Ans. \$ 8.

4. If $27\frac{1}{2}$cwt. of sugar be sold at \$ 12.50 per cwt., and there is gained 17 per cent., what was the prime cost per cwt.?

5. Sold wood at \$ $6.12\frac{1}{2}$ per cord, and by so doing lost $12\frac{1}{2}$ per cent. a cord. What was the original cost per cord? Ans. \$ 7.00.

414. To find the gain or loss per cent. when the cost and selling price are given.

Ex. 1. If I buy cloth at \$ 4, and sell it at \$ 5 per yard, what per cent. do I gain? Ans. 25 per cent.

OPERATION.

\$ 5 — \$ 4 = \$ 1 ; 1.00 ÷ 4 = .25, or 25 per cent., Ans.

Since the difference between the selling price and the prime cost is \$ 1, the gain is $\frac{1}{4}$ of the cost, or, expressed decimally, is .25 of the cost, or 25 per cent. of it.

RULE. — *Divide the number denoting the gain or loss by that denoting the prime cost, and the quotient, expressed decimally, will be the gain or loss per cent. required.*

EXAMPLES.

2. Bought cloth at \$ 7 per yard, and sold it at \$ 6.12$\frac{1}{2}$. What per cent. did I lose? Ans. 12$\frac{1}{2}$ per cent.
3. Bought a chaise for \$ 200, and sold it for \$ 225. What per cent. did I gain? Ans. 12$\frac{1}{2}$ per cent.
4. I sell a house that cost me \$ 2500 at an advance of \$ 500. What per cent. do I gain?
5. Bought 24 yards of cloth for \$ 64.864$\frac{32}{37}$, and sold it at \$ 2.50 per yard. What per cent. is the loss?

Ans. 7$\frac{1}{2}$ per cent.

415. The selling price of goods and the rate per cent. of gain or loss being given, to find what the gain or loss per cent. would be, if sold at another price.

Ex. 1. If I sell cloth at \$ 5 per yard, and thereby gain 25 per cent., what would have been my gain if I had sold it at \$ 7 per yard? Ans. 75 per cent.

OPERATION.

\$ 5.00 ÷ 1.25 = \$ 4 ; \$ 7 — \$ 4 = \$ 3.
3.00 ÷ 4 = .75, or 75 per cent., Ans.

We find the prime cost of the cloth per yard, when sold at \$ 5, by Art. 413, to be \$ 4. We then find what would have been the gain per cent. on the cost, if it had been sold at \$ 7 per yard, by Art. 414, and obtain 75 per cent. as the answer.

RULE. — *Find the prime cost* (*Art.* 413), *and then the gain or loss per cent. on this cost at the proposed selling price.*

EXAMPLES.

2. If I sell cloth at $ 7 per yard, and thereby gain 75 per cent., do I gain or lose if I sell the same at $ 3 per yard?

Ans. Lose 25 per cent.

3. Sold wheat at $ 1.25 per bushel, and lost 15 per cent. What per cent. should I have gained had I sold it for $ 1.647$\frac{1}{17}$ per bushel?

Ans. 12 per cent.

4. Sold wheat at $ 1.647$\frac{1}{17}$ per bushel, and gained 12 per cent. What per cent. should I have lost had I sold it for $ 1.25 per bushel?

5. I sold a horse for $ 75, and by so doing I lost 25 per cent.; whereas, I ought to have gained 30 per cent. How much was he sold under his real value?

Ans. $ 55.00.

6. When tea, sold at a loss of 25 per cent., brings $ 1.25 per pound, what would be the gain or loss should it bring $ 1.40 per pound?

Ans. 16 per cent. loss.

MISCELLANEOUS EXAMPLES.

1. Bought stocks whose par value is $ 100 per share, and sold the same at a premium of 15 per cent., and thereby gained $ 120. How many shares were there?

Ans. 8 shares.

2. Mining stock of a par value of $ 250 per share was bought at 9 per cent. premium, and afterwards sold at a loss of $ 25 per share on the price paid. At what rate per cent. was it sold?

Ans. 1 per cent. discount.

3. How much greater income may be realized yearly from $ 19200 invested in 7 per cent. stocks purchased at 96 per cent., than from the same amount in 5 per cent. stocks purchased at 80 per cent.?

4. The par value of the shares in a certain manufacturing company is $ 250 each, and the regular yearly dividends are $ 15 a share. At what price should the shares be bought that the investment may pay 10 per cent. interest?

Ans. $ 150 each.

5. A broker charges me $1\frac{1}{2}$ per cent. brokerage for purchasing some uncurrent bank-bills at 20 per cent. discount. Of these bills four of \$ 50 each become worthless, but the remainder I dispose of at par, and thus make by the operation \$ 364. What was the amount of the bills? Ans. \$ 3000.

6. A commission merchant sold a consignment of cotton for \$ 5640, and after deducting his commission, and \$ 76.50 for freight, storage, &c., remitted to his consignor \$ 5422.50 as the net proceeds. What per cent. was his commission?

Ans. $2\frac{1}{2}$ per cent.

7. A horse which I bought for 30 per cent. less than his real worth, having become injured, I sold him for 25 per cent. less than what he cost, and thereby lost \$ 55 of his original value. What did I get for the horse? Ans. \$ $60.78\frac{18}{19}$.

8. A watch, which cost me \$ 30, I have sold for \$ 35, on a credit of 8 months. What did I gain by the bargain?

Ans. \$ $3.653\frac{11}{13}$.

9. Bought a hogshead of molasses for \$ 112; but 15 gallon; having leaked out, the remainder was sold at \$ $2.216\frac{2}{3}$ a gallon. What per cent. is the loss? Ans. 5 per cent.

10. A hogshead of molasses was bought for a certain sum; but 15 gallons having leaked out, the remainder was sold at \$ $2.216\frac{2}{3}$ a gallon, and thereby the loss was 5 per cent. on the cost. What was the cost? Ans. \$ 112.

11. Bought 50 barrels of flour at \$ 9.00 per barrel, but a part of it having been damaged, half of it was sold at a loss of 10 per cent., and the remainder at \$ 9.50 per barrel. How much was lost by the operation? Ans. \$ 10.

12. My agent purchases flour for me, which, with his commission at $2\frac{1}{4}$ per cent., cost \$ 6135, and I dispose of it at 20 per cent. advance upon the price he paid. What did I make by the operation, allowing freight and storage to have been \$ 31.63?

13. I have remitted to my correspondent a certain sum of money, which he invested in iron; and having reserved to himself $2\frac{1}{2}$ per cent. on the purchase, which amounted to \$ 90, he purchased the iron at \$ 95 per ton. Required the sum remitted, and the quantity of iron purchased.

Ars. { Sum remitted, \$ 3690.
Iron purchased, 37T. 17cwt. 3qr. $14\frac{8}{19}$lb.

PARTNERSHIP, OR COMPANY BUSINESS.

416. PARTNERSHIP is the association of two or more persons in business, with an agreement to share the profits and losses in proportion to the amount of the capital stock, or the value of the labor and experience of each.

Partners are the persons associated in business.

Company, or *Firm*, is the general name of the business association.

Capital, or *Joint Stock*, is the money or property invested in the company or firm.

Dividend is the profit or loss on the shares of the capital, or joint stock.

NOTE. — Partnership is an application of the principle of distributive or partitive proportion (Art. 339).

417. To find each partner's share of the profit or loss, when there is no regard to time.

Ex. 1. Three men, A, B, and C, enter into partnership for two years, with a capital of \$ 1080. A puts in \$ 240, B \$ 360, and C \$ 480. They gain \$ 54. What is each man's part of the gain?

OPERATION.

A's stock, \$ 240	\$ 240 × .05 =	\$ 12, A's gain.
B's " 360	360 × .05 =	18, B's gain.
C's " 480	480 × .05 =	24, C's gain.
Entire stock, \$ 1080	Proof,	\$ 54, entire gain.

\$ 54.00 ÷ 1080 = \$ 0.05, gain on \$ 1.

Since the entire stock is \$ 1080 and the entire gain \$ 54, the gain on every \$ 1 of stock will be as many dollars as 54 contains times 1080, or \$ 0.05 on every \$ 1 of stock. Then, each man's stock multiplied by .05 gives his part of the entire gain. The same result also may be obtained, as follows,

BY PROPORTION.

\$ 1080 : \$ 240 :: \$ 54 : \$ 12, A's gain.
\$ 1080 : \$ 360 :: \$ 54 : \$ 18, B's gain.
\$ 1080 : \$ 480 :: \$ 54 : \$ 24, C's gain.
Proof, \$ 54, entire gain.

RULE. — *The entire gain or loss, divided by the number denoting the*

entire stock, will give the gain or loss on each dollar of stock; and each partner's stock, multiplied by the number denoting the gain or loss on \$ 1, will give his share of the entire gain or loss. Or,

As the whole stock is to each partner's stock, so is the whole gain or loss to each partner's gain or loss.

EXAMPLES.

2. Jones, Weston, and Sprague traded in company, with a capital of \$ 10000; Jones put in \$ 3000, Weston \$ 2000, and Sprague \$ 5000; they gained \$ 4000. What was each man's part of the gain?

Ans. Jones's part, \$ 1200; Weston's part, \$ 800; Sprague's part, \$ 2000.

3. Two merchants, C and D, engaged in trade; C put in \$ 6780, and D put in \$ 12000; they gain \$ 2000. What was each man's part?

Ans. C's part, 722.044; D's part, \$ 1277.956.

4. Harvey, Blake, and Horsford entered into partnership with a capital of \$ 11000, of which Harvey put in \$ 2500, Blake \$ 3000, and Horsford \$ 5500; they lost by trading 5 per cent. on their capital. What was each partner's share of the loss?

5. Elliott, Mayhew, and Griswold engaged together in a speculation. Of the money employed Elliott furnished \$ 500; Mayhew \$ 350, and Griswold a cart and two horses; they gained \$ 332.50, of which Griswold's part was \$ 120. What were Elliott's and Mayhew's parts of the gain, and what was the value of Griswold's part of the stock?

Ans. Elliott's gain, \$ 125; Mayhew's gain, \$ 87.50; Griswold's stock, \$ 480.

6. A, B, and C traded in company; A put in \$ 5000, B put in \$ 6500, and C put in \$ 7500; they gain 40 per cent. on their capital, but receive the whole amount of their gains in bills, for which they are obliged to allow a discount of 10 per cent. How much was each man's net gain?

Ans. A's gain, \$ 1800; B's gain, \$ 2340; C's gain, \$ 2700.

7. A, B, C, and D are in partnership, with a joint capital of \$ 40,000; on dividing their profits, it is found that A's share is \$ 2000, B's share \$ 4500, C's share \$ 2500, and D's share \$ 1500. What was each partner's stock?

8. A, B, and C were associated in trade; A's part of the general stock was $ 2000, B's part $ 3000, and C's part $ 7500. On dividing the profits, it was found that A's and B's gain together amounted to $ 1000, which was $ 500 less than C's gain. What was the gain of each?

Ans. A's gain, $ 400; B's gain, $ 600; C's gain, $ 1500.

9. A, B, and C own a ship together, which cost them $ 30000; of which A paid an unknown sum, B paid $1\frac{1}{2}$ as much, and C paid $1\frac{1}{4}$ as much. The profits were 25 per cent. of the cost of the ship. What was each man's gain?

Ans. A's gain, $ 2000; B's gain, $ 3000; C's gain, $ 2500.

10. Walker, Edwards, and Armstrong are partners, whose respective shares of joint stock are as the fractions $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$. They gain $ 50000. If, on dividing the profits, Armstrong relinquishes his part of the gain, how much will each of the others receive?

Ans. Walker, $ 28571.42$\frac{6}{7}$; Edwards, $ 21428.57$\frac{1}{7}$.

418. To find each partner's share of the profit or loss, when the stock is employed for different periods of time.

Ex. 1. A and B are associated in trade. A has furnished of the joint stock $ 420 for 5 months, and B has furnished $ 350 for 8 months; their net profits are $ 84. What is each man's share of the gain? Ans. A's share, $ 36; B's, $ 48.

OPERATION.

$\$420 \times 5 = \$2100$ $\quad \$2100 \times .01\frac{5}{7} = \$36$, A's gain.
$\$350 \times 8 = \underline{\$2800}$ $\quad \$2800 \times .01\frac{5}{7} = \underline{\$48}$, B's gain.
$\$4900$ $\quad$ Proof, $ 84, entire "

It is evident that $ 420 for 5 months is the same as $\$420 \times 5 = \$2100$ for 1 month, since $ 2100 would gain as much in 1 month as $ 420 in 5 months; and $ 350 for 8 months is the same as $\$350 \times 8 = \$2800$, for 1 month. The question then is the same as if A had furnished $ 2100 and B $ 2800 for equal times. Then, if $\$2100 + \$2800 = \$4900$, gain $ 84, $ 1 will gain $\frac{1}{4900}$ of $\$84 = \$0.01\frac{5}{7}$, and $\$2100 \times .01\frac{5}{7} = \$36$, A's gain; $\$2800 \times .01\frac{5}{7} = \$48$, B's gain. The same results may be obtained, as follows,

BY PROPORTION.

$\$4900 : \$2100 :: \$84 : \$36$, A's gain.
$\$4900 : \$2800 :: \$84 : \underline{\$48}$, B's gain.

Proof, $ 84, entire gain.

RULE. — *Multiply each partner's stock by the time it was in trade, and divide the entire gain or loss by the sum of the several products; by the quotient multiply the product of each partner's stock and time, and the result will be his share of the gain or loss.* Or,

Multiply each partner's stock by the time it was in trade; then, as the sum of these products is to each product, so is the whole gain or loss to each partner's gain or loss.

EXAMPLES.

2. Goodwin commenced business January 1, with a capital of \$ 3200; May 1, he took Blunt into partnership, with a capital of \$ 4200; and at the end of the year they had gained \$ 240. What was each partner's share of the gain?

Ans. Goodwin's gain, \$ 128; Blunt's gain, \$ 112.

3. Three men hire a pasture in common, for which they are to pay \$ 26.40. A put in 24 oxen for 8 weeks, B put in 18 oxen for 12 weeks, and C put in 12 oxen for 10 weeks. What ought each to pay?

4. Barclay, Hickman, and Oliver are partners. Barclay furnishes of the capital \$ 300 for 5 months, Hickman \$ 400 for 8 months, and Oliver \$ 500 for 3 months; they gain \$ 200. After paying \$ 50 for advertising and \$ 50 for agency, what will be each partner's share of the net profits?

Ans. Barclay's share, \$ 24.19$\frac{11}{31}$; Hickman's, \$ 51.61$\frac{9}{31}$; Oliver's, \$ 24.19$\frac{11}{31}$.

5. A, B, and C engaged in partnership, with a joint capital of \$ 1000, A putting in stock for 7 months, B for 8 months, and C for 12 months. Of the profits A's part was \$ 21; B's, \$ 40; and C's, \$ 24. Required the capital each put in.

Ans. A, \$ 300; B, \$ 500; C, \$ 200.

6. White and Daniels traded in company, with a joint stock of \$ 6300. White's money having been employed 12 months, and Daniel's 8 months, on dividing profits, each had gained exactly the same sum. How much of the capital did each furnish?

7. Three men engage in partnership for 20 months. A at first put into the firm \$ 4000, at the end of four months he put in \$ 500 more, and at the end of 16 months he took out \$ 1000; B at first put in \$ 3000, at the end of 10 months he took out \$ 1500, and at the end of 14 months he put in

$3000; C at first put in $2000, at the end of 6 months he put in $2000 more, at the end of 14 months he put in $2000 more, and at the end of 16 months he took out $1500; they had gained by trade $4420. What is each man's share of the gain?

Ans. A's gain, $1680; B's gain, $1260; C's gain, $1480.

8. Grover and Thorndike are associated in trade, Grover contributing of the capital $12000, and Thorndike $18000. At the end of 6 months they reduce the joint stock $5000, by each withdrawing an equal sum. 3 months afterwards Thorndike withdraws $6000, and Grover $1000. The business proving a losing one, they dissolve copartnership at the end of the year. Required what part of the stock then remaining, which was only $15000, belonged to each of the partners?

Ans. To Grover, 7276.69\frac{93}{103}$; Thorndike, 7723.30\frac{10}{103}$.

9. John Jones, Samuel Eaton, and Joseph Brown formed a partnership, under the firm of Jones, Eaton, & Co., with a capital of $10,000; of which Jones put in $4000, Eaton put in $3500, and Brown put in $2500; but at the end of 6 months Jones withdrew $2000 of his stock, and at the end of 8 months Eaton withdrew $1500 from the firm; but at the end of 10 months Brown added $2000 to his stock. At the end of 2 years they found their gains to be $1041.80. What was the share of each man?

Ans. Jones's gain, 300.51\frac{12}{13}$; Eaton's gain, 300.51\frac{12}{13}$; Brown's gain, 440.76\frac{2}{13}$.

10. James Bradshaw commenced trade, January 1, 1856, with a capital of $10000, and after some time formed a partnership with John Parkman, who contributed to the joint stock $2800. In course of time they admitted into the firm Joseph Delano, with a stock of $3600. On making a settlement, January 1, 1857, it was found that Bradshaw had gained $2250; Parkman, $420; and Delano, $405. How long had Parkman's and Delano's money been employed in trade, and what rate of interest per annum had each of the partners gained on their stock?

Ans. Parkman's, 8 months; Delano's, 6 months. Gained 22½ per cent. interest.

BANKRUPTCY.

419. BANKRUPTCY refers to business failures and inability to meet pecuniary liabilities.

A *bankrupt*, or *insolvent*, is one who fails or becomes unable to pay his debts.

An *assignee* is a person selected to take charge of the property and effects of a bankrupt, to convert the same into cash, and, after deducting the necessary expenses of settling, to divide the net proceeds, as the law requires, among the creditors.

The distribution is generally made *pro rata*, each creditor receiving according to his respective demand, or just claim.

420. To find each creditor's dividend or share of the net proceeds of an insolvent estate.

Ex. 1. A bankrupt owes to A $ 500, to B $ 1200, and to C $ 4300; and the net cash proceeds of his estate amount to only $ 1500. How much does he pay on $ 1, and what dividend does each creditor receive?

Ans. 25 cents on a dollar. A receives $ 125; B, $ 300; and C, $ 1075.

OPERATION.

A's claim,	$ 500	$ 500 × .25 =	$ 125, A's share.
B's "	$ 1200	$ 1200 × .25 =	$ 300, B's share.
C's "	$ 4300	$ 4300 × .25 =	$ 1075, C's share.
Entire claims,	$ 6000	Proof,	$ 1500, entire proc'ds.

$ 1500.00 ÷ 6000 = $ 0.25, or 25 cents on $ 1.

RULE. — *Divide the net proceeds of the insolvent estate by the number denoting the total amount of its indebtedness, to find the sum it pays on each dollar of the debts.*

Multiply each man's claim by the sum the estate pays on a dollar, to obtain each man's dividend.

EXAMPLES.

2. Clarke, Soule, & Co. have failed. Their liabilities are $ 63500, their assets have a cash value of $ 52384, and the expenses of settling are $ 1584. How much can they pay on a dollar, and what dividend should John Dayton receive, whose claim is $ 8361.55? Ans. 80 cents on a dollar; $ 6689.24.

3. A merchant failing in trade owes A $ 600, B $ 760, C $ 840, and D $ 800. The net proceeds of his effects are $ 2275. What dividend does each of his creditors receive?

Ans. A, $ 455; B, $ 576.33$\frac{1}{3}$; C, $ 637; D, $ 606.66$\frac{2}{3}$.

4. J. Bonney owes A $ 400, B $ 300, and C $ 1000. His effects are worth $ 600. What sum can he pay each of his creditors?

5. A manufacturing company becomes insolvent. Its indebtedness amounts to $ 300000. Its assets consist of factory buildings and machinery worth $ 180000, stock worth $ 40000, and bills receivable good for $ 12875. The charges of the court of insolvency and the assignee will amount to 3$\frac{1}{2}$ per cent. on the amount distributed to the creditors. How much will the company pay on a dollar, and what will be Amos Henderson's dividend on a claim of $ 1360.60?

Ans. 75 cents on a dollar; $ 1020.45.

TAXES.

421. A TAX is a sum of money assessed on individuals by government, corporations, societies, districts, &c.

Taxes for government purposes are imposed on property, and in most States on persons.

A *poll* or capitation tax is one without regard to property, on the person of each male citizen liable by law to assessment. A person thus liable is termed a *poll*.

422. Immovable property, such as lands, mills, houses, &c., is called *real estate*. All other property, such as money, notes, mortgages, cattle, shipping, furniture, &c., is called *personal property*.

423. In assessing taxes, it is necessary to have an inventory of taxable property; and, if a levy on the polls is to be included, there should be also a complete list of taxable polls.

424. The method of assessing taxes, though not precisely the same in all the States, is yet in most of them virtually the same.

In some of the States, however, the public schools are supported in whole or in part by *rate bills;* that is, the expenses of the schools, in whole or in part, are apportioned among the inhabitants patronizing the same, according to the number of days of each pupil's attendance.

NOTE. — In Massachusetts one sixth part of the whole sum to be raised is assessed upon the polls; provided the whole poll tax assessed in any one year upon any individual for town, county, and State purposes, except highway taxes, shall not exceed two dollars.

425. To assess a state, county, town, or other tax.

Ex. 1. The inhabitants of a certain town are to be taxed \$4109. The real estate of the town is valued at \$493000, and the personal property at \$177000. There are 506 polls, each of which is to be taxed \$1.50. What is the tax on each dollar of property? What is J. B. Tewksbury's tax, whose real estate is valued at \$3700, and his personal at \$2300, he paying for 2 polls?

OPERATION.

\$1.50 × 506 = \$759, amount assessed on the polls.
\$493000 + \$177000 = \$670000, amount of taxable property.
\$4109 — \$759 = \$3350, amount to be assessed on property.
\$3350 ÷ 670000 = \$.005, tax to be assessed on each dollar.
\$3700 + \$2300 = \$6000, Tewksbury's taxable property.
\$6000 × .005 = \$30, Tewksbury's property tax.
\$1.50 × 2 = \$3, Tewksbury's poll tax.
\$30 + 3 = \$33, amount of Tewksbury's tax.

RULE. — *Multiply the tax on each poll by the number of taxable polls; and the product, subtracted from the whole sum to be raised, will give the sum to be raised on the property.*

The sum to be raised on property, divided by the whole taxable property, will give the sum to be paid on each dollar of property taxed.

Each man's taxable property, multiplied by the number denoting the sum to be paid on \$1, with his poll tax added to the product, will give the amount of his tax.

NOTE. — The operation of assessing taxes may be facilitated by finding the tax on \$2, \$3, &c. at the rate of taxation on \$1, before making the assessment on the inhabitants of the town, &c., and arranging the numbers as in the following

TABLE.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	$0.005	$20	$0.10	$300	$1.50	$4,000	$20.00
2	0.010	30	0.15	400	2.00	5,000	25.00
3	0.015	40	0.20	500	2.50	6,000	30.00
4	0.020	50	0.25	600	3.00	7,000	35.00
5	0.025	60	0.30	700	3.50	8,000	40.00
6	0.030	70	0.35	800	4.00	9,000	45.00
7	0.035	80	0.40	900	4.50	10.000	50.00
8	0.040	90	0.45	1,000	5.00	20,000	100.00
9	0.045	100	0.50	2,000	10.00	30,000	150.00
10	0.050	200	1.00	3,000	15.00	40,000	200.00

EXAMPLES.

2. What is Samuel Forster's tax, by the above table, whose property, real and personal, is valued at $1310, and who pays for 7 polls a tax of $1.50 each? Ans. $17.05.

3. What is the tax of a non-resident, having property in the same town, worth $415.35?

4. A tax of $14018.90 is to be levied on a certain city. The property is valued at $3506300; and there are 3500 polls, each of which is taxed $1 each. What is the assessment on a dollar, and what is A's tax, who has property worth $29010, and pays for 2 polls? Ans. $.003 on $1; A's tax, $89.03.

426. To find what sum must be assessed to yield a given net amount.

Ex. 1. What sum must be assessed to yield a net amount of $6240, the collectors receiving $2\frac{1}{2}$ per cent. commission for collecting? Ans. $6400.

OPERATION.

$\$6240 \div .975 = \$6400.$

Since the commission is $2\frac{1}{2}$ per cent., the net value of each dollar of assessment will be $0.975, and the amount to be assessed to net $6240 will therefore be as many dollars as 6240 contains times .975. Hence,

The given net amount divided by 1 *decreased by the rate of commission, expressed decimally, will give the amount to be assessed.*

EXAMPLES.

2. What sum must be assessed to net $10450, allowing the collector receives 5 per cent. for collecting? Ans. $11000.

3. Allowing the net sum to be raised by a society to be \$ 9700, and the allowance for collection to be 3 per cent., what is the gross amount to be assessed, and how much will be the cost of collection?

Ans. Assessment, \$ 10000; cost of collection, \$ 300.

4. The taxable property in a certain district containing 450 polls is \$ 756000. It is proposed to raise \$ 18000 for building a union school-house. If the poll tax be limited to \$ 1.50 a poll, and the cost of collecting be 3 per cent., what will be the required assessment on a dollar, and how much will be A's tax, who pays for 3 polls, and has property to the amount of \$ 15600? Ans. Tax on \$ 1, \$ .0236; A's tax, \$ 372.66.

427. To apportion school expenses according to each pupil's attendance, or to make out rate bills.

Ex. 1. The expenses of a certain district school are \$ 464, and the aggregate of attendance 9280 days. What is A's tax, who has sent pupils amounting to 157 days?

OPERATION.

$$\$ 464.00 \div 9280 = \$ 0.05$$
$$\$ 0.05 \times 157 = \$ 7.85.$$

Since 9280 days' attendance cost \$ 464, 1 day's attendance will cost as many dollars as 464 contains times 9280, and 157 days will cost 157 times as much as 1 day. Hence,

The whole expenses of the school, less the public money, if any, divided by the number of days' attendance, will give the rate for each day's attendance.

The rate for each day, multiplied by the number of days of attendance of each patron's pupils, will give the amount of his rate bill.

EXAMPLES.

2. If a district expends for a teacher's salary \$ 500, for his board \$ 150, for repairs of school-house \$ 30; the public money being \$ 350, and the whole number of days of attendance 5500, what is the rate per day, and what is A's bill, who sends 2 pupils 60 days each, and 1 pupil 30 days?

Ans. Rate per day, \$ 0.06; A's bill, \$ 9.00.

3. In a certain district the teacher's wages amounted to \$ 150, the fuel cost \$ 18.50, the public money received was \$ 63.50, and the number of days' attendance was 3000. What was A's rate bill, whose pupils attended 121 days; and B's, whose pupils attended 173 days?

GENERAL AVERAGE.

428. GENERAL AVERAGE signifies a contribution ratably made to a general loss by the three great mercantile interests, vessel, freight, and cargo,—when, on account of a common peril, one or more of these has been voluntarily sacrificed, in whole or in part, to effect the preservation of the rest.

Jettison is the term applied to the part of the vessel or cargo cast overboard, or otherwise sacrificed, to save the remainder.

429. *Particular average* means an average to which contribute only "some parts of the property when severed from the whole; as in the case of a boat-load saved from peril by a sacrifice, to which only the boat and its contents contribute." Practically, it hardly needs to be discriminated from general average, since it is adjusted upon its contributory interest by the application of the same principles.

430. To constitute a valid claim to general average, there are three essentials, all of which must be present: 1. *A common peril impending at the time; 2. A voluntary and premeditated sacrifice of some property for the purpose of saving other property; 3. The success of the endeavor.* (Parsons on Mercantile Law.)

431. In adjusting a general average, the property sacrificed, as well as that saved, is regarded as contributory to the general loss. The entire value of the freight, however, is not contributory; $\frac{1}{2}$ of its value in New York, and $\frac{1}{3}$ of its value in other parts of the United States, being reserved for seamen's wages.

The goods, whether lost, injured, or saved, are valued at the price they would have brought in ready money on the vessel's arriving at her port of *destination*, unless the average is adjusted at the port of *lading*, when they are valued at the invoice price.

In compensating for the expenses incurred in repairing damages done to the vessel, only $\frac{2}{3}$ of the cost is allowed, as in general the new material is $\frac{1}{2}$ better than the old.

432. To adjust the general average of losses at sea.

Ex. 1. The ship America sailed on May 16, 1857, for New

Orleans, with an assorted cargo. In consequence of a violent gale in the Gulf of Mexico, the captain was obliged to throw overboard a portion of the cargo, amounting in value to \$ 4465.50, and the necessary repairs of the vessel cost \$ 423. The contributory interests were as follows: vessel, \$ 30000; gross freight, \$ 6225; cargo shipped by R. S. Davis & Co., \$ 3650; by Henry Mason, \$ 6500; by G. T. Sampson, \$ 2000; by J. Francis & Son, \$ 550; by Morton Brothers, \$ 5450; and by Sanborn & Carter, \$ 8500. Of the cargo thrown overboard, there belonged to Henry Mason the value of \$ 3000, and to Morton Brothers the remainder, \$ 1465.50. The cost of detention in port in consequence of repairs was \$ 116.50. How ought the loss to be apportioned among the contributory interests?

OPERATION.

CONTRIBUTORY INTERESTS.		LOSS FOR GENERAL BENEFIT.	
Vessel,	\$ 30000	Thrown overboard,	\$ 4465.50
Cargo,	26650	Cost of detention,	116.50
Freight, less $\frac{1}{3}$,	4150	Repairs, less $\frac{1}{3}$,	282.00
Entire contrib. int.,	\$ 60800	Entire loss,	\$ 4864.00

\$ 4864.00 ÷ 60800 = .08, the loss per cent.

\$ 30000 × .08 = \$ 2400, am't payable by vessel.
4150 × .08 = 332, am't payable by freight.
3650 × .08 = 292, am't payable by R. S. Davis & Co.
6500 × .08 = 520, am't payable by Henry Mason.
2000 × .08 = 160, am't payable by G. T. Sampson.
550 × .08 = 44, am't papable by J. Francis & Son.
5450 × .08 = 436, am't payable by Morton Brothers.
8500 × .08 = 680, am't payable by Sanborn & Carter.

Proof, \$ 4864, entire amount payable.

\$ 2400.00 — \$ 398.50 = \$ 2001.50, balance payable by vessel.
3000.00 — 520.00 = 2480.00, bal. rec'ble by H. Mason.
1465.50 — 436.00 = 1029.50, bal. rec'ble by Morton Bro's.

Since the vessel lost \$ 116.50 + \$ 282, = \$ 398.50, that amount is deducted in finding the net amount the vessel must contribute to the general loss. Henry Mason lost \$ 520; the amount payable by him is therefore made so much less on that account; and Morton Brothers also, having lost \$ 436, have the amount of their contribution lessened by that sum.

RULE. — *Multiply each contributory interest by the loss per cent., and the product will be its contribution to the general loss.*

Ex. 2. The ship Hope, in her passage from Liverpool to New York, was crippled in a storm, in consequence of which the captain had $ 6500 worth of the cargo thrown overboard, and the necessary repairs of the vessel cost $ 1050. The charges for board of seamen, pilotage, and dockage amounted to $ 142. The contributory interests were: vessel, $ 31500; gross amount of freight, $ 4160; cargo shipped by Manning & Brother, $ 2145; by Anderson & Fisk, $ 1460; by Smidt & Huber, $ 960; by Greenwood, Laporte, & Co., $ 670; and by Allermann, Ritter, & Herr, $ 1000. In adjusting the general average in New York, the deduction made from the gross amount of freight on account of seamen's wages was one half. Required the several shares of the general loss.

EQUATION OF PAYMENTS.

433. EQUATION OF PAYMENTS is the process of finding the *average* or *mean* time when the payments of several sums, due at different times, may all be made at one time, without loss either to the debtor or creditor.

434. A strictly accurate method of determining, by *Arithmetic*, the true average time for the payment of more than *two* sums, due at different times, it is believed, has not yet been discovered; and, even when there are *only two* sums, the accurate method is the translation of an *Algebraic Formula.* The true equated time, however, may be readily found by *Algebra.*

Mercantile usage, however, gives its sanction to another method, and one which is not entirely correct, though, when the sums are small and the terms short, the variation from the exact truth is practically of no consequence. But this method, being far the most convenient of application, is adopted among business men.

435. To find the average or mean time of paying several debts due at different dates.

Ex. 1. A owes B $ 19, $ 5 of which is to be paid in 6 months, $ 6 in 7 months, and $ 8 in 10 months. What is the average time of paying the whole? Ans. 8 months.

OPERATION.

$$\begin{array}{lll} 5 \times 6 = 30 & & \\ 6 \times 7 = 42 & & \\ \underline{8} \times 10 = \underline{80} & & \\ 19 & 19)\,152\,(8 \text{ months.} & \\ & \quad\ \underline{152} & \end{array}$$

The interest of $ 5 for 6 months is the same as the interest of $ 1 for 30 months; and of $ 6 for 7 months, the same as of $ 1 for 42 months; and of $ 8 for 10 months, the same as of $ 1 for 80 months. Hence, the interest of all the sums to the time of payment is the same as the interest of $ 1 for 30mo. + 42mo. + 80mo. = 152 months. Now, if $ 1 require 152 months to gain a certain sum, $ 5 + $ 6 + $ 8 = $ 19 will require $\frac{1}{19}$ of 152 months; and 152mo. ÷ 19 = 8 months, the average or mean time for the payment of the whole.

2. Purchased goods of Kendall & White at different times, and on various terms of credit, as by the statement annexed. What is the mean time of payment?

January 1,	a bill amounting to	$ 375.50	on 4 months.
" 20,	" "	168.75	on 5 months.
February 4,	" "	386.25	on 4 months.
March 11,	" "	144.60	on 5 months.
April 7,	" "	386.90	on 3 months.

OPERATION.

Due May 1,	$ 375.50			
" June 20,	168.75	× 50	=	8437.50
" June 4,	386.25	× 34	=	13132.50
" Aug. 11,	144.60	× 102	=	14749.20
" July 7,	386.90	× 67	=	25922.30
	$ 1462.00			62241.50 days.

$$62241.50 \div 1462.00 = 42\tfrac{1675}{2924} \text{ days.}$$

May 1 + 43 days = June 13, Ans.

We first find the time when each of the bills will become due. Then, since it will shorten the operation and not change the result, we *take the first time when any bill becomes due*, instead of its *date*, for the point from which to compute the average time. Now, since May 1 is the period from which the average time is computed, no time will be reckoned on the first bill, but the time for the payment of the second bill extends 50 days beyond May 1, and we multiply its amount by 50. Proceeding in the same manner with the remaining

bills, we find the average time of payment to be 43 days, nearly, from May 1, or on June 13.

RULE. — *Multiply each payment by its own term of credit, and divide the sum of the products by the sum of the payments.*

NOTE 1. — When the date of the average time of payment is required, as in Example 2, *find the time when each of the sums becomes due. Multiply each sum by the number of days intervening between the date of its becoming due and the earliest date on which any sum becomes due. Then proceed as in the rule, and the quotient will be the average time required, in days forward from the date of the earliest sum becoming due.*

NOTE 2. — In the result, it is customary, if there be a fraction of a day less than $\frac{1}{2}$, to reject it; but if more than $\frac{1}{2}$, to reckon it as 1 day.

In practice the work may be somewhat abridged, without varying materially the result, by disregarding, in performing the multiplications, the cents in the several sums, when they are less than 50, and by calling them \$ 1, when more than 50.

When a payment is made at the time of purchase, it has no product, but it must be added with the other products in finding the average time.

NOTE 3. — The method of the rule, or that generally adopted by merchants, as has been intimated (Art. 384), is not perfectly correct. For if I owe a man \$ 200, \$ 100 of which I am to pay *down*, and the other \$ 100 in two years, the equated time for the payment of both sums would be one year. It is evident that, for deferring the payment of the first \$ 100 for one year, I ought to pay the *amount* of \$ 100 for that time, which is \$ 106; but for the \$ 100 which I pay a year *before* it is due, I ought to pay the *present worth* of \$ 100, which is \$ $94.33\frac{51}{53}$, and \$ 106 + \$ $94.33\frac{51}{53}$ = \$ $200.33\frac{51}{53}$; whereas, by the mercantile method of equating payments, I only pay \$ 200.

EXAMPLES.

3. There is owing a merchant \$ 1000; \$ 200 of it is to be paid in 3 months, \$ 300 in 5 months, and the remainder in 10 months. What is the equated time for the payment of the whole sum? Ans. 7mo. 3d.

4. I have bought a farm for \$ 6500; \$ 2000 of which is to be paid down, \$ 500 in one year, and the remainder in 2 years. But if a note for the whole amount had been preferred, in what time would it have become due?

5. A owes B \$ 300, of which \$ 50 is to be paid in 2 months, \$ 100 in 5 months, and the remainder in 8 months. What is the equated time for the payment of the whole sum? Ans. 6 months.

6. I have sold H. W. Hathaway several bills of goods, at different times, and on various terms of credit, as by the following statement. What is the average time for the payment of the whole?

Jan. 1,	a bill amounting to		$ 600,	on 4 months.
Feb. 7,	"	"	370,	on 5 months.
March 15,	"	"	560,	on 4 months.
April 20,	"	"	420,	on 6 months.

Ans. July 11.

7. Purchased goods of J. D. Martin, at different times, and on various terms of credit, as by the statement annexed. What is the equated time of paying for the same?

March 1, 1855,	a bill amounting to		$ 675.25,	on 3 months.
July 4,	"	"	376.18,	on 4 months.
Sept. 25,	"	"	821.75,	on 2 months.
Oct. 1,	"	"	961.25,	on 8 months.
Jan. 1, 1856,	"	"	144.50,	on 3 months.
Feb. 10,	"	"	811.30,	on 6 months.
Mar. 12,	"	"	567.70,	on 5 months.
April 15,	"	"	369.80,	on 4 months.

Ans. March 16, 1856.

436. To find the time of paying the balance of a debt, when partial payments have been made before the debt is due.

Ex. 1. I have bought of Leonard Johnson goods to the amount of $ 1728, on 6 months' credit. At the end of one month I pay him $ 300, and at the end of 5 months, $ 800. How long, in equity, after the expiration of 6 months, should the balance remain unpaid? Ans. 3mo. 20d.

OPERATION.

$$
\begin{array}{rcr}
300 \times 5 & = & 1500 \\
800 \times 1 & = & 800 \\
\hline
1100 & & 2300
\end{array}
$$

$\$1728 - \$1100 = \$628;$

$2300 \div 628 = $ 3mo. 20d.

The interest on the $ 300 for 5 months is equal to the interest of $ 1 for 1500 months, and the interest of $ 800 for 1 month is equal to that of $ 1 for 800 months; and thus the interest on both partial payments, at the expiration of the 6 months, is equal to the interest of $ 1 for $1500 + 800 = 2300$ months. To equal this credit of interest, the balance of the debt, which we find to be $ 628, should remain unpaid, after the 6 months, $\frac{1}{628}$ of 2300 months, or 3 months and 20 days.

RULE. — *Multiply each payment by the time it was made before it becomes due, and divide the sum of the products by the balance remaining unpaid; and the quotient will be the required time.*

EXAMPLES.

2. A merchant has $144 due him, to be paid in 7 months, but the debtor agrees to pay one half ready money, and two thirds of the remainder in 4 months. What time should be allowed for paying the balance? Ans. 2y. 10mo.

3. March 23, 1856, I sold John Morse goods to the amount of $8000 on a credit of 8 months. April 5, he paid me $1200; July 4, $1500; September 25, $1800; October 1, $1000; November 20, $500. When, in equity, should I receive the balance?

4. There is due to a merchant $800, one sixth of which is to be paid in 2 months, one third in 3 months, and the remainder in 6 months; but the debtor agrees to pay one half *down*. How long may the debtor retain the other half so that neither party may sustain loss? Ans. $8\frac{2}{3}$ months.

5. I have sold Charles Fox goods to the amount of $3051, on a credit of 6 months, from September 25, 1856. October 4, he paid $476; November 12, $375; December 5, $800; January 1, 1857, $200. When, in equity, ought I to receive the balance? Ans. October 8, 1857.

AVERAGING OF ACCOUNTS.

437. AN Account Current is a statement of the mercantile transactions of one person with another, when immediate payments are not made.

An account is marked Dr. on the left, to indicate that the person with whom the account is kept is *debtor* for the items on that side; and is marked Cr. on the right, to indicate that he is *creditor* for the items on that side.

Accounts current are generally made up or settled at the end of every six months or year.

438. To find the equated time when the balance of an account current will be due.

Ex. 1. In the following account when did the balance become due, the merchandise articles being on 6 months' credit?

Ans. December 22, 1856.

Dr. *Messrs. James Dutton & Co. in account with David Hale.* *Cr.*

1856.			1856.		
Jan. 4.	To merchandise,	$ 96.51	Jan. 30.	By cash,	$ 240.00
" 18.	" "	57.67	Apr. 3.	" "	48.88
Feb. 4.	" cash paid draft,	80.00	May 22.	" "	50.00
" "	" merchandise,	38.96	July 7.	" Note,* June 22, 6mo.	410.01
" 9.	" cash paid draft,	50.26			
Mar. 3.	" merchandise,	154.46			
" 24.	" "	42.30			
Apr. 9.	" "	23.60			
May 15.	" "	28.46			
" 21.	" "	177.19			
		$ 748.89			$ 748.89

Errors excepted. Settled as above, Boston, July 7, 1856.

DAVID HALE,
By JOHN DAVIS.

FIRST OPERATION.

1856. Debits.				
Due July	4,	97 × 151	=	14647
" "	18,	58 × 165	=	9570
" Feb.	4,	80		
" Aug.	4,	39 × 182	=	7098
" Feb.	9,	50 × 5	=	250
" Sept.	3,	154 × 212	=	32648
" "	24,	42 × 233	=	9786
" Oct.	9,	24 × 248	=	5952
" Nov.	15,	28 × 285	=	7980
" "	21,	177 × 291	=	51507
		$ 749		139438 days.

$139438 \div 749 = 186\frac{124}{749}$ days.

Debits due 186 days from February 4, or on August 8, 1856.

Credits.				
Due Jan.	30,	240		
" Apr.	3,	49 × 64	=	3136
" May	22,	50 × 113	=	5650
		$ 339		8786 days.

$8786 \div 339 = 25\frac{311}{339}$ days.

Credits due 26 days from January 30, or on February 25, 1856.

Difference between February 25 and August 8 = 165 days.

$ 749 — $ 339 = $ 410, balance.

$339 \times 165 = 55935$;

$55935 \div 410 = 136\frac{35}{82}$ days.

186 days forward from August 8, 1856 = December 22, 1856.

On equating each side of the account (Art. 435), we find the debits became due 186 days from February 4, or on August 8; and the credits became due 26 days from January 30, or on February 25.

If the account had been settled on February 25, it is evident the debits would have been paid 165 days, or the time from February 25 to August 8, before having become due. This would have been a loss of interest to the debit side of the account, and a corresponding gain to the credit side. Now, as the settlement should be one of equity, we find how long it will take the balance, $ 410, to gain the same interest that $ 339 would gain in the 165 days. If it take $ 339 to gain a certain interest in 165 days, it would take $ 1 to gain the same interest 339 times 165 days = 55935 days; and it would take $ 410 to gain the same amount of interest $\frac{1}{410}$ of 55935 days = 136

* Included only to illustrate the manner of settling an account.

days nearly. Hence, the balance became due 136 days forward from August 8, 1856, or on December 22, 1856.

The time was counted *forward* from the average date of the larger amount, since it became due *last;* but had that amount become due first, the time would have been counted backward from its average time.

SECOND OPERATION.

Debits.

Due Feb. 4,	80 × 5 =	400	
" " 9,	50 × 10 =	500	
" July 4,	97 × 156 =	15132	
" " 18,	58 × 170 =	9860	
" Aug. 4,	39 × 187 =	7293	
" Sept. 3,	154 × 217 =	33418	
" " 24,	42 × 238 =	9996	
" Oct. 9,	24 × 253 =	6072	
" Nov. 15,	28 × 290 =	8120	
" " 21,	177 × 296 =	52392	
	$749	143183	days.

Credits.

Due Jan. 30,	240		
" April 3,	49 × 64 =	3136	
" May 22,	50 × 113 =	5650	
	$339	8786	days.

$749 — $339 = $410, balance of the items.

143183 — 8786 = 134397, balance of products.

134397 ÷ 410 = $327\frac{327}{410}$ days.

328 days forward from January 30, 1856 = December 23, 1856.

In the second operation, we take the earliest date on which any sum becomes due in the account, for the starting-point from which to reckon the days, by which to find the several products belonging either to the debit or credit side (Art. 435, Note 1). The sum of the debit products, 143183, denotes the number of days required for $1 to gain as much interest as all the items of debit would gain in the times of their becoming due, and the sum of the credit products, 8786, denotes the number of days required for $1 to gain as much interest as all the items of debit would gain in the times of their becoming due. The difference between the sums of debit and credit products is 134397, and the difference between the debit and credit items is $410. Then, if it requires 134397 days for $1 to gain a certain interest, it will require $410 to gain the same amount $\frac{1}{410}$ of 134397 days = 328 days nearly. 328 days forward from January 30, 1856 = December 23, 1856, the time of the balance of the account becoming due; thus varying one day in the result, on account of the fractions.

RULE 1. — *Find the average time of each side becoming due.*

Multiply the amount of the smaller side by the number of days between the two average dates, and divide the product by the balance of the account.

The quotient will be the time of the balance becoming due, counted from the average date of the larger side, FORWARD *when the amount of that side becomes due* LAST, *but* BACKWARD *when it becomes due* FIRST. Or,

RULE 2. — *Multiply each sum of debit and credit by the number of days intervening between the date of its becoming due, and the earliest date on which any sum in the account becomes due.*

Then, the difference between the sums of debit and credit products, divided by the difference between the debit and credit item, will give the

time, to be counted from the earliest date of any sum in the account becoming due, FORWARD *when the larger sum of products is on the* LARGER *side of the account, but* BACKWARD *when it is on the* SMALLER *side.*

NOTE. — The CASH VALUE of a balance of an account drawing interest, or whose items are on different terms of credit, depends upon the time of settlement, and is therefore either larger or smaller than the difference between the debit and credit items.

The average time of a balance becoming due being known, its *cash value* may be found, *when the balance is due* BEFORE *the time of settlement, by adding to the balance interest up to the time of settlement; and when due* AFTER *that time, by deducting from the balance interest for the time intervening between the time of settlement and the time of the balance becoming due.* The deduction of interest, in the latter case, is the mercantile method, instead of that of finding the true present worth by deducting the true discount.

EXAMPLES.

2. Required the time when the balance of the following accounts becomes subject to interest, allowing that each item was due from its date.

Dr. *D. Wadsworth in account with S. Adams.* *Cr.*

1855.			1855.		
July 4,	To balance,	$ 875.90	Aug. 10,	By cash,	$ 316.00
Aug. 20,	" merchandise,	815.58	Sept. 1,	" "	675.00
" 29,	" "	178.25	" 25,	" merchandise,	512.25
Sept. 25,	" "	887.20	Nov. 20,	" cash,	161.75
Dec. 5,	" "	418.70	Dec. 1,	" "	100.00

Ans. August 6, 1855.

3. When did the balance of the following account become due, the merchandise items being on 6 months?

Dr. *John Greene in account with F. Johnson.* *Cr.*

1856.			1856.		
March 1,	To merchandise,	$ 720.75	April 1,	By cash,	$ 700.00
" 20,	" "	815.30	May 30,	" merchandise,	569.89
April 11,	" "	587.80	July 20,	" cash,	500.00
" 30,	" "	300.00	Sept. 25,	" "	100.00
June 15,	" "	625.25	" 30,	" merchandise,	750.20
July 18,	" "	560.00	Oct. 30,	" "	329.96
Aug. 30,	" "	684.90	Nov. 20,	" "	500.00
Sept. 25,	" "	365.30			

4. Allowing that each item of the following account draws interest from its date, at what time would the balance become due, and how much ready money should in equity discharge the same, September 21, 1856, interest being at 6 per cent.?

Dr. *I. Bradley in account with T. B. Fuller.* *Cr.*

1856.			1856.		
March 1,	To merchandise,	$ 36.25	April 1,	By cash,	$ 48.25
April 7,	" "	18.15	May 20,	" "	90.10
June 15,	" "	48.26	June 17,	" "	12.50
July 21,	" "	91.20	July 4,	" "	20.00
Aug. 1,	" "	30.00	" 10,	" "	25.00

Ans. November 21, 1856; cash value of balance, $ 27.73.

5. Required the time when the balance of the following account became subject to interest, allowing the merchandise to have been on 8 months; and the cash value of the balance on November 28, 1857, provided it drew 6 per cent. interest from the time of its becoming due.

Dr. *N. Chandler, 2d, in account with T. E. Lanman.* *Cr.*

1856.			1857.		
May 1,	To merchandise,	$ 300.00	Jan. 1,	By cash,	$ 500.00
July 7,	" "	759.96	Feb. 18,	" merchandise,	481.75
Sept. 11,	" "	417.20	Mar. 19,	" cash,	750.25
Nov. 25,	" "	287.70	April 1,	" draft,	210.00
Dec. 20,	" "	571.10	May 25,	" cash,	100.00

Ans. July 28, 1857; cash value of balance, $ 299.88.

6. The following account was settled May 24, 1857; how long previous to date was the balance by average due, and what was the cash value of the balance at the time of settlement?

Dr. *David Taggart in account with George Perry.* *Cr.*

1856.			1856.		
Jan. 1,	To m'dise on 5mo.,	$ 560.00	March 7,	By m'dise on 6mo.,	$ 350.00
Feb. 11,	" " on 6mo.,	846.00	April 17,	" " " "	820.00
Mar. 20,	" " on 6mo.,	728.00	June 20,	" cash,	100.00
July 30,	" cash,	400.00	Aug. 15,	" "	800.00
Sept. 12,	" m'dise on 3mo.,	560.00	Sept. 18,	" m'dise on 6mo.,	630.00
Dec. 18,	" cash,	600.00	Oct. 28,	" cash,	400.00
1857.			Nov. 1,	" m'dise on 4mo.,	750.00
May 10,	" "	500.00			

Ans. May 19, 1856; cash value of balance, $ 364.93.

439.° To find the true balance of an account current whose items draw interest.

Ex. 1. Required the true balance of the following account, on November 1, 1857, the time of settlement, allowing that each item drew interest from its date, at the rate of 6 per cent.

Ans. $ 430.04.

Dr. *Wesley Scott in account with Seth Wilson.* *Cr.*

1857.			1857.		
Jan. 17,	To merchandise,	$ 144 05	Feb. 1,	By cash,	$ 200.00
Mar. 20,	" "	374.95	March 9,	" "	300.00
May 7,	" "	500.00	April 1,	" "	600.00
July 4,	" cash,	100.00	" 5,	" merchandise,	180.00
" 7,	" merchandise,	600.00	June 12,	" "	700.00
Sept. 25,	" "	250.00	Aug. 19,	" cash,	400.00
Nov. 1,	" *bal. new acc't,	430.04	Nov. 1,	" *balance of int.,	19.04
		$ 2399.04			$ 2399.04

Errors excepted. Philadelphia, November 1, 1857.

SETH WILSON.

OPERATION.

Debits.		Credits.	
Due Jan. 17, 144 × 288 =	41472	Due Feb. 1, 200 × 273 =	54600
" March 20, 375 × 226 =	84750	" March 9, 300 × 237 =	71100
" May 7, 500 × 178 =	89000	" April 1, 600 × 214 =	128400
" July 4, 100 × 120 =	12000	" " 5, 180 × 210 =	37800
" " 7, 600 × 117 =	70200	" June 12, 700 × 142 =	99400
" Sept. 25, 250 × 37 =	9250	" Aug. 19, 400 × 74 =	29600
$ 1969	6) 306.672	$ 2380	6) 420.900
	$ 51.112		$ 70.150

$ 2380 — $ 1969 = $ 411, balance of items.
$ 70.15 — $ 51.11 = $ 19.04, balance of interest.
$ 411 + $ 19.04 = $ 430.04, true balance.

The time in days intervening between the date of each item and the time of settlement is, evidently, the number of days each item is on interest. Then, if each item be multiplied by the number denoting its number of days, and divided by 6000, the result will be its interest (Art. 356, Note 1), and the sum of the interest of the several items of debit will be the aggregate interest of the debit side, and the same principle will hold in finding the aggregate interest of the credit side. But the same result may be obtained in a shorter way by dividing the sums of the debit and credit products by 6000, which is done most readily by pointing off three decimal places at the right of each sum (Art. 78), and dividing by 6, as in the operation. Then the balance of items and the balance of interest we add together, and thus obtain the true balance.

2. Required the true balance of the following account, on October 1, 1857, the time of settlement, allowing the rate of interest to be 6 per cent. Ans. $ 189.81.

* Introduced only to illustrate the manner of balancing the account.

Dr. *Grant & Barker in account and interest with J. Ritchie.* *Cr.*

1857.			1857.		
Mar. 1,	To balance of old acc't,	$ 100.15	Mar. 15,	By m'dise on 6mo.,	$ 160.00
" 15,	" cash paid draft,	40.00	April 1,	" cash,	100.00
Mar. 20,	" merch'dise on 6mo.,	180.85	June 1,	" draft,	120.00
Oct. 1,	" *balance,	189.81	Aug. 10,	" m'dise on 2mo.,	80.00
			Sept. 1,	" cash paid draft,	50.00
			Oct. 1,	" *balance of int.,	.81
		$ 510.81			$ 510.81

OPERATION.

Debits.	Debit products.	Credit products.
Due March 1, 100 × 214 =	21400	
" " 15, 40 × 200 =	8000	
" Sept. 20, 181 × 11 =	1991	
$ 321	31391	
Credits.		
Due April 1, 100 × 183 =		18300
" June 1, 120 × 122 =		14640
" Sept. 1, 50 × 30 =		1500
" " 15, 160 × 16 =		2560
" Oct. 10, 80 × 9 =	720	
$ 510	32111	37000
321		32111
$ 189 balance of items.		6) 4.889
		$ 0.81 bal. of interest.

$ 189 + $ 0.81 = $ 189.81, true balance.

We find the several products by multiplying, as in working the preceding example. The products belonging to items becoming due previous to settlement we arrange as belonging to their respective sides. One item, however, becomes due 9 days *after* the time of settlement, thus requiring its side to be diminished by the interest of that item for the 9 days, or that the opposite side should be increased by that interest. We, therefore, write the product of this item with the debit products. We may now find the interest of each side separately, and then subtract that of the one side from that of the other for the balance of interest; but we obtain the desired result by a shorter method by finding the interest corresponding to the difference of the sums of the debit and credit products. Hence the general

RULE. — *Multiply each item of debit and credit by the number of days intervening between its becoming due and the time of settlement.*

Place the product of each item becoming due BEFORE *the time of settlement on its own side of the account, and place the product of each item*

* Introduced only to illustrate the manner of settlement.

becoming due AFTER *the time of settlement on the side opposite to its own.*

The difference of the sums of the debit and credit products, with three places pointed off on the right for decimals, divided by 6, *will give the balance of interest.*

Place the balance of interest on its own side of the account, and the difference then between the two sides will be the true balance.

NOTE 1.— The interest as found by the rule is that at 6 per cent. From which the interest at any other rate may be found by aliquot parts.

NOTE 2.— Besides the above method of settling accounts drawing interest, accountants often make use of tables constructed to aid in equating and balancing accounts.

EXAMPLES.

3. Alfred Hicks is in account and interest with Keen & Lee, as follows: Debtor, January 1, 1857, to merchandise, on 6 months, $ 156.10; February 3, to cash paid draft, $ 100; March 20, to merchandise, on 4 months, $ 316.90; March 30, to merchandise, on 4 months, $ 162.00; May 15, to cash paid draft, $ 100; August 20, to merchandise, on 6 months, $ 213.00. Creditor, February 1, by cash, $ 120.00; March 20, by merchandise, on 4 months, $ 420.16; May 1, by merchandise, on 6 months, $ 300; July 1, by merchandise, on 4 months, $ 50; September 10, by merchandise, on 4 months, $ 99.84. Required the true balance, if settled on December 1, 1857, interest being at 6 per cent. Ans. $ 61.36.

4. Required the true balance due on the following account, on March 25, 1857, each item drawing 7 per cent. interest from its date. Benjamin Lyman in account and interest with John Russell: Debtor, July 4, 1856, to merchandise, $ 200; September 8, to merchandise, $ 300; September 25, to merchandise, $ 250; October 1, to merchandise, $ 600; November 20, to merchandise, $ 400; December 12, to merchandise, $ 500; January 15, 1857, to merchandise, $ 100; March 11, to merchandise, $ 120. Creditor, July 20, 1856, by cash, $ 300; August 15, by cash, $ 350; September 1, by cash, $ 400; November 1, by cash, $ 320; December 6, by merchandise, $ 600; December 20, by cash, $ 100; February 1, 1857, by cash, $ 200; February 28, by merchandise, $ 150.

Ans. $ 50.64.

ACCOUNTS OF STORAGE.

440. ACCOUNTS of storage of property contain an entry of the number of articles received and delivered, with the date of each transaction.

Storage is usually reckoned by the month of 30 days, at a certain price per barrel, bale, box, &c.

The number of articles on which storage is chargeable for one month, or any other time agreed upon, is usually determined by an average.

441. To find the average of storage for a month, or any other time.

Ex. 1. What will be the cost for the storage of flour at 6 cents per barrel, which was received and delivered as follows: Received May 1, 1857, 1000 barrels; May 26, 2000 barrels. Delivered May 16, 500 barrels; June 1, 1000 barrels; June 12, 1100; July 2, 400. Ans. $114.

OPERATION.

1857.		bbl.		d.		prod.
May 1,	Rec.	1000	×	15	=	15000
" 16,	Deliv.	500				
	Bal.	500	×	10	=	5000
" 26,	Rec.	2000				
	Bal.	2500	×	5	=	12500
June 1,	Deliv.	1000				
	Bal.	1500	×	11	=	16500
" 12,	Deliv.	1100				
	Bal.	400	×	20	=	8000
July 2,	Deliv.	400		3|0	)	5700|0
Chargeable for 1 month,						1900

1900bbl. × .06 = $114, cost of storage.

The storage of 1000bbl. for 15d. + 500bbl. for 10d. + 2500bbl. for 5d. + 1500bbl. for 11d. + 400bbl. for 20d., is the same as the storage of 57000bbl. for 1d., or 1900bbl. for a month of 30 days. And the storage of 1900bbl. at 6 cents each equals $114, the answer required.

In practice, it is customary, when the number of articles upon which storage is chargeable, as found, contains a fraction *less than a half*, to reject the fraction; but if it is *more than a half*, to regard it as an entire article.

RULE. — *Multiply the number of barrels, or other articles, by the number of days they are in store, and divide the sum of the products by 30, or the number of days in any term agreed upon. The quotient will give the number of barrels, or other articles, on which storage is chargeable for that term.*

EXAMPLES.

2. What is the cost of storage of tea at 3 cents a chest per month, received and delivered as follows: Received, May 16, 1857, 4560 chests. Delivered, May 30, 564 chests; June 1, 904 chests; July 9, 1000 chests; August 3, 1500 chests; and August 16, the balance.

3. Received, and delivered, on account of Richard Gordon, sundry bales of cotton, as follows: Received, January 1, 1857, 2310 bales; January 16, 120 bales; February 1, 300 bales. Delivered, February 12, 1000 bales; March 1, 600 bales; April 3, 400 bales; April 10, 312 bales. Required the number of bales remaining in store on May 1, and the cost of storage up to that date, at the rate of 5 cents a bale per month. Ans. In store, 418 bales; cost, $ 306.90.

MISCELLANEOUS EXAMPLES.

1. The stocks of three partners, A, B, and C, are $ 3500, $ 2200, and $ 2500, and their gains $ 1120, $ 880, and $ 1200, respectively, and B's stock continued in trade 2 months longer than A's. Required how long the money of each was in trade.

Ans. A's money, 8mo.; B's, 10mo.; C's, 12mo.

2. A merchant failed for $ 15000. On settling up, his net assets, equably distributed, gave only $ 540 to a creditor whose demand was $ 660 more than that sum. How much did the bankrupt pay on a dollar, and how much did he owe more than he could pay?

Ans. 45 cents on a dollar; owed $ 8250 more than he could pay.

3. A and B pay a poll tax each of $ 1.50, and a property tax at the rate of 7 mills on a dollar. A's entire tax is $ 64.50, which is just $ 14 more than B's entire tax. What is the taxable property of each?

Ans. A's property, $ 9000; B's property, $ 7000.

4. The expenses of a district school are, for fuel, $ 20, for

repairs of the school-house \$ 30, and for teacher's salary \$ 150, and the public money amounts to \$ 50. If the rate bills require to be made out at the rate of 3 cents a day for each pupil's attendance, what was the aggregate attendance?

Ans. 5000 days.

5. J. Kimball had goods to the value of \$ 7000 on board a vessel, which, from stress of weather, required cargo to the amount of \$ 4000 to be thrown overboard, and of which the value of \$ 3000 belonged to Kimball. If, in adjusting the general average, the several contributory interests pay 5 per cent., to what sum will Kimball's loss be reduced?

Ans. \$ 350.

6. A owes B \$ 150, \$ 50 to be paid in 4 months, and \$ 100 in 8 months. B owes A \$ 250 to be paid in 10 months. It is agreed between them that A shall make present payment of his whole debt, and that B shall pay his so much sooner as to balance the favor. Required the time at which B must pay the \$ 250. Ans. 6 months.

7. A debt is to be paid $\frac{1}{3}$ down, $\frac{1}{4}$ in 6 months, $\frac{1}{6}$ in 8 months, and the balance in 12 months. If the payments were all converted into one, on what credit should it be? Ans. 5mo. 25d.

8. A merchant proposes to admit a young man into business with him, on condition that, if he put into the stock \$ 2000, his pay shall be \$ 800 a year, or if he put in \$ 4000, he shall have \$ 1100 a year; in this offer what was allowed for his services only? Ans. \$ 500.

9. A gentleman is owing three notes to George Shannon, one of \$ 100 due in 4 months, another of \$ 100 due in 8 months, and a third of \$ 200 due in 12 months. Should the three notes be converted into two for the same amount, the one to run just twice as long as the other, when ought they to be made payable? Ans. The one in 6mo., and the other in 12mo.

10. An account settled January 1, 1858, showed a balance of debts to the amount of \$ 360, and a balance of interest in favor of the credit side to the amount of \$ 3.78; how long in equity ought the balance of debts to remain unpaid after the day of settlement, exactly to offset this balance of interest, allowing the rate of interest to be 6 per cent.? Ans. 63 days.

INSURANCE.

442. INSURANCE is a contract of indemnity, by which one party engages, for a stipulated sum, to insure another against a risk or loss to which he is exposed.

The *insurer* or *underwriter* is the party taking the risk.

The *premium* is the sum paid for insurance, and is generally reckoned at a certain per cent. of the value of the property insured.

The *policy* is the written obligation or contract.

NOTE.—Insurance is generally made by an incorporated joint-stock company, and sometimes by individuals. When each person insured becomes a member and proprietor in the profit or loss of the concern, it is called a *mutual* insurance company. Many companies, as a security against fraud, do not insure property for its full value.

FIRE AND MARINE INSURANCE.

443. *Fire Insurance* is that which indemnifies damage and loss caused by fire or lightning.

444. *Marine Insurance* is that which indemnifies damage and loss caused by the perils peculiar to navigation.

445. To compute the premium of insurance on any given amount at a specified rate.

Ex. 1. What would be the premium for insuring a house, valued at $5728, at $1\frac{3}{4}$ per cent.? Ans. $100.24.

OPERATION.

$$\$5728 \times .01\tfrac{3}{4} = \$100.24.$$

RULE.—*Find the percentage of the given sum at the rate of insurance, and the result is the premium.*

NOTE.—When the amount insured and the premium are known, *the rate of insurance* may be found by Art. 350.

EXAMPLES.

2. What is the premium for insuring a house valued at $896, at 12 per cent.? Ans. $107.52.

3. How much would be required to be paid to effect the insurance of a brig valued at $17,289, at $1\frac{1}{4}$ per cent.?

Ans. 216.11\frac{1}{4}$.

4. My ship Keystone State is valued at $ 35000, and her cargo at $ 75000. I procure an insurance on $\frac{3}{5}$ of the value of the ship, at $3\frac{1}{4}$ per cent., and on $\frac{2}{3}$ of her cargo, at $2\frac{1}{2}$ per cent. What is the amount of premium? Ans. $ 1932.50.

5. My library, worth $ 3675, I got insured by paying $4\frac{7}{8}$ per cent.; and the policy cost me $ 1. The library having been destroyed by fire, what was my actual loss? Ans. $ 180.15$\frac{5}{8}$.

6. The premium for insuring $ 9870 was $ 690.90. What was the rate per cent.?

7. White & Bigelow effect an insurance on their store and goods, worth $ 47600, for 5 years. The first year they are to pay $4\frac{1}{4}$ per cent.; the second year, $3\frac{7}{8}$ per cent.; the third year, $4\frac{3}{8}$ per cent.; the fourth year, 5 per cent.; and the fifth year, $5\frac{1}{8}$ per cent. What is the whole they are to pay for the insurance? Ans. $ 11,007.50.

446. To find what sum must be insured, at a given rate, to cover both property and premium.

Ex. 1. For what sum must a policy be taken out, to cover both property and premium, the value of the property being $ 2475, and the rate of insurance 10 per cent.?

OPERATION.

$$\$2475 \div .90 = \$2750.$$

Since the sum to be covered by the insurance includes both the property and the premium, and as the premium is 10 per cent., or .10 of that sum, the property, evidently, must be .90 of the sum for which the policy is to be taken out.

RULE. — *Divide the value of the property to be covered by* 1 *decreased by the rate of insurance expressed decimally, and the quotient will give the whole sum to be covered.*

NOTE. — When the rate of insurance and premium are known, the *amount insured* may be found by Art. 350.

EXAMPLES.

2. A manufacturing company own a factory valued at $ 26250. For what sum must a policy be taken out to cover the property and premium of insurance, the rate being $12\frac{1}{2}$ per cent.? Ans. $ 30,000.

3. A store and its goods are worth $ 6370. What sum must be insured, at 2 per cent., to cover both property and premium? Ans. $ 6500.

4. The premium for insuring a school-house, at the rate of $1\frac{1}{4}$ per cent., was $ 50. For what sum was it insured?

Ans. $ 4000.

5. If a policy covering property and premium be taken for $ 600, at 10 per cent., what is the value of the property covered? Ans. $ 540.

6. A merchant adventured $ 1000 from Boston to New Orleans, at 3 per cent.; thence to Chili, at 5 per cent.; thence to Canton, at 6 per cent.; and thence to Boston, at 7 per cent. For what sum must he take out a policy, to cover his adventure the voyage round? Ans. $ 1241.348.

LIFE INSURANCE.

447. Insurance on life is a contract which stipulates for the payment of a certain sum of money on the death of one or more individuals, in consideration of an immediate payment, or an annual premium, being made by the insured.

A temporary insurance on life is a contract to pay a certain sum in case a given individual dies within a given number of years.

448. The amount of premium required of the insured as a security for the payment of a certain sum at his death by the insurer, is based *upon the expectation of life of the insured, and on the rate of interest or net profit the insurer may be able to make by investing the premium.*

449. By expectation of life is meant the average number of years of life that remains to any individual of a given age, as determined by the rates of mortality.

450. The Carlisle Table of the Expectation of Life, which is in general use in England, has also been taken as a guide by some American companies in fixing their rates of insurance.

Other companies have been guided in fixing their premiums by a table prepared by Dr. Wigglesworth, with a special reference to mortality in this country, and which the Supreme Court of Massachusetts has adopted as a rule in estimating the value of life estates.

451. The expectation of life, according to the Carlisle Table

and according to that prepared by Dr. Wigglesworth, is shown in the following

TABLE.

Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table Expectation.	Wigglesworth Table. Expectation.	Age.	Carlisle Table. Expectation.	Wigglesworth Table. Expectation.
0	38.72	28.15	24	38.59	32.70	48	22.80	22.27	72	8.16	9.14
1	44 68	36.78	25	37.86	32.33	49	21.81	21.72	73	7.72	8.69
2	47.55	38.74	26	37.14	31.93	50	21.11	21.17	74	7.33	8.25
3	49.82	40.01	27	36.41	31.50	51	20.39	20.61	75	7.01	7.83
4	50.76	40.73	28	35.69	31.08	52	19.68	20.05	76	6 69	7.40
5	51.25	40.88	29	35.00	30.66	53	18.97	19.49	77	6.40	6.99
6	51.17	40.69	30	34.34	30.25	54	18 28	18.92	78	6.12	6.59
7	50.80	40.47	31	33.68	29.83	55	17.58	18.35	79	5.80	6.21
8	50.24	40.14	32	33.03	29.43	56	16.89	17.78	80	5.51	5.85
9	49.57	39.72	33	32.36	29.02	57	16.21	17.20	81	5.21	5.50
10	48.82	39.23	34	31.68	28.62	58	15.55	16.63	82	4.93	5.16
11	48.04	38.64	35	31.00	28.22	59	14.92	16.04	83	4 65	4.87
12	47.27	38.02	36	30.32	27.78	60	14.34	15.45	84	4.39	4.66
13	46.51	37.41	37	29.64	27.34	61	13.82	14.86	85	4.12	4.57
14	45.75	36.79	38	28.96	26.91	62	13.31	14.26	86	3.90	4.21
15	45.00	36.17	39	28.28	26.47	63	12.81	13.66	87	3.71	3.90
16	44.27	35.76	40	27.61	26.04	64	12.30	13.05	88	3.59	3.67
17	43.57	35.37	41	26.97	25.61	65	11.79	12.43	89	3.47	3.56
18	42 87	34.98	42	26.34	25.19	66	11.27	11.96	90	3 28	3.73
19	42.17	34.59	43	25.71	24.77	67	10.75	11.48	91	3.26	3.32
20	41.46	34.22	44	25.09	24.35	68	10.23	11.01	92	3.37	3.12
21	40.75	33.84	45	24.46	23.92	69	9.70	10.50	93	3.48	2.40
22	40.04	33.46	46	23.82	23.37	70	9.18	10.06	94	3.53	1.98
23	39.31	33.08	47	23.17	22.83	71	8.65	9.60	95	3.53	1.62

Against any age given in the table may be found the expectation of age corresponding to it.

452. The transactions of life insurance companies extending as they do over a term of years, the value of money and the average rates of interest, no less than the expectation of life, have an important influence in fixing their rates of insurance.

453. The premiums of life insurance are generally reckoned at a certain sum on $100, payable annually in advance.

The rates of annual premium for insuring a healthy life for one year, for seven years, or for the whole period of life, in the sum of $100, by the Massachusetts Hospital Life Insurance Company, of Boston, and by the Girard Life Insurance Annuity and Trust Company, of Philadelphia, are given in the following

TABLE.

Age next Birthday.	Mass. Hospital Life Insurance Company.			The Girard Life Insurance Company.			Age next Birthday.
	1 year.	7 years.	For Life.	1 year.	7 years.	For Life.	
15	.83	.85	1.44	.77	.88	1.56	15
16	.84	.86	1.47	.84	.90	1.60	16
17	.85	.87	1.51	.86	.91	1.65	17
18	.86	.88	1.54	.89	.92	1.69	18
19	.87	.90	1.58	.90	.94	1.73	19
20	.88	.91	1.62	.91	.95	1.77	20
21	.89	.92	1.66	.92	.97	1.82	21
22	.90	.93	1.70	.94	.99	1.88	22
23	.91	.95	1.74	.97	1.03	1.93	23
24	.92	.96	1.79	.99	1.07	1.98	24
25	.93	.98	1.84	1.00	1.12	2.04	25
26	.95	.99	1.89	1.07	1.17	2.11	26
27	.96	1.01	1.94	1.12	1.23	2.17	27
28	.98	1.03	2.00	1.20	1.28	2.24	28
29	.99	1.05	2.06	1.28	1.35	2.31	29
30	1.01	1.07	2.12	1.31	1.36	2.36	30
31	1.03	1.09	2.18	1.32	1.42	2.43	31
32	1.05	1.11	2.25	1.33	1.46	2.50	32
33	1.07	1.14	2.32	1.34	1.48	2.57	33
34	1.09	1.16	2.40	1.35	1.50	2.64	34
35	1.11	1.19	2.48	1.36	1.53	2.75	35
36	1.14	1.21	2.56	1.39	1.57	2.81	36
37	1.16	1.24	2.65	1.43	1.63	2.90	37
38	1.19	1.28	2.75	1.48	1.70	3.05	38
39	1.22	1.31	2.85	1.57	1.76	3.11	39
40	1.24	1.36	2.95	1.69	1.83	3.20	40
41	1 27	1.41	3.07	1.78	1.88	3.31	41
42	1.31	1.47	3.19	1.85	1.89	3.40	42
43	1.35	1.54	3.32	1.89	1.92	3.51	43
44	1.40	1.62	3.45	1.90	1.94	3.63	44
45	1.47	1.71	3.60	1.91	1.96	3.73	45
46	1.54	1.80	3.75	1.92	1.98	3.87	46
47	1.62	1.90	3.92	1.93	1.99	4.01	47
48	1.71	2.02	4.09	1.94	2.02	4.17	48
49	1.81	2.14	4.27	1.95	2.04	4.49	49
50	1.91	2.28	4.46	1.96	2.09	4.60	50
51	2.03	2.42	4.67	1.97	2.20	4.75	51
52	2.15	2.59	4.89	2.02	2.37	4 90	52
53	2.29	2.76	5 12	2.10	2.59	5.24	53
54	2.44	2.95	5.36	2.18	2.89	5.49	54
55	2.60	3.15	5.62	2.32	3.21	5.78	55
56	2.78	3.38	5.89	2.47	3.56	6.05	56
57	2.96	3.62	6.19	2.70	4.20	6.27	57
58	3.17	3.87	6.50	3.14	4.31	6.50	58
59	3.39	4.17	6.83	3.67	4.63	6.75	59
60	3.64	4.50	7.18	4.35	4.91	7.00	60

According to the table, a healthy man who is 42 years old next birthday, by paying the Massachusetts Hospital Insurance Company $ 1.31, would secure to his family, heirs, or to whom-

soever he desires, $ 100, should he die in one year, and in the same proportion for a larger sum. And if he would obtain a life insurance of the Girard Life Insurance Annuity and Trust Company, he must pay annually $ 3.40. The New York Life Insurance Company has a schedule of rates like that of the Girard Company, given in the table.

454. To compute the premium of life insurance for any given amount.

Ex. 1. What premium will the Massachusetts Hospital Life Insurance Company require for the insurance of a life one year for $ 1728, the person being thirty years of age next birthday? Ans. $ 17.45.

OPERATION.

$\$1728 \times .0101 = \$17.45.$

By the table we find the premium on $ 100 to be $ 1.01. Therefore $\$\frac{1.01}{100} = \$.0101$ = the premium on $ 1 for one year, and $ 1728 × .0101 = the premium on $ 1728 for the same time. Hence,

Compute the premium on the sum to be insured at a rate proportionate to the given premium on $ 100.

Examples.

2. What amount of premium must S. C. Kendall pay annually to the Massachusetts Hospital Life Insurance Company, to effect an insurance on his life for 7 years for $ 8000, his age being 33 years? Ans. $ 91.20.

3. Robert Vaux, 60 years of age, wishes to engage in a very profitable speculation; and being obliged to borrow the necessary funds, he effects an insurance on his life for 7 years, for $ 78000, at the office of the Girard Life Insurance Company. Required the amount of the annual premium. Ans. $ 3829.80.

4. What will be the yearly premium for insuring a person's life, who is 15 years old, for $ 2000 for 7 years, at the New York Life Insurance Company?

5. A gentleman 45 years of age, being bound on a long and dangerous voyage, and wishing to secure a competence for his family, obtains an insurance for life of the Girard Life Insurance Company, for $ 12000. By an act of Providence he dies in the third year. What is the net gain to his family?

Ans. $ 10657.20.

6. Richard Sears, 50 years old, effects an insurance for life for $ 5000, for which he pays an annual premium of $ 4.60 on each $ 100 insured. If he should die at the age of 80 years, how much less will be the amount of insurance than the payments, allowing the latter to be without interest? Ans. $ 1900.

7. A gentleman, 56 years old, gets his life insured for $ 4000, at the office of the Kentucky Mutual Life Insurance Company, by paying an annual premium of $ 5.20 on each $ 100 insured; and dies at the age of 60 years. Reckoning interest on his payments at simple interest, what is gained by the insurance? Ans. $ 3043.20.

8. Alexander Murray, 28 years of age, effects an insurance on his life for $ 10000, at the office of the Massachusetts Hospital Life Insurance Company. If the company loan the premium at 6 per cent. compound interest, and he should die at the age of 40 years, who will gain by the insurance?

Ans. The insured gains $ 6626.01.

CUSTOM-HOUSE BUSINESS.

455. Duties or customs are sums of money required by government to be paid on imported goods.

Ports of entry are ports into which merchandise may be imported, or from which it may be exported.

At each port of entry is an establishment, called a *custom-house*, at which certain officers, appointed by government, attend to the collection of the duties.

456. Duties are either *specific* or *ad valorem*.

A *specific* duty is a certain sum paid on a ton, hundred-weight, yard, gallon, &c., without regard to the cost of the article.

An *ad valorem* duty is a certain percentage paid on the actual cost of the goods in the country from which they are imported.

NOTE. — As evidence of the cost value of merchandise subject to duties, the importer, owner, or consignee is required to produce an invoice or manifest, if one has been received, made out in the currency of the place or country whence imported, and containing a true statement of the actual cost of such goods in such foreign currency. When the currency of a place has a depreciated value compared with that of the United States, it is necessary that a consular certificate showing the rate of depreciation should be attached to the invoice. When, however, an invoice has not been received, the fact must be testified to under oath, and then the imported articles will be entered at an appraised value.

457. Under the tariff of 1861, duties are specific on some articles; and on others, either *ad valorem*, or specific and *ad valorem*.

458. ALLOWANCES are deductions required to be made before estimating the duties, on account of the weight of the cask, box, bag, &c. in which an article is imported, or on account of breakage, leakage, waste, or other damage.

Tare is the allowance made for the weight of the cask, box, &c. containing the commodity.

Draft is the allowance made for waste in the weighing of goods.

Leakage is the allowance made for waste on liquids imported in casks.

Gross weight is the weight of the commodity together with the cask, box, bag, &c. containing it.

Net weight is what remains after all allowances have been made.

459. No allowances for tare, draft, breakage, &c. are applicable to imports subject to *ad valorem* duties, except actual tare, or weight of a cask or box, and actual drainage, leakage, or damage. The collector may cause these to be ascertained, when he has any doubt as to what they are.

NOTE. — When the tariff laws of the United States require the collection of specific duties, the allowance for draft is on 112lb., 1lb.; above 112 lb. and not exceeding 224lb., 2lb.; above 224lb. and not exceeding 336lb., 3lb.; above 336lb. and not exceeding 1120lb., 4lb.; above 1120lb. and not exceeding 2016lb., 7lb.; above 2016lb., 9lb. The allowance for tare is deducted after the draft has been deducted. The allowance for breakage is 10 per cent. on all beer and porter in bottles; and 5 per cent. on all other liquors imported in bottles; and a dozen bottles of common size are estimated to contain $2\frac{1}{4}$ gallons. The allowance for leakage is 2 per cent. on liquors imported in casks. In making the allowances for tare or leakage, a fraction, when equal to, or greater than, one half, is reckoned 1; when less, it is omitted.

Specific duties are calculated by deducting all allowances to be made from the given quantity of merchandise, and multiplying the remainder by the duty on a unit of the given quantity.

460. To calculate *ad valorem* duties.

Ex. 1. What is the duty on 2565lb. of sugar, invoiced at $ 256.50, at 24 per cent. *ad valorem?* Ans. $ 61.56.

OPERATION.

$$\$\,256.50 \times .24 = \$\,61.56, \text{ duty.}$$

RULE. — *Find the percentage on the invoiced value of the goods, at the given rate of tariff, and the result will be the ad valorem duty.*

NOTE 1. — Other questions in *ad valorem* duties beside those requiring the finding of the amount of duty are likewise solved by some one of the rules in percentage.

NOTE 2. — In custom-house calculations 2240lb. are considered a ton, and 112lb. a hundred-weight.

EXAMPLES.

2. What is the duty at 8 per cent. on an importation of books invoiced at $ 4350? Ans. $ 348.00.

3. Required the duty at 19 per cent. on 7890 pounds of cordage, invoiced at 15 cents per pound. Ans. $ 224.865.

4. What is the duty at 24 per cent. on an invoice of woollen goods, which cost in London 986£., the pound sterling being valued at $ 4.84? Ans. $ 1145.3376.

5. $ 112.50 duty is paid on an importation of window-glass whose invoice value was $ 750. What was the rate per cent. of duty?

6. Robinson & Brother of New York have imported wines from Havre, invoiced as follows: 60 baskets Champagne at 70 francs per basket; 36 baskets port at 35 francs per basket; 50 casks of sherry, each 31 gallons, at 4 francs per gallon. The allowance for breakage on the wine in baskets is 5 per cent.; and the actual waste of that in casks is 1 gallon to a cask. Required the duties at 30 per cent., allowing the value of a franc to be $18\frac{6}{10}$ cents. Ans. $ 624.2346.

7. Paid $53.76 duties, at the rate of 8 per cent., on 60 casks of raisins, after the deduction of 12lb. to a cask for tare. Allowing the gross weight of each cask of raisins to have been 112lb., what was their invoice value per pound?

Ans. $11\frac{1}{5}$ cents.

8. A portion of the cargo of the ship Cuba from Havana was invoiced as follows: 40 hogsheads of molasses, 63 gallons each, at 3 reals plate per gallon; 24 boxes of brown sugar, 400lb. each, at 1 real vellon per pound; 260 boxes of oranges, at $ 2 per box; and 410 boxes of cigars, at $7 per box. The tare on the sugar was 10 per cent., and the leakage of the molasses 2 per cent. Allowing the value of a real plate to be 10 cents, and a real vellon 5 cents, and the rate of duties on the molasses and the sugar to have been 24 per cent., on the oranges 8 per cent., and on the cigars 30 per cent., what was the whole amount of duties?

9. 270 tons of railroad iron, invoiced at $ 50 per ton, cost, when the duties were paid, exclusive of other charges, $ 16740. What was the rate per cent. of duty? Ans. 24 per cent.

10. A merchant of Baltimore makes an importation of goods invoiced at $ 20560. On goods invoiced at $ 3000 the duties were at the rate of 4 per cent.; on goods invoiced at $ 4200 the duties were at the rate of 8 per cent.; on goods invoiced at $ 2100 the duties were at the rate of 15 per cent.; goods invoiced at $ 6000 were free of duty; and on the remainder the duties were at the rate of 30 per cent. What was the whole amount of the duties?

11. Willard, Fairbanks, & Co., of Boston, import from Liverpool 10 pieces of Brussels carpeting, 40 yards each, purchased at 5s. per yard, duty 24 per cent.; 200 yards of hair-cloth, at 4s. per yard, duty 19 per cent.; 100 woollen blankets, at 2s. 6d., duty 15 per cent.; and shoe-lasting to the cost of 60£., duty 4 per cent. Required the whole amount of duty, allowing the value of the pound sterling to be $ 4.84. Ans. $ 173.635.

12. A merchant imported from Bremen 32 pieces of linen of 32 yards each, on which he paid for the duties, at 24 per cent., $ 122.88, and other charges to the amount of $ 40.96. What was the invoice value per yard, and the cost per yard after duties and charges were paid?

Ans. Invoice value per yard, $ 0.50; cost per yard, $ 0.66.

COINS AND CURRENCIES.

461. COINS are pieces of metal legally stamped, and issued for circulation as money.

The currency of a state or country is its money or circulating medium of trade.

462. The former currency of this country had the denominations of sterling money, viz. pounds, shillings, and pence. On the adoption by Congress, in 1786, of the present decimal currency, with the dollar as its unit, there were in circulation colonial notes, or bills of credit, which had depreciated in value. This depreciation, being greater in some States than in others, gave rise to the difference in the States as to the number of shillings equivalent to a dollar, as shown in the following

TABLE.

\$ 1 in	New Eng. States, Virginia, Kentucky, Tennessee,	= 6s. = $\frac{3}{10}$£., called New Eng. currency; of which 1£. = \$ $3\frac{1}{3}$; 1s. = $16\frac{2}{3}$cts.
\$ 1 in	New York, Ohio, Michigan, North Carolina,	= 8s. = $\frac{2}{5}$£., called New York currency; of which 1£. = \$ $2\frac{1}{2}$; 1s. = $12\frac{1}{2}$cts.
\$ 1 in	Pennsylvania, New Jersey, Delaware, Maryland,	= 7s. 6d. = $\frac{3}{8}$£., called Pennsylvania currency; of which 1£. = \$ $2\frac{2}{3}$; 1s. = $13\frac{1}{3}$cts.
\$ 1 in	Georgia, South Carolina,	= 4s. 8d. = $\frac{7}{30}$£., called Georgia currency; of which 1£. = \$ $4\frac{2}{7}$; 1s. = $21\frac{3}{7}$cts.

NOTE 1. — The State currencies are now merely nominal, accounts being nowhere kept in them. Articles, however, are sometimes priced in them, but much less often now than formerly.

NOTE 2. — \$ 1 equals about $4\frac{16}{121}$s. = $\frac{25}{121}$£. of English or sterling money; and consequently 1£. = \$ 4.84; 1s. = \$ $0.24\frac{1}{5}$. Also \$ 1 = 5s. = $\frac{1}{4}$£. of Nova Scotia, New Brunswick, Newfoundland, and Canada, called Canada currency; of which 1£. = \$ 4; 1s. = 20 cents.

463. The *legal tender* in payment of debts in the United States is gold and silver.

NOTE. — The gold coins of the United States of coinage prior to 1834 are a legal tender for the payment of all debts, at the rate of $94\frac{3}{5}$ cents per penny-

weight, or about $ 10.66 for each eagle; and all the gold coins of a subsequent coinage are a legal tender of payment in any sums whatever, according to their nominal values.

Of the *silver* coins coined prior to April 1, 1853, the *three-cent pieces* are a legal tender of payment in any sums of thirty cents and under, and the dollars, half-dollars, quarter-dollars, dimes, and half-dimes are a legal tender of payment in any sums whatever, at their nominal value, and all the silver coins of a subsequent coinage are a legal tender in sums not exceeding five dollars.

464. Besides the coins of the country, foreign coins, whose value has been fixed by law, and bank-notes, redeemable in specie, pass as money. However, by the act of Congress passed February, 1857, all former acts authorizing the currency of foreign gold or silver coin, and declaring the same legal tender in payment of debts, were repealed.

465. The *intrinsic* value of foreign coins is the value depending upon the weight and purity of the metal of which they are made; their *legal* value is that fixed by law; their *commercial* value is the price they will bring in the market; and their *exchange* value is the nominal price assigned to them in reckoning exchange between one country and another.

466. The United States mint, at present, purchases standard silver at $ 1.22$\frac{1}{2}$ per ounce, and fine silver at $ 1.36$\frac{1}{8}$ per ounce.

NOTE. — At these rates of purchase, five-franc pieces yield about 98 cents each; Mexican and South American dollars, 106$\frac{1}{4}$ cents each; old Spanish dollars, 105 cents each; half-dollars of the United States coined before 1837, 52$\frac{1}{8}$ cents each; half-dollars of the United States coined since 1837, and previous to the change of standard in 1853, 52$\frac{1}{8}$ cents each; German florins, 41$\frac{1}{10}$ cents each; Prussian and Hanoverian thalers, 72 cents each; best manufactured American plate, from 120 to 122 cents per ounce; and genuine British plate, 125$\frac{1}{8}$ cents per ounce.

467. By the act of 1843 the gold coins of Great Britain, if not less than $\frac{915\frac{1}{2}}{1000}$ in fineness, were rated at 94$\frac{6}{10}$ cents per pennyweight, and the gold coin of France of not less than $\frac{899}{1000}$ in fineness, at 92$\frac{9}{10}$ cents per pennyweight. By a previous act, the gold coins of Portugal and Brazil of not less than 22 carats fine were rated at 94$\frac{8}{10}$ cents per pennyweight, and the gold coins of Spain, Mexico, and Colombia, of a fineness of 20 carats 3$\frac{7}{16}$ grains, at the rate of 89$\frac{9}{10}$ cents per pennyweight.

NOTE. — The relative value of gold and silver in the United States, at pres-

ent, is nearly as $14\frac{7}{8}$ to 1; in England, as $14\frac{29}{100}$ to 1; in France and in Russia, as 15 to 1; in Spain, as 16 to 1; in China, as $14\frac{1}{4}$ to 1; and in Portugal, as $13\frac{56}{100}$ to 1.

468. The value of certain foreign coins and currencies, as fixed by law in the collection of duties at the United States custom-houses, is shown in the following

TABLE.

Pound Ster. of G. Britain,	\$ 4.84	Florin of South of Germ.,	\$ 0.40
Pound Ster. of Br. Prov., Nova Scotia, N. Bruns., and Newfoundland,	4.00	Ounce of Sicily,	2.40
Dollar of Mexico, Peru, Chili, and Cen. Amer.,	1.00	Pagoda of India,	1.84
Specie Dollar of Sweden and Norway,	1.06	Star Pagoda of Madras,	1.84
Specie Dollar of Denmark,	1.05	Tael of China,	1.48
Rix Dollar of Bremen,	$.78\frac{3}{4}$	Millrea of Portugal,	1.12
Thaler of Bremen of 72 grotes,	.71	Millrea of Azores,	$.83\frac{1}{3}$
Rix Dollar, or Thaler of Prussia and Northern States of Germany,	.69	Ducat of Naples,	.80
Ruble, silver, of Russia,	.75	Sicca Rupee of Bengal, or of Bombay,	.50
Florin of the Austrian Empire and city of Augsburg,	$.48\frac{1}{2}$	Rupee of British India,	$.44\frac{1}{2}$
Florin or Guilder of Netherlands,	.40	Mark Banco of Hamburg,	.35
		Franc of France and Belg.,	$.18\frac{6}{10}$
		Livre Tournois of France,	$.18\frac{1}{2}$
		Leghorn Livre,	.16
		Lira of Lombardo-Venetian Kingdom,	.16
		Lira of Tuscany,	.16
		Lira of Sardinia,	$.18\frac{6}{10}$
		Real Plate of Spain,	.10
		Real Vellon of Spain,	.05

REDUCTION OF CURRENCIES.

469. Reduction of Currencies is the process of finding the value of the denominations of one currency in the denominations of another.

470. To find the value of the denominations of one currency in the denominations of another, when the values of a unit of each are known.

Ex. 1. What is the value of 18£. 4s. 6d. of the New England currency in United States money? Ans. \$ 60.75.

OPERATION.

18£. 4s. 6d. = 18.225£.

$18.225 \div \frac{3}{10} = \$\,60.75$.

We reduce the shillings and pence to the decimal of a pound, and annex it to the pounds. We then divide the sum by $\frac{3}{10}$, because 6s., or a dollar in this currency, is $\frac{3}{10}$ of a pound.

Ex. 2. What is the value of $ 60.75 in New England currency? Ans. 18£. 4s. 6d.

OPERATION.

$60.75 \times \frac{3}{10} = 18.225£.$
$18.225£. = 18£. 4s. 6d.$

Since 6s., or a dollar of this currency, is $\frac{3}{10}$ of a pound, we multiply the given sum by the fraction $\frac{3}{10}$, and find the value of the decimal in shillings and pence.

RULE. — *Multiply or divide, as the case may require, by the value of the unit of the given currency expressed in United States money.*

NOTE. — When one currency is to be changed to another, and neither of them is United States money, the value of one of them may be found in United States money, and the value of that result be found in the other currency.

EXAMPLES.

3. Change 46£. 16s. 6d. of the currency of New York to United States money. Ans. $ 117.06$\frac{1}{4}$.

4. Change $ 1032 to the currency of Pennsylvania. Ans. 387£.

5. Find the value of $ 515.70 in Canada currency. Ans. 128£. 18s. 6d.

6. Change 160.50 francs to United States money.

7. Change $ 728.41 to English money. Ans. 150£. 9s. 11$\frac{61}{121}$d.

8. Find the value of 12£. 12s. of the currency of Georgia in United States money. Ans. $ 54.

9. Find the value of 128£. 18s. 6d. of Canada in United States money. Ans. $ 515.70.

10. Find the value of 740.45 rubles, silver, of Russia, in United States money. Ans. $ 555.33$\frac{3}{4}$.

11. Find the value of 46£. 16s. 6d. of New York currency in the currency of New England. Ans. 35£. 2s. 4$\frac{1}{2}$d.

12. Change 151 millreas of Portugal to real plate of Spain.

13. Find the value of 1000 specie dollars of Norway in francs.

14. A merchant of Quebec bought in London 30 pieces of broadcloth of 30 yards each, at 15 shillings per yard; what is the amount of his bill in Canada currency? Ans. 816£. 15s.

15. A merchant of Prussia bought in Naples silks to the amount of 410 ducats; what was the amount of his purchase in thalers? Ans. 475$\frac{35}{63}$ thalers.

EXCHANGE.

471. EXCHANGE, in commerce, is the paying or receiving money in one place for an equivalent sum in another, by means of *drafts*, or *bills of exchange*.

472. A BILL OF EXCHANGE is a written order, to some person at a distance, to pay a certain sum, at an appointed time, to another person, or to his order.

The *maker* or *drawer* of a bill is the person who signs it.

The *buyer*, *taker*, or *remitter* of a bill is the person in whose favor it is drawn.

The *drawee* of a bill is the person on whom it is drawn, who is also called the *acceptor*, after he has accepted it.

The *indorser* of a bill is the person who indorses it.

The *holder* or *possessor* of a bill is the person in whose legal possession it may be at any time.

473. Most mercantile payments are made in bills of exchange, since it is generally more convenient to discharge debts by means of them than by cash remittances. For example, suppose A, of Boston, is creditor to B, of Baltimore, $100; and C, of Boston, is debtor to D, of Baltimore, $100; both these debts may be discharged by means of one bill. Thus, A draws for this sum on B, and sells his bill to C, who remits it to D, and the latter receives the amount, when due, from B. Here, by a transfer of claims, the Boston debtor pays the Boston creditor, and the Baltimore debtor the Baltimore creditor; and no money is sent from one place to the other. The same would take place if D, of Baltimore, drew on C, of Boston, and sold his bill to B, of Baltimore, who should send it to A, of Boston; the effect, in either case, being merely a transfer of debt and credit.

Bills of exchange pass from hand to hand, until due, like any other circulating medium.

474. The terms of a bill vary according to the agreement between parties, or the custom of countries. Some bills are drawn *at sight;* others, at a certain number of days, or months, *after sight* or *after date;* and some, at *usance*, which is the customary or usual term between different places.

Days of Grace.

Days of grace are a certain number of days granted the acceptor for paying the bill, after the term of a bill has expired. The usual time allowed in this country is three days.

Note 1. — In some States three days of grace are allowed on all bills of exchange payable at sight, or at a future day certain; but in other States sight drafts are excepted, as in New York, Pennsylvania, New Jersey, Maryland, Virginia, Missouri, Illinois, Michigan, Connecticut, Rhode Island, Delaware, &c.
No days of grace are allowed on bills written payable on demand, or which have no time of payment expressed in them.

Note 2. — In reckoning when a bill, payable after date, becomes due, the day on which it is dated is not included; and if it be a bill payable after *sight*, the day of presentment is not included. When the term is expressed in months, calendar months are understood; and when a month is shorter than the preceding, it is a rule not to go in the computation into a third month.
Thus, if a bill be dated the 28th, 29th, 30th, or 31st of January, and payable one month after date, the term expires on the last day of February, to which the days of grace must, of course, be added; and therefore the bill becomes due on the 3d of March.

Indorsing Bills.

475. An indorsement of a bill is the act by which the holder of it transfers his right to another. It is usually made on the back of the bill, and must be in writing.

476. Bills payable to order are transferred only by indorsement and delivery, but bills payable to bearer are transferred by either mode. On transfer by delivery, the person making it ceases to be a party to the bill; but on a transfer by indorsement, he is to all intents and purposes chargeable as a new drawer.

477. An indorsement may take place any time after the bill is issued, even after the day of payment has elapsed.

Note. — An indorsement may be restrictive, giving authority to the indorsee to receive the money for the indorser, but not to transfer the bill to another. The indorsement for a part of the money only is not valid, except with regard to him who makes it. The drawer and acceptor are not bound by it. After the payment of a part, however, a bill may be indorsed over for the residue. The indorsement is said to be in *blank*, when the indorser simply writes his name upon the back of the bill to make it transferable by delivery; and is said to be *special*, when the indorser directs the money to be paid to some particular person, or to his order. If the indorser would avoid all liability, he must qualify his indorsement by the words, "without recourse," or by others of the same import.

Accepting Bills.

478. An *acceptance* is an engagement to pay a bill according to the tenor of the acceptance, which may be either absolute or qualified.

479. An *absolute* acceptance is an engagement to pay a bill according to its request, which is commonly done by the drawee writing his name at the bottom, or across the body of the bill, with the word *accepted.*

480. A *qualified* acceptance is when a bill is accepted conditionally. But the holder is not obliged to receive a conditional or partial acceptance. He may act as if an acceptance had been entirely refused.

Note. — When a bill is drawn for the account of a third person, and is accepted as such, and he fails without making provision for its payment, the acceptor must discharge the bill, and can have no recourse against the drawer. When a holder, at his own risk, takes a conditional or partial acceptance, he must give immediate notice to all other parties to the bill, or he can have no resort to them in default of payment.

All bills payable at sight, or at a day certain, or on demand, should be presented within a reasonable time, or the holder may, from his default, be the loser.

Protesting Bills.

481. The holder of a bill, when acceptance or payment has been refused, should give regular and immediate notice to all the parties to whom he intends to resort for payment; since, if a loss should be incurred, on account of unnecessary delay, by the failure of any of the parties, he would be obliged to bear the loss. Such a notice of non-acceptance or non-payment, when made by a public officer called a notary or notary public, or by any other legal mode, is called a *protest.*

482. When the parties to a bill, which the drawee has failed to accept or pay, live in different countries, a protest is indispensably necessary, as this instrument is admitted in foreign countries as legal proof of the fact of the refusal, and that the holder intends to recover any damages which he may sustain in consequence.

483. In case of non-payment, when the parties to a bill live in the same country, it is not necessary, although frequently

practised, that there should be a regular protest by a public notary. But a notice simply of non-payment is sufficient to entitle the holder to claim interest.

484. The damages incurred by non-acceptance and non-payment of a foreign bill, besides interest commencing from the day of demand, consist usually of the exchange or re-exchange, commission, and postage, together with the expenses of protest and interest. The damages of protested bills in general, however, are regulated in a great measure by the local laws and usages of the different States and countries.

Liabilities of the Parties.

485. The drawer, acceptor, and each and every indorser of a bill, are liable to the payment of it; and though the holder can have but one satisfaction, yet, till such satisfaction is actually had, he may sue any of them, or all of them, either at the same time or in succession, and obtain judgment against them all, till satisfaction be made.

486. Nothing will discharge an indorser from his engagement, but the absolute payment of the money; not even a judgment recovered against the drawer, or any previous indorser, or any execution against any of them, unless the money be paid in consequence.

NOTE. — When acceptance is refused, and the bill is returned by protest, an action may be commenced immediately against the drawer, though the regular time of payment be not arrived. His debt, in such a case, is considered as contracted the moment the bill is drawn.

In order, however, to make the indorsers liable, it is proper that the holder should present the bill for payment on the day it becomes due.

Par of Exchange.

487. The *intrinsic par of exchange* is the value the coins of one country have, when compared with those of another, with respect both to weight and fineness.

488. The *commercial par of exchange* is the value the coins of one country sell for in the markets of another; and is, therefore not a fixed, but a variable value.

Course of Exchange.

489. The *course of exchange* is the variable price of the money of one country, which is paid for a fixed sum of money of another country.

490. The fluctuations of exchange are occasioned by various circumstances, both political and commercial, but in general bills rise or fall in their prices, like any other salable articles, according to the relation existing for the time being between the demand and the supply.

491. The limits within which the fluctuations of exchange range, correspond with the cost of making remittances in cash. Therefore, in the time of peace, exchange seldom remains long unfavorable to any country. When unfavorable, it has a tendency to correct itself, by giving an unusual stimulus to exportation, and by throwing obstacles in the way of importation; and when favorable, it produces the same effect, by restricting exportation and facilitating importation.

INLAND BILLS.

492. An Inland Bill of exchange, or draft, is one of which the drawer and drawee are residents of different parts of the same country.

Inland bills are seldom bought or sold at the precise sum specified upon their face, but, according to the course of exchange, are subject to a discount, or command a premium.

493. To compute inland exchange.

Ex. 1. What is the value of the following bill of exchange or draft, at $\frac{1}{8}$ per cent. premium? Ans. $ 2563.20.

$ 2560. *New York, April* 14, 1857.

At sight, pay to Dura Wadsworth, or order, two thousand five hundred and sixty dollars, value received, and charge the same to the account of Cameron, Bashford, & Co.

To Messrs. Lawrence & Aspinwall, *Merchants, Boston.*

OPERATION.

$$\$2560 \times 1.00\tfrac{1}{8} = \$2563.20.$$

RULE. — *Multiply the face of the bill or draft by* 1 *increased by the rate per cent. of premium, or by* 1 *decreased by the rate per cent. of discount, expressed decimally. The product will be the value of the given bill or draft.*

NOTE. — When there is interest to be computed, it must be reckoned on the face of the bill or draft. When other than the value or cost of the bill or draft is to be found, proceed as in percentage.

EXAMPLES.

2. What is the value of the following bill, or draft, at $\frac{1}{4}$ of 1 per cent. premium? Ans. $ 1955.37.

$ 1950$\frac{50}{100}$. *Chicago, August* 3, 1857.

Thirty days after date, please pay to the order of Robert S. Davis & Co., one thousand nine hundred fifty $\frac{50}{100}$ *dollars, value received, and charge the same to our account.*

KEENE & LEE.

To GEORGE REED, *Broker, Boston.*

3. What is the cost of a draft on Philadelphia, at $\frac{1}{2}$ per cent. premium, the face being $ 2000? Ans. $ 2010.

4. What must be the face of a draft, at 2 per cent. discount, to cost $ 1744.40?

3 5. What is the cost of a 60 days' bill on Pittsburg to the amount of $ 600, at 1 per cent. discount, and interest off at 6 per cent.? Ans. $ 587.70.

6. What is the cost of a 30 days' bill on Boston, at $\frac{3}{8}$ per cent. premium, and interest off at 6 per cent., the face of the bill being $ 9256.40? Ans. $ 9240.20.

7. What must be the face of a 30 days' bill which will yield $ 9240.20 when sold at $\frac{3}{8}$ per cent. premium, and interest off at 6 per cent.?

8. A 15 days' draft yielded $ 1190.184 when sold at $1\frac{1}{2}$ per cent. discount, and interest off at 6 per cent. What was the face of the draft? Ans. $ 1212.

FOREIGN BILLS.

494. A FOREIGN BILL of exchange is one of which the drawer and drawee are residents of different countries.

495. Foreign bills are usually drawn in sets; that is, at

the same time there are drawn two or more bills of the same tenor and date, each containing a condition that it shall continue payable only while the others remain unpaid. Each bill of a set is remitted in a different manner, and when one of the set has been accepted and paid, the others become worthless.

In reckoning the value of foreign bills of exchange, an acquaintance with the moneys of account of foreign countries is required.

496. The *moneys of account* of the principal places of commercial importance, with the par value of the unit, expressed in United States money, are shown in the following

TABLE.

Cities and Countries.	Denominations of Money.	Value.
London, Liverpool, &c.,	12 pence = 1 shilling; 20s. = 1 pound =	$4.84
Paris, Havre, &c.,	100 centimes = 1 franc =	0.186
Amsterdam, Hague, &c.,	100 cents = 1 guilder or florin =	0.40
Bremen,	5 swares = 1 grote; 72gr. = 1 rix dollar =	0.78¾
Hamburg, Lubec, &c.,	12 pfennings = 1 schilling; 16s. = 1 mark banco =	0.35
Berlin, Dantzic,	12 pfennings = 1 groschen; 30gr. = 1 thaler =	0.69
Belgium,	100 centimes = 1 franc =	0.186
St. Petersburg,	100 kopecks = 1 ruble =	0.75
Stockholm,	12 rundstycks = 16 skillings; 48s. = 1 rix dollar specie =	1.06
Copenhagen,	16 skillings = 1 mark; 6m. = 1 rix dollar =	1.05
Vienna, Trieste, &c.,	60 kreutzers = 1 florin =	0.48½
Naples,	10 grani = 1 carlino; 10car. = 1 ducat =	0.80
Venice, Milan, &c.,	100 centesimi = 1 lira =	0.16
Florence, Leghorn, &c.,	100 centesimi = 1 lira =	0.16
Genoa, Turin, &c.,	100 centesimi = 1 lira =	0.186
Sicily,	20 grani = 1 taro; 30 tari = 1 ounce =	2.40
Portugal,	1000 reas = 1 millrea =	1.12
Spain,	34 maravedis = 1 real vellon =	0.05
	68 maravedis = 1 real plate =	0.10
Constantinople,	100 aspers = 1 piaster =	0.05
British India,	12 pice = 1 anna; 16 annas = 1 rupee =	0.44½
Canton,	100 candarines = 1 mace; 10m. = 1 tael =	1.48

NOTE 1. — *Grani* is the plural of *grano*, *carlini* of *carlino*, *centesimi* of *centesimo*, *lire* of *lira*, *tari* of *taro*.

NOTE 2. — The moneys of account in *Brazil* are of the same denominations as in Portugal; in *Mexico*, *Central America*, *New Granada*, *Chili*, *Venezuela*, *Bolivia*, *Peru*, and *Buenos Ayres*, accounts are kept in pesos, or dollars, of 8 reals each; in *Cuba* accounts are kept in dollars equal 8 reals plate, or 20 reals vellon, equal $ 1; in *British American Possessions* accounts are kept in the denominations of sterling money, but of a depreciated value compared with the currency of the same denominations in England, except in *Canada*, where, by an act of the Colonial Parliament, passed in 1857, the decimal currency of the United States has been adopted.

497. The exchange value, in United States money, of the pound sterling of Great Britain is that of its former legal value, or $\$ 4\frac{4}{9} = \$ 4.44\frac{4}{9}$, which is considerably below either its intrinsic or commercial value. The commercial value is generally about 9 per cent. more than this exchange, or nominal par value.

Thus, nominal par value being	=	$\$ 4.44\frac{4}{9}$
Add premium at 9 per cent.	=	.40
The commercial par value will be	=	$\$ 4.84\frac{4}{9}$.

Therefore, when the nominal exchange between the United States and Great Britain exceeds 9 per cent. premium, it is above true par; when less, it is below true par.

498. The quotations of the rates of exchange of the United States on England have always reference to the old par value of the pound sterling in United States money. The course of exchange of the United States on France is so many francs and centimes payable in France for a dollar paid here; on Holland it is so many cents a guilder (of Netherlands); on Hamburg, so many cents a mark banco; on Bremen, so many cents a rix dollar; and so on.

NOTE 1. — The course of exchange of London on France is so many francs and centimes payable in France for 1£.; on Amsterdam, it is so many florins for 1£.; on Hamburg, so many schillings for 1£.; on Vienna and Trieste, so many florins and creutzers; on Spain, so many pence paid in England for 1 dollar of plate (= $8\frac{8}{17}$ reals plate) payable in Spain; on Lisbon, so many pence for 1 millrea; on Naples, so many pence for 1 ducat; and so on.

NOTE 2. — The value of 1£. sterling from 7 to $10\frac{3}{4}$ premium on the old par value of $\$ 4.44\frac{4}{9}$, is shown in the following

TABLE.

7	per cent.	premium,	$4.756	9	per cent.	premium,	$4.844
$7\frac{1}{4}$	"	"	4.767	$9\frac{1}{4}$	"	"	4.856
$7\frac{1}{2}$	"	"	4.778	$9\frac{1}{2}$	"	"	4.867
$7\frac{3}{4}$	"	"	4.789	$9\frac{3}{4}$	"	"	4.878
8	"	"	4.80	10	"	"	4.889
$8\frac{1}{4}$	"	"	4.811	$10\frac{1}{4}$	"	"	4.90
$8\frac{1}{2}$	"	"	4.822	$10\frac{1}{2}$	"	"	4.911
$8\frac{3}{4}$	"	"	4.833	$10\frac{3}{4}$	"	"	4.922

NOTE 3. — In this country the quotations of foreign exchanges are usually for bills payable 60 days after sight, and of inland or domestic exchanges, for bills payable at sight.

499. To compute foreign exchange.

Ex. 1. What should be paid for the following bill at $9\frac{1}{2}$ per cent. premium? Ans. \$ 486.666.

Exchange for £ 100. *Philadelphia, May* 21, 1857.

Sixty days after sight of this, my first Bill of Exchange, (second and third of the same date and tenor unpaid,) pay to Langdon Shannon, or order, one hundred pounds sterling, value received, with or without further advice.

WILLIAM P. BROWN.

To MESSRS. PEABODY & Co., *Bankers, London.*

OPERATION.

$$100 \times \tfrac{40}{9} = \$444.44;\ \$444.444 \times 1.095 = \$486.666.$$

2. What is the cost at Amsterdam of a bill on New York for \$ 340.67, exchange being at \$ 0.38 to the guilder?

Ans. 896.50 guilders.

OPERATION.

$$340.67 \div 38 = 896.5 = 896 \text{ guilders } 50 \text{ cents.}$$

3. What must be paid in Boston for a bill on Paris for 3676 francs, exchange being 5 francs 20 centimes to the dollar?

Ans. \$ $706.92\frac{4}{13}$.

4. What is the value of a bill on Hamburg for 3000 marks 10 schillings, exchange being at \$ 0.35 to a mark banco?

Ans. \$ $1050.21\frac{7}{8}$.

5. How large a bill can be purchased on Liverpool for \$ 81727.75, exchange being at $9\frac{1}{2}$ per cent. premium?

6. Paid \$ 14400.12 for a bill on Havre for 79000 francs; how much was exchange below par? Ans. 2 per cent.

7. What is the cost of a draft on St. Petersburg for 5763 rubles 75 kopecks, exchange being at 74 cents a ruble?

Ans. \$ 4265.175.

8. What must the face of a bill on Lisbon be which costs \$ 550.66, exchange being at \$ 1.10 a millrea?

Ans. 500 millreas 600 reas.

9. What is the cost at Berlin of a draft on Philadelphia for \$ 10000, exchange being at \$ 0.68 a thaler?

Ans. 14705 thalers 26 groschen $5\frac{1}{17}+$ pfennings.

10. How much must be paid in Chicago for a bill on Stockholm amounting to 400 specie rix dollars 12 skillings?

Ans. \$ 424.265.

11. What is the value in St. Louis of a draft on Dantzic for 300 thalers 20 groschen 10 pfennings, exchange being at $ 0.69 a thaler? Ans. $ 207.47$\frac{1}{12}$.

12. What must be the face of a draft on Calcutta which costs in Boston $ 5694, when exchange is at $ 0.40 a company rupee? Ans. 14235 rupees.

13. How much must be paid in Naples for a draft on Baltimore amounting to $ 615.60, the value of a ducat in exchange being $ 0.80? Ans. 769 ducats 5 carlini.

14. There was paid in New Orleans $ 7300 for 1500£. draft on Liverpool; at what per cent. of premium was it purchased? Ans. 9$\frac{1}{2}$ per cent.

15. What must be the face of a draft on Paris that can be bought in London for 868£. 17s. 6d., exchange at 23 francs 60 centimes a pound sterling? Ans. 20505 francs 45 centimes.

16. How much must be paid in Genoa for a bill on New York whose face is $ 2640, when exchange is at $ 0.18 a lire? Ans. 14666 lire 66$\frac{2}{3}$ centesimi.

17. When a bill on Paris for 88128 francs costs $ 17280, at what per cent is the rate of exchange above par?

18. Robert Anderson, of Cincinnati, has consigned a cargo of pork, valued at 17000£., to Richard Arkwright & Son, Liverpool. Robert S. Davis & Co., being about to import an invoice of books, have purchased of Anderson a bill of exchange, at 8$\frac{1}{2}$ per cent. premium, for the value of the said cargo. What should they pay for the bill? Ans. $ 81977.777.

ARBITRATION OF EXCHANGES.

500. Arbitration of Exchanges is the process of finding the proportional exchange of two places by means of one or more given intermediate exchanges.

Exchange effected thus, through one or more intermediate exchanges, is called *circular* exchange.

The exchange when made through a single intervening ex-

change is called *simple arbitration*, and when through two or more intervening exchanges is called *compound arbitration.*

501. Since the actual course or rate of exchange between any two places is almost always, from various circumstances, different from the arbitrated course, the object of arbitration is to enable an individual in one place to ascertain whether he can most advantageously draw and remit directly between his own place and another, or circuitously through other places.

502. Exchange of merchandise, and the different weights and measures of different countries, may be arbitrated in the same manner as bills of exchange and currencies.

503. The value, in the standards of the United States, of the principal weights and measures of the most important commercial places, is shown in the following

TABLE.

ENGLAND.	Value.
Avoirdupois pound,	1lb.
Old wine gallon,	1gal.
Imperial gallon,	1.20gal.
Old ale gallon,	1.22gal.
Old Winchester bushel,	1bu.
Imp. corn bu. (= 8 imp. gal.),	1.03bu.
Quarter of grain (=8 imp.bu.),	8.25bu.
Imperial yard,	36in.
FRANCE.	
Kilogramme,	2.20lb.
Quintal = 100kil.,	220.54lb.
Millier = 1000kil.,	2205.48lb.
Litre,	2.11pt.
Velt,	2gal.
Decalitre = 10 litre,	2.64gal.
Hectolitre = 100 litre,	26.42gal.
Hectolitre = 100 litre,	2.84bu.
Metre,	39.36in.
HOLLAND AND BELGIUM.	
Pond,	1.08lb.
Fr. kilogramme,	2.20lb.
Last, marine, = 2000 p.	4410lb.
Vat = 100kan = 1 hectol. Fr.,	26.42gal.
Ahm of wine,	41gal.
Mudde=100 kop=1 hectolitre,	2.84bu.
Last of grain,	85.25bu.
Amsterdam ell,	27in.
HAMBURG.	
Pound,	1.06lb.
100 pounds,	106.8lb.
Ahm of wine,	38.25gal.
Fuder = 6 ahms,	229.5gal.
Last of grain,	89.64bu.
Stock = 1½ last,	134.4bu.
Brabant ell,	27.58in.
DENMARK AND NORWAY.	
Pound,	1.10lb.
Centner = 100 pounds,	110.28lb.
Viertel of wine,	2.04gal.
Anker of wine,	10gal.
Ahm = 4 ankers,	40gal.
Fuder of wine,	237.16gal.
Toende or barrel of grain,	3.95bu.
Last = 12 toende	47.50bu.
Danish ell,	24.66in.
SWEDEN.	
Pound,	.93lb.
Pound of iron,	.75lb.
Anker of wine,	10.35gal.
Eimer of wine,	20.75gal.
Ahm = 2 eimers,	41.50gal.
Pipe = 3 ahms,	124.25gal.
Tun or barrel of grain,	4.16bu.
Ell,	23.36in.
BREMEN.	
Pound,	1.09lb.
Centner,	116lb.
Viertel of wine,	1.93gal.
Anker = 5 viertels,	9.65gal.
Oxhoft = 6 ankers,	58gal.
Scheffel of grain,	2bu.
Last = 40 scheffels,	80.70bu.
Ell,	22.75in.

Trieste.

	Value.
Pound,	1.23lb.
100 pounds,	123.60lb.
Eimer of wine,	15gal.
Staro of grain,	234bu.
Ell for silks,	25.20in.
Ell for woollens,	26.60in.

Constantinople.

Quintal,	124.45lb.
Alma for liquids,	1.37gal.
Kisloz of grain,	.94bu.
Pik, commercial,	27in.

Calcutta.

Maund,	74.66lb.
Bazaar maund,	82.13lb.
Guz,	36in.

Russia.

Pound,	.90lb.
Pood = 40 pounds,	36lb.
100 pounds,	90.26lb.
Wedro of wine,	3.25gal.
Sorokovy = 40 wedros,	130gal.
Chetwert of grain,	5.95bu.
Arsheen,	28in.
Sashen,	7ft.

Prussia.

Pound,	1.03lb.
100 pounds, Dantzic,	103.8lb.
Quintal = 110 pounds,	113.42lb.
Eimer of wine,	18.14gal.
Ahm,	39.66gal.
Scheffel of grain,	1.52bu.
Last of grain,	91bu.
Berlin Ell,	25.5in.
Prussian Ell,	26.28in.

Portugal.

Pound or arratel,	1.01lb.
Arroba = 22 arratels,	22.26lb.
Quintal = 4 arrobas,	89.05lb.
100 pounds or arratels,	101.19lb.
Almude of wine,	4.37gal.
Tonelado = 52 almude,	227.25gal.
Moyo,	23.03bu.
Vara,	43.20in.

Spain.

	Value.
Pound,	1.01lb.
Arroba = 25 pounds,	25.38lb.
Quintal = 4 arrobas,	101.52lb.
Cantaro or arroba of oil,	3.75gal.
Cantaro or arroba of wine,	4.25gal.
Moyo of wine = 16 arrobas,	68gal.
Botta = 38ar. of wine = 38½ar. of oil,	127.5gal.
Fanega of grain,	1.57bu.
Cahiz = 12 fanegas,	18.91bu.
Vara or yard,	33.37in.

Cuba.

Quintal,	101.75lb.
Arroba of wine,	4.1gal.
Fanega of grain,	3bu.
Vara,	33.34in.

Naples.

Rottolo,	1.96lb.
Cantaro grosso=100 rottolo,	196.50lb.
Cantaro piccolo,	106lb.
Salma of oil,	42.75gal.
Carro of wine	264gal.
Carro of grain,	52.20bu.
Canna,	83in.

Sicily.

Cantaro grosso,	192.50lb.
Cantaro sottile,	175lb.
100 Sicilian pounds,	70lb.
Tonna,	9.38gal.
Salma grossa,	9.48bu.
Salma generale,	7.62bu.
Conna or yard,	86.40in.

Genoa.

Cantaro grosso,	76.87lb.
Cantaro sottile,	69.89lb.
Mezzarola,	39.25gal.
Mina of grain,	3.50bu.
Canna piccola,	87.50in.
Canna grossa,	116.70in.

Venice.

100 pounds, pesso grosso,	105.18lb.
100 pounds, pesso sottile,	66.42lb.
Miro of oil,	4.02gal.
Anfora of wine,	1.37gal.
Staja of grain,	2.27bu.
Moggio = 4 staji,	9.08bu.
Braccio for silks,	24.84in.
Braccio for woollens,	26.64in.

China.

Catty,	1.33lb.
Pecul,	133.33lb.
Covid,	14.62in.

Note. — The weights and measures of Mexico, Central America, and of the republics of South America are the same generally as those of Spain; of Brazil, the same as those of Portugal; of the British North American Provinces, the same, in general, as in England; and of Hayti, the weights are the same as in the United States, except about 8 per cent. heavier, and the measures the same as in France.

504.° To compute arbitration of exchanges.

Ex. 1. When the exchange between New York and London is at a premium of 9 per cent., and that between London and Paris 25 francs to a pound sterling, how much must be paid in New York for a bill on Paris for 1000 francs?

OPERATION.

$$\begin{aligned} \$40 &= 9£. \\ 109£. &= 100£. \\ 1£. &= 25\,\text{fr.} \\ 1000\,\text{fr.} &= \$ \text{—} \end{aligned}$$

$$\frac{\overset{16}{\cancel{40}} \times 109 \times 1 \times \overset{10}{\cancel{1000}}}{9 \times \cancel{100} \times \cancel{25}} = \$\,193.77+ \text{ Ans.}$$

Since $\$4\frac{4}{9}$ = 1£. of the nominal par of exchange (Article 497), $40=9£.; and 109£. of the same value = 100£. at 9 per cent. premium.

We *write the terms of equivalent value as antecedent and consequent, and proceed as in conjoined proportion* (Art. 341).

NOTE. — When it is required to find which of several routes of exchange is the most advantageous, the rate of exchange by each route may be determined first, and the results then compared.

EXAMPLES.

2. When exchange at Lisbon on Paris is at the rate of 5 francs 95 centimes per millrea, and at Paris on the United States at 5 francs 20 centimes per dollar, how much must be paid in Lisbon to cancel a demand in New York for $ 3500?

Ans. 3058 millreas $823\frac{9}{17}$ reas.

3. A merchant in Boston wishes to pay 2000£. in Liverpool. Exchange on Liverpool he finds is at 10 per cent. premium, on Paris 5 francs 20 centimes to a dollar, and on Hamburg 35 cents to a mark banco; and the exchange between France and England at the same time is 24 francs to a pound sterling, and that of Hamburg on England $13\frac{3}{4}$ marks banco to a pound sterling. Which is the most advantageous course of remittance, that direct to Liverpool, or that through Paris or Hamburg?

4. A merchant of St. Louis wishes to pay a debt of $ 5000 in New York. The direct exchange is $1\frac{1}{2}$ per cent. in favor of New York, but on New Orleans it is $\frac{1}{2}$ per cent. discount; and between New Orleans and New York at $\frac{1}{4}$ per cent. premium. How much would he save by the circular exchange compared with the direct?

Ans. $ $87.56\frac{1}{4}$.

5. A merchant in Boston owes a debt of 9760 thalers in Bremen, to pay which he purchases a bill on London, at a premium of 9 per cent., and remits the same to his agent in England, on whom his creditor is requested to draw. If the exchange between London and Bremen be at the rate of 34d. sterling per thaler, and the charges for brokerage $\frac{1}{2}$ per cent., how much must have been the cost of the bill in New York? Ans. $ 6731.74+.

6. When exchange between New Orleans and Hamburg is at 34 cents per mark banco, and between Hamburg and St. Petersburg is 2 marks 8 schillings per ruble, how much must be paid in St. Petersburg for a bill on New Orleans for $ 650? Ans. 764 rubles $70\frac{10}{17}$ kopecks.

7. When exchange in Philadelphia on Boston is at $\frac{1}{4}$ per cent. premium, and on Chicago at 2 per cent. discount, if the exchange between Chicago and Boston is at par, how much better is the circuitous route of exchange between Philadelphia and Boston than the direct?

8. A merchant, about to import broadcloth, finds he can obtain the quality desired in Amsterdam at 8 guilders per Amsterdam ell; in Berlin, at 3 thalers 15 groschen per Berlin ell; and in England, at 15 shillings per yard. Exchange being on Amsterdam at 40 cents per guilder, on Berlin at 66 cents per thaler, and on England at $9\frac{1}{2}$ per cent. premium, and the freight being the same in each case, from which place can he make the importation to the best advantage? Ans. Berlin.

9. When exchange between Washington and London is at 8 per cent. premium, and between London and Paris 25.25 francs per pound sterling, what sum in Washington is equal to 7000 francs in Paris?

10. A merchant in London remits to Amsterdam 1000£. at the rate of 18d. per guilder, directing his correspondent at Amsterdam to remit the same to Paris at 2 francs 10 centimes per guilder, less $\frac{1}{2}$ per cent. for his commission; but the exchange between Amsterdam and Paris happened to be, at the time the order was received, at 2 francs 20 centimes per guilder. The merchant at London, not apprised of this, drew upon Paris at 25 francs per pound sterling. Did he gain or lose, and how much per cent.? Ans. Gain, $16\frac{56}{75}$ per cent.

ALLIGATION.

505. ALLIGATION is a process employed in the solution of questions relating to the compounding or mixing of articles of different qualities or values.

It is of two kinds: *Alligation Medial*, and *Alligation Alternate.*

ALLIGATION MEDIAL.

506. ALLIGATION MEDIAL is the process of finding the mean or average rate of a mixture composed of articles of different qualities or values, the quantity and rate of each being given.

507. To find the average value of several articles mixed, the quantity and rate of each being given.

Ex. 1. A grocer mixed 2cwt. of sugar worth \$9 per cwt. with 1cwt. worth \$7 per cwt. and 2cwt. worth \$10 per cwt.; what is 1cwt. of the mixture worth? Ans. \$9.

$$\begin{array}{rcrcr} \$9 & \times & 2 & = & \$18 \\ 7 & \times & 1 & = & 7 \\ 10 & \times & \underline{2} & = & \underline{20} \\ & & 5) & & \$45 \\ & & & & \overline{\$9} \text{ Ans.} \end{array}$$

Since 2cwt. at \$9 per cwt. is worth \$18, 1cwt. at \$7 per cwt. is worth \$7, and 2cwt. at \$10 per cwt. is worth \$20; 2cwt. + 1cwt. + 2cwt. = 5cwt. is worth \$18 + \$7 + \$20 = \$45; and 1cwt. is worth as many dollars as 45 contains times 5, or \$9.

RULE. — *Find the value of each of the articles, and divide the sum of their values by the number denoting the sum of the articles. The quotient will be the average value of the mixture.*

EXAMPLES.

2. If 19 bushels of wheat at \$1.00 per bushel should be mixed with 40 bushels of rye at \$0.66 per bushel, and 11 bushels of barley at \$0.50 per bushel, what would a bushel of the mixture be worth? Ans. \$0.727$\frac{1}{7}$.

3. If 3 pounds of gold of 22 carats fine be mixed with 3 pounds of 20 carats fine, what is the fineness of the mixture? Ans. 21 carats.

4. If I mix 20 pounds of tea at 70 cents per pound with 15 pounds at 60 cents per pound, and 80 pounds at 40 cents per pound, what is the value of 1 pound of this mixture?

Ans. \$0.47$\frac{19}{23}$.

ALLIGATION ALTERNATE.

508. Alligation Alternate is a process of finding in what ratio, one to another, articles of different rates of quality or value must be taken, to compose a mixture of a given mean or average rate of quality or value.

509. To find the proportional quantities of the articles of different rates of value that must be taken to compose a mixture of a given mean rate of value.

Ex. 1. A merchant has spices, some at 18 cents a pound, some at 24 cents, some at 48 cents, and some at 60 cents. How much of each sort must be taken that the mixture may be worth 40 cents a pound?

Ans. 1lb. at 18c.; 1lb. at 24c.; 1lb. at 48c.; $1\frac{1}{2}$lb. at 60c.

FIRST OPERATION.

$$\text{Given mean, 40 cents.}\left\{\begin{array}{l} \text{1lb. at 18c., gain 22c.} \\ \text{1lb. at 24c., gain 16c.} \\ \text{1lb. at 48c., loss 8c.} \\ \text{1lb. at 60c., loss 20c.} \\ \hline \tfrac{1}{2}\text{lb. at 60c., loss 10c.} \end{array}\right. \begin{array}{l} \left.\begin{array}{l}\\ \\ \end{array}\right\} = \text{38c., gain.} \\ \left.\begin{array}{l}\\ \\ \\ \end{array}\right\} = \text{38c., loss.} \end{array}$$

PROOF.

1lb. at 18c.	= 18c.
1lb. at 24c.	= 24c.
1lb. at 48c.	= 48c.
$1\frac{1}{2}$lb. at 60c.	= 90c.
$4\frac{1}{2}$lb.	180c.

$$180c. \div 4\tfrac{1}{2} = 40c. \text{ per lb.}$$

Compared with the given mean value, by taking 1lb. at 18cts. there is a gain of 22cts., by taking 1lb. at 24cts. a gain of 16cts., by taking 1lb. at 48cts. a loss of 8cts., and by taking 1lb. at 60cts. a loss of 20cts. Now it is evident that the mixture, to be of the mean or average value given, should have the several items of gain and loss in the aggregate exactly offset one another. This balance we effect by taking $\frac{1}{2}$lb. more of the spice at 60cts.; and thus have a mixture of the required average value, by having taken, in all, 1lb. at 18cts., 1lb. at 24cts., 1lb. at 48cts., and $1\frac{1}{2}$lb. at 60cts. We prove the correctness of this result by dividing the value of the whole mixture by the number of pounds taken.

SECOND OPERATION.

$$40\left\{\begin{array}{l} 18 \\ 24 \\ 48 \\ 60 \end{array}\right. \quad \begin{array}{c} \frac{1}{22} \\ \frac{1}{16} \\ \frac{1}{8} \\ \frac{1}{20} \end{array} \text{ or } \begin{array}{c} 20 \\ 8 \\ 16 \\ 22 \end{array} \text{ or } \begin{array}{c} 10 \\ 4 \\ 8 \\ 11 \end{array} \ \&c.$$

Having arranged in a column the rates of the articles, with the given mean on the left, we connect together terms denoting the rate of the articles, so that a rate less than the given mean is united with one that is greater. We then proceed to find what quantity of each of the two kinds whose rates have been connected can be taken, in making a mixture, so that what shall be gained on the one kind shall be balanced by the loss on the other.

By taking 1lb. at 18cts. the gain will be 22cts.; hence it will require

$\frac{1}{22}$lb. to gain 1ct.; and by taking 1lb. at 60cts. the loss will be 20cts.; hence it will require $\frac{1}{20}$lb. to lose 1ct. Therefore, the gain on $\frac{1}{22}$lb. at 18cts. balances the loss on $\frac{1}{20}$lb. at 60cts. The proportions at these rates are, then, $\frac{1}{22}$ and $\frac{1}{20}$ or (by reducing to a common denominator) $\frac{20}{440}$ and $\frac{22}{440}$, or (by omitting the denominators, which do not affect the ratio) 20 and 22, which is obviously the same result as would be obtained by placing against each rate the difference between the rate with which it is connected and the mean rate. In like manner we determine the quantity that may be taken of the other two articles, whose rates are connected together.

We thus find that there may be taken $\frac{1}{22}$lb. at 18cts., $\frac{1}{16}$lb. at 24cts., $\frac{1}{8}$lb. at 48cts., and $\frac{1}{20}$lb. at 60cts.; or, 20lb. at 18cts., 8lb. at 24cts., 16lb. at 48cts., and 22lb. at 60cts. By dividing the last set by 2, we obtain another set of results, and by multiplying or dividing any of these results others may be found, all of which can be proved to satisfy the conditions of the question. Hence, examples of this kind admit of an indefinite number of answers.

RULE 1. — *Take a unit of each article of the proposed mixture, and note the gain or loss; and then take such additional quantity or quantities of the articles as shall equalize the gain and loss.* Or,

RULE 2. — *Write the rates of the articles in a column, with the mean rate on the left, and connect the rate of each article which is less than the given mean with one that is greater; the difference between the mean rate and that of each of the articles, written opposite to the rate with which it is connected, will denote the quantity to be taken of the article corresponding to that rate.*

NOTE. — When a rate has more than one rate connected with it, the sum of the differences written against it will denote the quantity to be taken. There will be as many different answers as there are different ways of connecting the rates; and, by multiplying and dividing, these answers may be varied indefinitely.

EXAMPLES.

2. How much barley at 45 cents a bushel, rye at 75 cents, and wheat at $1.00, must be mixed, that the composition may be worth 80 cents a bushel?

Ans. 1 bushel of rye, 1 of barley, and 2 of wheat.

3. A goldsmith would mix gold of 19 carats fine with some of 15, 23, and 24 carats fine, that the compound may be 20 carats fine. What quantity of each must he take?

Ans. 1oz. of 15 carats, 2oz. of 19, 1oz. of 23, and 1oz. of 24.

4. It is required to mix several sorts of wine at 60 cents, 80 cents, and $1.20, with water, that the mixture may be worth 75 cents per gallon; how much of each sort must be taken?

Ans. 1gal. of water, 1gal. of 60 cents, 9gal. of 80 cents, and 1gal. of $1.20.

510. When the quantity of one or more of the articles composing a mixture of a given mean value is given, to find the quantity of each of the others.

Ex. 1. How much gold of 15, 17, and 22 carats fine must be mixed with 5 ounces of 18 carats fine, so that the composition may be 20 carats fine?

Ans. 1oz. at 15 carats, 1oz. at 17 carats, 9oz. at 22 carats.

OPERATION.

$$20\left\{\begin{array}{l}\text{1oz. at 15, gain } 5\\ \text{1oz. at 17, gain } 3\\ \text{5oz. at 18, gain } 10\\ \text{1oz. at 22, loss } 2\\ \hline \text{8oz. at 22, loss } 16\end{array}\right.\quad \begin{array}{l}\left.\begin{array}{l}\\ \\ \\ \end{array}\right\}=18\\ \left.\begin{array}{l}\\ \\ \end{array}\right\}=18\end{array}$$

By taking 1oz. at 15 carats fine there is a gain of 5 carats, by taking 1oz. at 17 carats a gain of 3 carats, by taking 5oz., the given quantity, at 18 carats, a gain of 10 carats, and by taking 1oz. at 22 carats a loss of 2 carats; and to balance the gain and loss we take 8oz. additional, at 22 carats, a loss of 16. We have then for the result 1oz. at 15 carats, 1oz. at 17 carats, and 9oz. at 22 carats.

RULE. — *Take of the limited article or articles the quantity or quantities given, with a unit of each of the other articles of the proposed mixture, and note the gain or loss; and then take, if required, such additional quantity or quantities of the articles not limited, as shall equalize the gain and loss.*

EXAMPLES.

2. How much wine at $1.75 and at $1.25 per gallon must be mixed with 20 gallons of water, that the whole may be sold at $1.00 per gallon? Ans. 20 gallons of each.

3. How much wheat at $2.00 per bushel and at $1.80 per bushel must be mixed with 4 bushels at $2.20 per bushel and 10 bushels at $1.70 per bushel to make a mixture worth $1.90 per bushel? Ans. 9 bushels at $2.00; 1 bushel at $1.80.

4. How many pounds of sugar, at 8, 14, and 13 cents a pound, must be mixed with three pounds at 9½ cents, 4 pounds at 10½ cents, and 6 pounds at 13½ cents a pound, so that the mixture may be worth 12½ cents a pound?

Ans. 1lb. at 8cts.; 5½lb. at 13cts.; and 9lb. at 14cts.

5. How much barley at 45 cents a bushel must be mixed with 10 bushels of oats at 58 cents a bushel, to make a mixture worth 50 cents a bushel?

511. When the quantity and rate of a mixture, with the rates of the articles composing it, are given, to find the quantity of each article which is not limited.

Ex. 1. How many gallons of water must be mixed with wine at \$1.50 a gallon so as to make a mixture of 100 gallons worth \$1.20 a gallon? Ans. 20 gallons.

OPERATION.

$$1.20\left\{\begin{array}{l}\text{1gal. at 0.00, gain 1.20} \quad 1.20\\ \left.\begin{array}{l}\text{1gal. at 1.50, loss}\quad .30\\ \hline \text{3gal. at 1.50, loss}\quad .90\end{array}\right\}1.20\end{array}\right.$$

$1 + 4 = 5$gal.; $\frac{1}{5}$ of 100gal. $= 20$gal. Ans.

Representing the rate of the water by 0.00, we then find, as in Art. 509, the quantity required of each article, in composing a mixture of the given mean, to be 1 gallon of water and 4 gallons of the wine. Therefore, the quantity of water is to the whole quantity of the mixture as 1 to 5. Hence, in a mixture of 100 gallons at the mean rate given, the water must be $\frac{1}{5}$ of 100 gallons, or 20 gallons.

RULE. — *Find the proportional quantities of the several articles, as in Art.* 509, *or* 510, *as though the quantity of the mixture were not limited.*

Then take such a part of the given quantity of the mixture, as each of these proportional quantities is of their sum.

EXAMPLES.

2. A merchant has sugar at 8 cents, 10 cents, 12 cents, and 20 cents a pound; with these he would fill a hogshead that would contain 200 pounds. How much of each kind must he take, so that the mixture may be worth 15 cents a pound?

Ans. $33\frac{1}{3}$lb. of 8, 10, and 12cts., and 100lb. of 20cts.

3. How much wheat at \$2.00 and \$1.80 a bushel must be mixed with 4 bushels at \$2.20, and 10 bushels at \$1.70, so as to make a mixture of 48 bushels, worth \$1.90 per bushel?

Ans. 21 bushels \$2.00; 13 bushels at \$1.80.

4. How much gold of 15, 17, and 22 carats fine must be mixed with five ounces of 18 carats fine, to make a composition of 5 pounds, that shall be 20 carats fine?

5. A gentleman's servant having been ordered to purchase 20 animals for \$20, brought home sheep at \$4.00, lambs at \$0.50, and kids at \$0.25 each. Required the number of each kind. Ans. 3 sheep; 15 lambs; and 2 kids.

MISCELLANEOUS EXAMPLES.

1. A manufacturer employs a number of men at $ 1.20, and a number of boys at $ 0.80, per day; and the amount of the wages of the whole is the same as if each had $ 0.97½ per day. Required the number of men, that of the boys being 9.

Ans. 7 men.

2. What is the value of 5000 specie rix dollars 12 skillings of Sweden in United States money? Ans. $ 5300.265.

3.° Exchange between New Orleans and England being in New Orleans at 8 per cent. premium, and in Liverpool at 10 per cent. premium, if L. Sandford of Liverpool owes M. Lassale of New Orleans for cotton to the amount of 1500£. 15s. sterling, what will be the difference between Lassale drawing or Sandford remitting the amount?

4. If 17 gallons of spirits at $ 1.26 per gallon be mixed with 7 gallons at a different price, and 20 per cent. be made by selling the mixture at $ 1.56, what was the price of the latter kind per gallon? Ans. $ 1.39$\frac{5}{7}$ per gallon.

5. What is the value of 100 ounces 20 tari 10 grani of Sicily in lire and centesimi of Leghorn?

Ans. 1510 lire 25 centesimi.

6.° If 20 United States gallons equal 1 eimer of Sweden, 3 eimers of Sweden equal 4 eimers of Trieste, 24 eimers of Trieste equal 9 ahms Danish, and 33 ahms Danish equal 5 carri of Naples, which will cost the most in United States money, 170 eimers of Trieste of wine at 1 florin 45 kreutzers per gallon, or 12 carri of wine at 1 ducat of Naples a gallon?

7.° When exchange on England is at 8 per cent. premium, and freight at 12d. per United States bushel, how much can be paid per bushel for wheat in Baltimore, in answering an order from Liverpool limited to 60s. per imperial quarter?

Ans. $ 1.50$\frac{6}{11}$ per bushel.

8. A merchant mixes 11 pounds of tea with 5 pounds of an inferior quality, and gains 16 per cent. by selling the mixture at 87 cents per pound. Allowing that a pound of the one cost 12 cents more than a pound of the other, what was the cost of each kind per pound?

Ans. The one 78¾cts.; the other 66¾cts. per lb.

INVOLUTION.

512. INVOLUTION is the process of finding the powers of quantities.

A *power* of a number or quantity is the result obtained by taking that quantity a certain number of times as a factor.

513. The number from which a power is derived is called the *root* of that power.

The *first power* is the root, or the number involved.

The *second power* is the product of the root multiplied by itself once, or used twice as a factor.

The *third power* is the root used three times as a factor; &c.

514. The *index* or *exponent* of a power is a small figure written at the right, above the root, indicating the number of times it is employed as a factor. Thus, the second power of 4 is written 4^2, the third power of 9 is written 9^3, and the fourth power of $\frac{3}{4}$ is written $(\frac{3}{4})^4$.

NOTE. — In denoting the power of a fraction, the fraction is included in a parenthesis, in order that the exponent may be regarded as applying to the whole expression, and not to the numerator alone. When no index is written, the number itself is to be considered the first power. The second power is sometimes called the *square* of a number, the third power the *cube*, and the fourth power the *biquadrate*.

515. To raise a number to any required power.

$2 = 2$, the first power of 2, is written 2^1 or 2.
$2\times2 = 4$, the second power of 2, is written 2^2.
$2\times2\times2 = 8$, the third power of 2, " " 2^3.
$2\times2\times2\times2 = 16$, the fourth power of 2, " " 2^4.
$2\times2\times2\times2\times2 = 32$, the fifth power of 2, " " 2^5.

By examining the several powers of 2 in the examples, it is seen that each has been produced by taking the 2 as a factor as many times as there are units in the exponent of each power raised. Hence the

RULE. — *Multiply the given number into itself, till it has been used as a factor as many times as there are units in the exponent of the power to which the number is to be raised.*

NOTE 1. — The number of multiplications will always be *one less* than the number of units in the exponent of the power to be raised, since in the first

multiplication the root is used *twice;* once by being taken as the multiplicand, and once more as the multiplier.

NOTE 2. — A fraction is involved by involving both its numerator and its denominator.

EXAMPLES.

1. What is the 3d power of 8? Ans. 512.
2. What is the 5th power of 4? Ans. 1024.
3. What is the 3d power of $\frac{3}{4}$? Ans. $\frac{27}{64}$.
4. What is the 4th power of $2\frac{2}{3}$? Ans. $50\frac{46}{81}$.
5. What is the 5th power of $\frac{1}{3}$? Ans. $\frac{1}{243}$.
6. What is the 6th power of 5? Ans. 15625.
7. What is the 6th power of $1\frac{3}{5}$? Ans. $16\frac{12144}{15625}$.
8. What is the value of 7^{10}? Ans. 282475249.
9. What is the value of $.045^4$? Ans. .000004100625.

516. To raise a number to any required higher power, without producing all the intermediate powers.

Ex. 1. What is the 7th power of 5? Ans. 78125.

OPERATION.

$$\overset{1}{5},\ \overset{2}{25},\ \overset{3}{125};\quad \overset{3}{125} \overset{+}{\times} \overset{2}{25} \overset{+}{\times} \overset{2}{25} \overset{=}{=} \overset{7}{78125}.$$

We raise the 5 to the 2d and to the 3d power, and write above each power its exponent. Then, by adding the exponent 2 to itself, and increasing the sum by the exponent 3, we obtain 7, a number equal to the exponent of the required power; and by multiplying 25, the power belonging to the exponent 2, into itself, and the product thence arising by 125, the power belonging to the exponent 3, we obtain 78125, the required 7th power. Therefore,

The product of two or more powers of the same number is that power which is denoted by the sum of their exponents. Hence, the

RULE. — *Multiply together two or more powers of the given number, the sum of whose exponents is equal to the exponent of the power required, and the product will be that power.*

NOTE. — When the number to be involved contains a decimal, it is generally sufficient to retain in the result not more than six places of decimals; and the work may be accordingly contracted as in the multiplication of decimals (Art. 273).

EXAMPLES.

2. What is the 7th power of 8? Ans. 2097152.
3. What is the 9th power of 7? Ans. 40353607.
4. What is the 10th power of 6?

5. What is the 5th power of 195? Ans. 281950621875.
6. What is the 6th power of $\frac{2}{3}$? Ans. $\frac{64}{729}$.
7. Required the 2d power of 4698.
8. Required the 2d power of 6031. Ans. 36372961.
9. What is the 13th power of 7? Ans. 96889010407.
10. What is the 12th power of 6?
11. What is the 15th power of 9? Ans. 205891132094649.
12. What is the 4th power of 4.367? Ans. 363.691179+.
13. Involve the following numbers to the powers denoted by their respective exponents: $(2\frac{3}{4})^5$, 1.04^{15}, and $(3\frac{2}{7})^4$.

Ans. $157\frac{283}{1024}$; 1.800943+; $116\frac{1325}{2401}$.

EVOLUTION.

517. Evolution, or the extraction of roots, is the process of finding the roots of quantities. It is the reverse of involution.

518. The root of a quantity or number is such a factor as, being multiplied into itself a certain number of times, will produce that quantity or number.

The root takes the name of the power of which it is the correlative term. Thus, if the number is a *second* power, the root is called the *second* or *square* root; if it is the *third* power, the root is called the *third* or *cube* root; if it is the *fourth* power, its root is called the *fourth* or *biquadrate* root; and so on.

Rational roots are such as can be exactly obtained.

Surd roots are such as cannot be exactly obtained.

519. Roots are usually denoted by writing the radical sign, $\sqrt{}$, before the power, with the index of the root over it; in case, however, of the second or square root, the index 2 is omitted. Thus, the third root of 27 is denoted by $\sqrt[3]{27}$, the second root of 16 is denoted by $\sqrt{16}$, and the fourth root of $\frac{1}{2}$ is denoted by $\sqrt[4]{\frac{1}{2}}$.

Roots are sometimes denoted by a fractional index or exponent, of which the numerator indicates the power, or the number of times the number is to be taken as a factor, and the denominator indicates the root, or the number of equal factors into which that product is to be divided. Thus the square or second root of 12 is denoted by $12^{\frac{1}{2}}$, the fourth root of $\frac{2}{5}$ by $(\frac{2}{5})^{\frac{1}{4}}$, and the square of the cube root of 27, or the cube root of the square of 27, is denoted by $27^{\frac{2}{3}}$.

520. All the rational roots of whole numbers are also whole numbers, since every power of a fractional number is also a fractional number.

521. Prime numbers have no rational roots.

A composite number, to have a given rational root, must have the exponent of the power of each of its prime factors exactly divisible by the exponent of that root.

NOTE. — The number of composite numbers that have rational roots is comparatively small. The number of rational square roots of whole numbers from 1 to 250000 inclusive is only 500, and the number of rational cube roots of whole numbers from 1 to 8000000 inclusive is only 200.

522. The roots represented by the first ten numbers and their first six corresponding powers are shown in the following

TABLE.

1st Power,	1	2	3	4	5	6	7	8	9	10
2d Power,	1	4	9	16	25	36	49	64	81	100
3d Power,	1	8	27	64	125	216	343	512	729	1000
4th Power,	1	16	81	256	625	1296	2401	4096	6561	10000
5th Power,	1	32	243	1024	3125	7776	16807	32768	59049	100000
6th Power,	1	64	729	4096	15625	46656	117649	262144	531441	1000000

NOTE. — It will be observed by the table, that a rational square root can only be obtained from numbers ending in 1, 4, 5, 6, or 9; or in an even number of ciphers, preceded by one of these figures. It is true, also, that, if the square number ends in 1, its square root ends in 1 or 9; if in 4, its square root ends in 2 or 8; if in 9, its square root ends in 3 or 7; if in 6, its square root ends in 4 or 6; and if in 5, its square root ends in 5.

A perfect cube, however, may end in either of the nine digits, and in ciphers if the number of them is three or any multiple of three; also if the cube number ends in 1, its cube root will end in 1; if in 2, its cube root ends in 8; if in 3, its cube root ends in 7; if in 4, its cube root ends in 4; if in 5, its cube root ends in 5; if in 6, its cube root ends in 6; if in 7, its cube root ends in 3; if in 8, its cube root ends in 2; and if in 9, its cube root ends in 9.

EXTRACTION OF THE SQUARE ROOT.

523. The extraction of the square root of a number is the process of finding one of its two equal factors; or of finding such a factor as, when multiplied by itself, will produce the given number.

524. The method generally adopted for extracting the square root depends upon the following principles: —

1. *The square of any number has, at most, only twice as many figures as its root, and, at least, only one less than twice as many.* For the square of any number of a *single* figure consists of either *one* or *two* places of figures, as $1^2 = 1$, and $9^2 = 81$; the square of any number of *two* figures consists of either *three* or *four* places, as $10^2 = 100$, and $99^2 = 9801$; and the same law holds in regard to numbers of *three* or more figures. Therefore, when the square number consists of one or two figures, its root will consist of one figure; when of three or four figures, its root will consist of two figures; when of five or six figures, its root will consist of three figures; and so on. Hence, if a number be separated into as many *periods* as possible of *two figures* each, commencing at the right, to these periods respectively will correspond the *units, tens, hundreds,* &c. of the square root of the number.

2. *The square of a number consisting of* TENS *and* UNITS *is equal to the square of the tens, plus twice the product of the tens into the units, plus the square of the units.* Thus, if the tens of a number be denoted by a and the units by b, the square of the number will be denoted by $(a + b)^2 = a^2 + 2\,a\,b + b^2$. Then, by this formula, if $a = 3$, and $b = 6$, we have 3 *tens* + 6 *units* $= 30 + 6 = 36$; and

$$36^2 = (30 + 6)^2 = 30^2 + 2 \times (30 \times 6) + 6^2 = 1296.$$

Or, analytically,

$$\begin{array}{lclclcl}
a+b & = & 30+6 & = & 30+\ \ 6 & = & 36 \\
a+b & = & 30+6 & = & 30+\ \ 6 & = & 36 \\
\hline
(a+b)\times a & = & 30^2+30\times 6 & = & 900+180 & = & 1080 \\
(a+b)\times b & = & 30\times 6+6^2 & = & 180+36 & = & 216 \\
\hline
(a+b)^2 & = & 30^2+2\times(30\times 6)+6^2 & = & 900+360+36 & = & 1296
\end{array}$$

It is evident, as evolution is the reverse of involution (Art. 517), that from the process now given of obtaining a square may be deduced a method of extracting its root. Since the square of $(a+b)$ is $a^2+2ab+b^2$, the square root of $a^2+2ab+b^2$ must be $a+b$. Now it will be observed that a, the first term of the root, is the square root of a^2, the first term of the square; and if a^2 be subtracted, there will remain $2ab+b^2$, from which b, the second term of the root, is to be obtained. But $2ab+b^2$ is the same as $(2a+b)\times b$, therefore the remainder equals $(2a+b)\times b$. But as b, the units, is always much less than $2a$, twice the tens, we consider that $2a\times b$ is about equal to the whole remainder, and taking $2a$ (which we know) as the trial divisor, we obtain b, the units. But as the true divisor is $2a+b$, we add the units to twice the tens and multiply the sum by the units, which gives a product equal to the whole remainder, or $2ab+b^2$.

Since *every number* of more than one figure may be considered as composed of *tens* and *units*, we may have tens and units of *units*, tens and units of *tens*, tens and units of *hundreds*, &c. Hence, the principle just explained applies equally whether the root contains two or more than two figures.

525. To extract the square or second root of numbers.

Ex. 1. What is the square root of 1296? Ans. 36.

OPERATION.

$$\begin{array}{r|l|l} & 1\dot{2}9\dot{6} & 36 \\ & 9 & \\ \hline 66 & 396 & \\ & 396 & \\ \hline & 0 & \end{array}$$

Beginning at the right, we separate the number into periods of two figures each, by placing a point (·) over the right-hand figure of each period. Since the number of periods is two, the root will consist of two figures, *tens* and *units*. Then 1296 = the square of the tens plus twice the product of the tens into the units, plus the square of the units. The square of tens is hundreds, and must therefore be found in the hundreds of the number. The greatest number of tens whose square does not exceed 12 hundreds is 3, which we write as the tens figure of the root. We subtract the 9 hundreds, the square of the 3 tens, from the 12 hundreds, and there remain 3 hundreds; after which we write the figures of the next period, and the remainder is 396 = twice the product of the tens into the units plus the square of the units. We have then next to find a number which, added to twice the 3 tens of the root, and multiplied into their sum, shall equal 396. By dividing this remainder

by twice the three tens of the root, we may obtain the units, a number somewhat too large. But though it may be too large, it cannot be too small, since the remainder 396 contains twice the product of the tens into the units, and also the square of the units. We therefore make twice the three tens of the root = 6 tens, a trial divisor, with which we divide the 39 tens, exclusive of the 6 units, which cannot form any part of the product of the tens by the units. The quotient figure obtained, 6, must be the units figure of the root, or a number somewhat larger. To determine whether it expresses the real number of the units in the root, we annex it to the 6 tens, and multiply the number 66, thus formed, by it. The product is 396, which being subtracted, there is no remainder. Therefore 1296 is a perfect square, and 36 the root sought.

2. What is the square root of 278784? Ans. 528.

OPERATION.

```
      2 7 8 7 8 4 | 5 2 8
      2 5
 1 0 2 | 2 8 7
       | 2 0 4
1 0 4 8 | 8 3 8 4
        | 8 3 8 4
              0
```

PROOF.

$528 \times 528 = 278784$

Since there are three periods, the root will contain three figures; the first two may be considered as *tens and units of* TENS. As the square of tens cannot give less than hundreds, we must find that square in the two left-hand periods; and as we have *tens and units* of tens, their square = the square of the tens plus twice the tens into the units, plus the square of the units = 2787 (nearly). We proceed then with the first two periods exactly the same as when the root consists of but two figures, and thus take from the given number the square of the 52 tens, which leaves a remainder of 8384. We now consider the given number 278784, as the square of a number consisting of 52 tens and a certain number of units, which square will of course equal the square of the tens plus twice the tens into the units, plus the square of the units. But the square of the tens, or $(52)^2$ has already been taken from the given number, leaving a remainder, 8384, which must equal twice the tens into the units plus the square of the units. From this we readily obtain the units, just as when we had but two figures in the root.

RULE.— *Separate the given number into as many periods as possible of two figures each, by placing a point over the place of units, another over the place of hundreds, and so on.*

Find the greatest square in the left-hand period; write the root of it at the right of the given number after the manner of a quotient in division, and subtract the second power from the left-hand period.

Bring down the next period to the right of the remainder for a dividend, and double the root already found for a trial divisor. Find how often this divisor is contained in the dividend, exclusive of the right-hand figure, and write the quotient as the next figure of the root.

Annex the last root figure to the trial divisor for the true divisor, which multiply by the last root figure and subtract the product from the dividend. To the remainder bring down the next period for a new dividend.

Double the root already found for a new trial divisor, and continue the operation as before, till all the periods have been brought down.

NOTE 1. — When the product of any trial divisor exceeds its corresponding dividend, the last root figure must be made less.

If a dividend does not contain its corresponding divisor, a cipher must be placed in the root, and also at the right of the divisor; then, after bringing down the next period, this last divisor must be used as the divisor of the new dividend.

NOTE 2. — When there is a remainder after extracting the root of a number, periods of ciphers may be annexed, and the figures of the root thus obtained will be decimals.

NOTE 3. — If the given number is a decimal, or a whole number and a decimal, the root is extracted in the same manner as in whole numbers, except, in pointing off the decimals, either alone or in connection with the whole number, we place a point over every second figure toward the right, from the separatrix, filling the last period, if incomplete, with a cipher. The number of decimal places in the root will always equal the number of periods of decimals in the power.

NOTE 4. — If the given number is a common fraction, reduce it to its simplest form, if it is not so already, and extract the root of both terms, if they are perfect powers; otherwise, either find their product, extract its root, and divide the result by the denominator, or reduce the fraction to a decimal, and extract the root of the decimal.

NOTE 5. — When the given number is a mixed number, it may be changed to the form of a common fraction, or the fractional part may be reduced to a decimal, before attempting to extract the root.

EXAMPLES.

3. What is the square root of $\frac{144}{15129}$? Ans. $\frac{4}{41}$.

$$\sqrt{\tfrac{144}{15129}} = \tfrac{12}{123} = \tfrac{4}{41}.$$

4. What is the square root of $\frac{3}{80}$? Ans. .1936+.

$$3 \times 80 = 240;\ \frac{\sqrt{240}}{80} = .1936+.$$

$$\text{Or } \tfrac{3}{80} = .0375;\ \sqrt{.0375} = .1936+.$$

5. What is the square root of 3444736? Ans. 1856.
6. What is the square root of 998001? Ans. 999.
7. What is the square root of $\frac{343}{1008}$?
8. Extract the square root of 234.09? Ans. 15.3.
9. What is the square root of $42\frac{1}{4}$? Ans. $6\frac{1}{2}$.

10. What is the square root of .000729? Ans. .027.
11. What is the square root of 17.3056? Ans. 4.16.
12. What is the square root of $52\frac{9}{16}$? Ans. $7\frac{1}{4}$.
13. What is the square root of $95\frac{1}{16}$? Ans. $9\frac{3}{4}$.
14. What is the square root of $363\frac{1}{361}$? Ans. $19\frac{1}{19}$.
15. How much is $\sqrt{1.96}$? Ans. 1.4.
16. How much is $6561^{\frac{1}{2}}$? Ans. 81.
17. How much is $\sqrt{9^3}$? Ans. 27.
18. How much is 8^{2}? Ans. 64.
19. How much is one of the two equal factors of 9645192360241? Ans. 3105671.

526. When the square root is to be extracted to many places of figures, the work may be contracted thus: —

Having found in the usual way one more than half of the root figures required, the rest may be found by dividing the last remainder, with a single figure annexed instead of two, by the last divisor, and proceeding as in contracted division of decimals. (Art. 276.)

Examples.

1. What is the square root of 785 to five places of decimals? Ans. 28.01785.

CONTRACTED METHOD.

```
      7̇85̇|28.01785+
      4   |
     ------
  48 |385
     |384
     ------
5601 |10000
     |  5601
     --------
 5602|43990
     |39219
     --------
       4771
       4482
       ----
        289
        280
        ---
          9
```

COMMON METHOD.

```
         7̇85̇|28.01785+
         4   |
        ------
     48 |385
        |384
        ------
   5601 |10000
        |  5601
        --------
  56027 |439900
        |392189
        --------
 560348 |4771100
        |4482784
        ---------
5603565 |28831600
        |28017825
        ---------
           813775
```

The nature and extent of the contraction will be seen by comparing the contracted method with the common method.

2. Extract the square root of $6\frac{2}{5}$ to four places of decimals. Ans. 2.5298+.

3. Required the square root of 2 to five places of decimals. Ans. 1.41421+.

4. Required the square root of 3.15 to eight places of decimals. Ans. 1.77482393+.

5. Required the square root of 373 to seven places of decimals. Ans. 19.3132079+.

6. Extract the square root of 8.93 to eight places of decimals. Ans. 2.98831055+.

EXTRACTION OF THE CUBE ROOT.

527. The extraction of the cube root of a number is the process of finding one of its three equal factors; or, of finding a factor which, being multiplied into itself twice, will produce the given number.

528. The common method of extracting the cube root depends upon the following principles: —

1. *The cube of any number has, at most, only three times as many figures as its root, and, at least, only two less than three times as many.* For the cube of a number of a *single* figure consists of, at most, *three* figures, and, at least, two less than that number, as $1^3 = 1$, and $9^3 = 729$; the cube of a number of *two* figures consists of, at most, *six* figures, and, at least, two figures less than that number, as $10^3 = 1000$, and $99^3 = 970299$; and so on. Therefore, when a cube number consists of one, two, or three figures, its root will consist of one figure; when of four, five, or six figures, its root will consist of two figures, and so on; and if a number be separated into as many periods as possible of *three figures*, each commencing at the right, to these periods respectively will correspond the *units, tens, hundreds,* &c. of the cube root of that number.

2. *The cube of a number consisting of* TENS *and* UNITS *is equal to the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units.* Thus, if the tens of a number be denoted by a, and the units by b, the cube of the number

will be denoted by $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Then, by this formula, if $a = 3$, and b equal 6, we have 3 *tens* $+$ 6 units $= 30 + 6 = 36$, and $36^3 = (30+6)^3 = 30^3 + 3(30^2 \times 6) + 3(30 \times 6^2) + 6^3 = 46656$. Or, analytically,

$$
\begin{array}{lll}
a+b & = 30 + 6 & = 36 \\
a+b & = \underline{30 + 6} & = \underline{36} \\
(a+b) \times a & = 30^2 + 30 \times 6 & = 1080 \\
(a+b) \times b & = \underline{\qquad\quad 30 \times 6 + 6^2} & = \underline{216} \\
(a+b)^2 & = 30^2 + 2 \times 30 \times 6 + 6^2 & = 1296 \\
a+b & = \underline{30 + 6 \qquad\qquad\qquad\quad} & = \underline{36} \\
(a+b)^2 \times a & = 30^3 + 2 \times 30^2 \times 6 + 30 \times 6^2 & = 38880 \\
(a+b)^2 \times b & = \underline{\qquad\qquad 30^2 \times 6 + 2 \times 30 \times 6^2 + 6^3} & = \underline{7776} \\
(a+b)^3 & = 30^3 + 3\ (30^2 \times 6) + 3\ (30 \times 6^2) + 6^3 & = 46656
\end{array}
$$

It is evident, as evolution is the reverse of involution, that from this process of obtaining a cube may be deduced a method of extracting the cube root. Since the cube of $a + b$ is $a^3 + 3a^2b + 3ab^2 + b^3$, the cube root of $a^3 + 3a^2b + 3ab^2 + b^3$ must be $a + b$. Now a, the first term of the cube root, is the cube root of a^3, the first term of the cube; and if a^3 be subtracted, there will remain $3a^2b + 3ab^2 + b^3$, from which b, the second term of the root, is to be obtained. But $3a^2b + 3ab^2 + b^3$ is the same as $(3a^2 + 3ab + b^2) \times b$; therefore the remainder equals $(3a^2 + 3ab + b^2) \times b$. But as $3ab$, three times the tens into the units, plus b^2, the square of the units, is generally much less than $3a^2$, three times the square of the tens, we consider that $3a^2 \times b$ is about equal to the whole remainder, and taking $3a^2$ (which we know) as the trial divisor, we obtain b, the units. But as the true divisor is $3a^2 + 3ab + b^2$ we add three times the tens by the units plus the square of the units, and multiply the sum by the units, which gives a product equal the whole remainder, or $3a^2b + 3ab^2 + b^3$.

Since *every number* of more than one figure may be considered as composed of *tens* and *units*, we may have tens and units of *units*, tens and units of *tens*, tens and units of *hundreds*, &c. Hence, the principle just explained applies equally whether the root contains two or more than two figures.

529. To extract the cube or third root of numbers.

Ex. 1. What is the cube root of 46656? Ans. 36.

OPERATION.

$$
\begin{array}{lrl|l}
 & & 4\dot{6}65\dot{6} & 36 \\
 3^3 = & & 27 & \\
\text{Trial div., } 3 \times 30^2 = & 2700 & \overline{19656} & \\
3 \times 30 \times 6 = & 540 & & \\
6^2 = & \underline{36} & & \\
\text{True divisor,} & 3276 \times 6 = & 19656 & \\
 & & \overline{\quad 0} &
\end{array}
$$

PROOF.

$$36 \times 36 \times 36 = 46656$$

Beginning at the right, we separate the given number into periods, by placing a point over the units figure and over the third figure to the left. Since the number of periods is two, the root will consist of two figures, *tens* and *units*. Then 46656 = the cube of tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units. The cube of tens is thousands, and must therefore be found in the thousands of the number. The greatest number of tens whose cube does not exceed 46 thousands is 3, which we write as the tens figure of the root. We then subtract the 27 thousands, the cube of the 3 tens, from the 46 thousands, and there remain 19 thousands; and, annexing the next period, we have as the entire remainder, 19656, equal three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units, or the product of three times the square of the tens, plus three times the tens into the units, plus the square of the units, multiplied by the units. By dividing this remainder by three times the square of the tens of the root, we obtain the units, or a number somewhat too large. Although it may be too large, it cannot be too small, since the remainder 19656 contains not only three times the square of the tens into the units, but three times the tens into the square of the units, plus the cube of the units. We therefore make three times the square of the tens of the root, = 27 hundreds, a trial divisor, with which we divide the 196 hundreds of the remainder, disregarding the 56 units, since they cannot form any part of the product of the square of the tens by the units. The quotient figure obtained, 7, must be the units figure of the root, or a number somewhat larger.

But on undertaking to complete the divisor on the supposition that 7 is the true units figure of the root, we find a divisor too large for the remainder. We therefore take 6, a number one less, and to determine whether it expresses the real number of units in the root, we add to the 27 hundreds of the trial divisor three times the 3 tens of the root into the 6 units, plus the square of the 6 units; and multiplying the true divisor, 3276, thus formed, by the units, and subtracting the product, 19656, from the remainder, there is nothing left. Hence, 46656 is a perfect cube, and 36 its cube root.

2. What is the cube root of 12326391? Ans. 231.

FIRST OPERATION.

$$
\begin{array}{lrl|l}
 & & & 1\dot{2}\,3\,2\,\dot{6}\,3\,9\,\dot{1} \mid 2\,3\,1 \\
 & 2^3 = & & 8 \\
\text{Trial divisor, } & 3 \times 20^2 = & 1200 & 4\,3\,2\,6 \\
 & 3 \times 20 \times 3 = & 180 & \\
 & 3^2 = & 9 & \\
\text{True divisor, } & & 1389 \times 3 = & 4\,1\,6\,7 \\
\text{Trial divisor, } & 3 \times 230^2 = & 158700 & 1\,5\,9\,3\,9\,1 \\
 & 3 \times 230 \times 1 = & 690 & \\
 & 1^2 = & 1 & \\
\text{True divisor, } & & 159391 \times 1 = & 1\,5\,9\,3\,9\,1
\end{array}
$$

Since there are three periods the root will contain three figures, the first two of which may be considered as tens and units of tens. As the cube of tens cannot give less than thousands, we must find that cube in the two left-hand periods; and as we have *tens* and *units* of tens, their cube will equal the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units. This we apply to the first two periods exactly the same as when the root consists of but two figures, and thus take from the given number the cube of the 23 tens, which leaves a remainder of 159391. We now consider the given number, 159391, as the cube of a number consisting of 23 tens and a certain number of units, which cube will of course equal the cube of the tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units. But the cube of the tens, or $(23)^3$, has already been taken from the given number, leaving a remainder, 159391, which must equal three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units; or equal the product of three times the square of 23, plus three times 23 into the units, plus the square of the units, multiplied by the units. From this we readily obtain the units, just as when we had but two figures in the required root.

SECOND OPERATION.

$$
\begin{array}{lcl|l}
 & & & 1\dot{2}\,3\,2\,\dot{6}\,3\,9\,\dot{1} \mid 2\,3\,1 \\
2^3 & = & & 8 \\
\text{Trial divisor, } & = & 1200 & 4\,3\,2\,6 \\
63 \times 3 & = & 189 & \\
\text{True divisor, } & = & 1389 \times 3 = & 4\,1\,6\,7 \\
3^2 & = & 9 & 1\,5\,9\,3\,9\,1 \\
\text{Trial divisor, } & = & 158700 & \\
691 \times 1 & = & 691 & \\
\text{True divisor, } & & 159391 \times 1 = & 1\,5\,9\,3\,9\,1
\end{array}
$$

In the second operation the work is somewhat abridged by rendering shorter the method of finding each divisor after the first, of both kinds. Thus, the second true divisor is obtained by prefixing to the second root figure, 3, three times the part of the root preceding it, or $3 \times 2 = 6$, and adding 189, the product of the number 63 thus formed by the last root figure, 3, to the preceding trial divisor, or $1200 + 189 = 1389$, second true divisor. This is equivalent to adding to the trial divisor, in forming the true divisor, three times the tens into the units, plus the square of the units, or $3 \times 20 \times 3 + 3^2$. For the second trial divisor we annex two ciphers to the sum found by adding together the square of the last root figure, the last true divisor, and the number standing over it, or $9 + 1389 + 189 = 1587$, with two ciphers annexed $= 158700$, the second trial divisor. This is equivalent to taking for the trial divisor three times the square of the tens, or the part of the root already found, or 3×230^2. The next true divisor is found in like manner as was the second true divisor.

RULE 1. — *Separate the given number into as many periods as possible of three figures each, beginning at the units' place.*

Find the greatest cube in the left-hand period, and write its root as the first figure of the required root. From that period subtract the cube, and to the remainder bring down the next period for a dividend.

Multiply the square of the root figure by 3, and to the product annex two ciphers for a trial divisor, and see how often it is contained in the dividend, and write the result as the next figure of the root.

Add to the trial divisor three times the product of the tens figure of the root by the units figure with a cipher annexed, and the square of the last figure, for a true divisor.

Multiply the true divisor by the last figure of the root; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.

Multiply the square of the root figures already found by 3, and to the product annex two ciphers for a new trial divisor; and proceed as before until all the periods are brought down. Or,

RULE 2. — *Having found the first trial divisor and determined the second root figure as by the preceding rule,* —

Take three times the part of the root already found, except the last figure, to it annex the last figure of the root, multiply the result by the figure annexed, and write the product below the trial divisor, and add it to the same for a true divisor.

Multiply the true divisor by the last figure of the root: subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.

To the last true divisor and the number immediately over it, add the square of the last root figure, and to the sum annex two ciphers for a new trial divisor; then proceed as before.

NOTE 1. — The observations made in Notes 1, 2, 3, and 5, under the rule for the extraction of the square root (Art. 525), are equally applicable to the ex-

traction of the cube root, except that two ciphers must be placed at the right of a true divisor when it is not contained in its corresponding dividend, and in pointing off decimals each period must contain three figures.

NOTE 2.—If the given number is a common fraction, reduce it to its simplest form, if it is not so already, and extract the root of both terms, if they be perfect powers; otherwise, either find the product of the numerator by the square of the denominator, extract its root, and divide the result by the denominator; or reduce the fraction to a decimal, and extract the root of the decimal.

EXAMPLES.

3. What is the cube root of 77308776? Ans. 426.
4. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.
5. What is the cube root of 84.604519? Ans. 4.39.
6. What is the cube root of 54439939? Ans. 379.
7. What is the cube root of 60236288? Ans. 392.
8. Required the cube root of .726572699. Ans. .899.
9. Extract the third root of 109215352. Ans. 478.
10. What is the third root of $\frac{1331}{4913}$? Ans. $\frac{11}{17}$.
11. What is the value of $\sqrt[3]{\frac{59319}{68921}}$? Ans. $\frac{39}{41}$.
12. What is the value expressed by $\sqrt[3]{34965783}$? Ans. 327.
13. What is the value of $122615327232^{\frac{1}{3}}$? Ans. 4968.
14. What is the value of $436036824287^{\frac{1}{3}}$? Ans. 7583.

530. When the cube root is to be extracted to many places of decimals, the work may be contracted thus:—

Having found in the usual way one more than half of the root figures required, the rest may be found by dividing the last remainder by its corresponding true divisor, as in contracted division of decimals (Art. 276), *observing however at each step to reject two figures from the right of the divisor and one from the right of the remainder.*

EXAMPLE.

1. Required the cube root of 2 to four places of decimals. Ans. 1.2599+.
2. Find the third root of 11 to four places of decimals. Ans. 2.2239.
3. Extract the cube root of 3 to six places of decimals. Ans. 1.442249+.
4. Extract the cube root of 9 to fifteen places of decimals. Ans. 2.080083823051904.

EXTRACTION OF ANY ROOT.

531. *The root corresponding to any perfect power may be obtained by resolving that power into its prime factors, and multiplying together one of each number of equal factors denoted by the exponent of the required root.* Thus one of each *two* equal factors of the power will give the *second* or *square* root; one of each *three* equal factors will give the *third* or *cube* root; one of each *four* equal factors will give the *fourth* root; and so on.

532. *When the index or exponent of the root to be extracted is a composite number, the root may be obtained by successive extractions of the simpler roots denoted by the several factors of that exponent.* Thus the *fourth* root may be obtained by extracting the square root twice in succession; the *sixth* root by extracting the square root and then the cube root; and so on.

EXAMPLES.

1. Required the fourth root of 50625? Ans. 15.

BY FACTORS.

```
  5|50625
  5|10125
  5|2025
× 5|405
  3|81
  3|27
  3|9
  × 3
5 × 3 = 15, Ans.
```

BY SUCCESSIVE EXTRACTIONS.

```
    50625|225
    4
 42|106
   | 84
445|2225
   |2225
       0

    225|15, Ans.
    1
 25|125
   |125
      0
```

2. What is the square root of 998001? Ans. 999.
3. What is the cube root of 262144? Ans. 64.
4. What is the fourth root of 43046721?
5. What is the fifth root of 14348907? Ans. 27.
6. What is the sixth root of 11390625? Ans. 15.

533.° When the given number is an imperfect power, or otherwise, or the exponent denoting the root is prime, or otherwise, the required root may be found by an elegant process, perfect in principle, called from its inventor,

Horner's Method.

Ex. 1. Required the cube root of 92959677. Ans. 453.

OPERATION.

0	0	92959677	453
4	16	64	
4	16	28959	
4	32	27125	
8	4800	1834677	
4	625	1834677	
120	5425	0	
5	650		
125	607500		
5	4059		
130	611559		
5			
1350			
3			
1353			

The greatest cube contained in the left-hand period we find to be 64, whose root, 4, we write as the first figure of the required root. This figure, 4, we write under the cipher of the first column; and adding it to the cipher obtain 4, which sum multiplied by the 4 gives 16; and the result, 16, we write under the cipher of the second column, and by addition obtain 16, which sum, multiplied by the 4, gives 64; and the result, 64, we write in the last column under the left-hand period, 92, of the given number. The 64 subtracted from the 92 above it gives for a remainder 28. We next add the 4 to the last term, 4, of the first column, obtaining 8; and the result, 8, multiplied by the 4, gives 32, which we write under the last term, 16, of the second column, and, adding the same together, obtain 48. We next add the 4 to the last term, 8, of the first column, obtaining 12; and, annexing one cipher to the last term, 12, of the first column, obtaining 120, two ciphers to the last term of the second column, obtaining 4800, and to the remainder in the last column bringing down the next period, 959, obtaining 28959, we complete the work preparatory to the finding of the second root figure.

To determine that root figure, we take the last term, 4800, of the second column, for a trial divisor, and the last term, 28959, of the last column, for a dividend; and, dividing, 6 would appear to be the second figure of the root. This, on trial, however, is found to be too large; we therefore take 5, which answers. This 5 we add to the last term, 120, of the first column, obtaining 125; which sum, 125, multiplied by the 5, gives 625, and that product, added to the last term, 4800, of the second column, gives 5425; and this result, 5425, multiplied by the 5, gives 27125, which, written in the last column and subtracted from the figures above it, gives a remainder 1834. Then we add the 5 to the last term, 125, of the first column, obtaining 130; and the result, 130, multiplied by the 5, gives 650, which we write under the last term, 5425, of the second column, and by addition obtain 6075. We next add the 5 to the last term, 130, of the first column, obtaining 135; and, annexing one cipher to the last term, 135, of the first column, obtaining 1350, two ciphers to the last term, 6075, of the second column, obtaining 607500, and to the remainder

in the last column bringing down the next period, 677, obtaining 1834677, the work is completed preparatory to finding the third root figure.

To determine that figure, as before, we divide the last term of the third column by the last term of the second. We thus obtain 3, which, added to the last term of the first column, gives 1353, which sum, multiplied by the 3, gives 4059; and that product, being added to the last term of the second column, gives 611559; and that sum, multiplied by the 3, gives 1834677, which being exactly as large as the last term of the third column, on being written under it and subtracted there is no remainder. The given number is therefore a perfect power, and the cube root sought is 453.

In practice, the work may be performed with less figures, by writing down in the several columns only the results.

RULE. — *Commence as many columns as there are units in the exponent of the root to be extracted, by writing the given number as the head of the right-hand column, and a cipher as the head of each of the others.*

Separate the given number into as many periods as possible of as many figures each as the exponent of the root requires; and having found the nearest root of the left-hand period, write it as the first figure of the required root.

Write this figure in the first column, and, having added it to what stands above it, multiply the sum by the same figure, and write the product in the second column; add, in like manner, in the second column, and multiply the sum by the same figure, writing the product in the third column; and so proceed, writing the last product in the last column, and subtracting it from what stands above it.

Then add the same figure to the last term of the first column, multiply the sum by the same figure, and add the product to the last term of the second column; and so on, writing the last product in the last column but one. Repeat the process, stopping each time with one column farther to the left, till the last product shall fall in the second column.

Add the figure found for the root to the last term of the first column; annex one cipher to the last number in the first column, two ciphers to the last number in the second column, and so on; and to the last number in the last column bring down the next period for a dividend.

Take the last term of the column next to the last for a trial divisor, and see how often it is contained in the dividend, and write the result as the next figure of the root.

Add this figure to the last term of the first column, multiplying the sum by the same figure, add the product to the second column, and so on; proceed as before, till all the periods have been brought down, or an answer sufficiently exact has been obtained.

NOTE 1. — When any dividend will not contain its corresponding trial divisor, write a cipher in the root, bring down to the dividend another period, annex an additional cipher to the last term of the first column, two additional

ciphers to the last term of the second column, and so on; and use the same trial divisor as before, increased however by the additional ciphers.

NOTE 2. — When the given number does not have an exact root, periods of ciphers may be annexed.

NOTE 3. — When the root is required to many places of decimals, the work may be contracted by rejecting one figure at the right from the number in the column next to the last, two from the number in the column next farther to the left, and so on, and otherwise proceeding as directed in the rule, except that the new figure of the root is not added to the first column. As soon as all the figures are rejected in the number of the first column, the remainder of the work may be performed as in contracted division of decimals (Art. 276).

EXAMPLES.

2. Required the fourth root of 1.016397 to eight places of decimals. Ans. 1.00407427.

OPERATION.

0	0	0	1.01639700	1.00407427
1	1	1	1	
1	1	1	1639700	
1	2	3	1609632	
2	3	4000000	30068	
1	3	2408·	28338	
3	60000	402408	1730	
1	2·	2416	1619	
400	602	404824	111	
	2		81	
	604		30	
	2		28	
	606		2	

3. What is the cube root of 41673648563? Ans. 3467.

4. What is the cube root of 43614208?

5. What is the cube root of 1.05 to six places of decimals? Ans. 1.016397.

6. What is the fifth root of 184528125? Ans. 45.

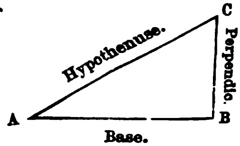
7. Required the fourth root of 100 to six places of decimals? Ans. 3.162278.

8. Required the fifth root of the fourth power of 9 to seven places of decimals? Ans. 5.7995466.

APPLICATIONS OF POWERS AND ROOTS.

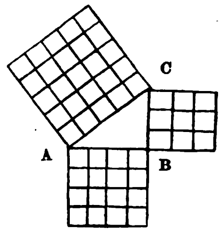
534. A TRIANGLE is a figure having three sides and three angles. When one of the sides of a triangle is perpendicular to another side, the opening between them is called a *right angle*, and the triangle is called a right-angled triangle.

The lowest side, A B, is called the *base* of the triangle A B C, the side B C the *perpendicular*, the longest side, A C, the *hypothenuse*, and the angle at B is a *right angle*. Also, the line B C, being perpendicular to the base, is the altitude.



535. The *square* described upon the hypothenuse of a right-angled triangle is equivalent to the sum of the squares described upon the other two sides.

Thus, if the hypothenuse A C be 5 feet, the base A B 4 feet, and the perpendicular B C 3 feet, then $5^2 = 4^2 + 3^2$, or $25 = 16 + 9$.



536. To find the *hypothenuse*, the base and perpendicular being given.

Add together the square of the base and the square of the perpendicular, and extract the square root of the sum.

Thus, if the base be 4 and the perpendicular 3, the hypothenuse will equal $\sqrt{4^2 + 3^2} = \sqrt{25} = 5$.

537. To find the *perpendicular*, the base and hypothenuse being given.

Subtract the square of the base from the square of the hypothenuse, and extract the square root of the remainder.

Thus, if the base be 4 and the hypothenuse 5, the perpendicular will equal $\sqrt{5^2 - 4^2} = \sqrt{9} = 3$.

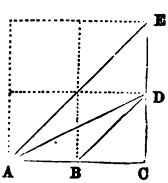
538. To find the *base*, the hypothenuse and perpendicular being given.

Subtract the square of the perpendicular from the square of the hypothenuse, and extract the square root of the remainder.

Thus, if the perpendicular be 3 and the hypothenuse 5, the base will equal $\sqrt{5^2 - 3^2} = \sqrt{16} = 4$.

539. All *triangles* having the same base are to each other as their altitudes.

All *similar triangles*, and other similar *rectilineal figures*, are to each other as the squares of their homologous or corresponding sides.

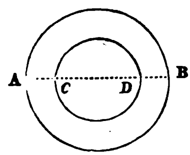


Thus, the triangles A C E and A C D, having the same base, A C, are to each other as the altitude E C of the one is to the altitude D C of the other.

Also, the triangles A C E and B C D, having their corresponding angles the same, and their sides in direct proportion, are said to be similar, and are to each other as the squares of their corresponding sides, or as $(A\ E)^2$ is to $(B\ D)^2$, $(A\ C)^2$ is to $(B\ C)^2$, and $(C\ E)^2$ is to $(C\ D)^2$. Likewise the larger square, of which A C is one of the equal sides, is to the smaller square, of which B C is one of the equal sides, as $(A\ C)^2$ is to $(B\ C)^2$.

540. All *circles* (Art. 143) are to each other as the squares of their diameters, semidiameters, or circumferences.

The *circumference* of a circle is the line which bounds it; and the *diameter* is a line drawn through the centre, and terminated by the circumference; as A B and C D.



Then, the larger circle, of which A B is the diameter, is to the smaller, of which C D is the diameter, as $(A\ B)^2$ is to $(C\ D)^2$, &c.

541. To find the *side*, *diameter*, or *circumference* of any surface, which is similar to a given surface.

State the question as in Proportion, and square the given sides, diameters, or circumferences, and the square root of the fourth term of the proportion will be the answer required.

Thus, if 12 feet be the length of a side of a triangle whose area is 72 square feet, the length of the corresponding side of a

similar triangle whose area is 32 square feet would be found as follows:

$72 : 32 :: 12^2 = 144 : 64$; $\sqrt{64} = 8$ feet, length required.

542. To find the *area* of any surface which is similar to a given surface.

State the question as in Proportion, and square the given sides, diameters, or circumferences, and the fourth term of the proportion will be the answer required.

Thus, if 72 square feet be the area of a triangle of which 12 feet is one of the sides, the area of a similar triangle of which the corresponding side is 8 feet would be found as follows:

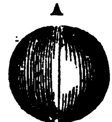
$12^2 = 144 : 8^2 = 64 :: 72$ sq. ft. : 32 sq. ft., area required.

543. To find the *side* of a square equal in area to any given surface.

Find the square root of the given area, and that root will be the side of the area required.

544. A *sphere* is a solid bounded by a continued convex surface, every part of which is equally distant from the point within called the centre.

A



B

The *diameter* of a sphere is a straight line passing through the centre, and terminated by the surface; as A B.

545. A CONE is a solid having a circle for its base, and tapering uniformly to a point, called the *vertex*.

C

A B D

The *altitude* of a cone is its perpendicular height, or a line drawn from the vertex perpendicular to the plane of the base, as B C. The *diameter* of its base is a straight line drawn through the centre of the plane of the base from one side of the circle to the other; as A D.

546. *Spheres* are to each other as the cubes of their diameters, or of their circumferences.

Similar *cones* are to each other as the cubes of their altitudes, or of the diameters of their bases.

All *similar solids* are to each other as the cubes of their homologous or corresponding sides, or of their diameters.

NOTE. — Cones and other solids are said to be similar when their corresponding parts are in direct proportion to each other.

547. To find the *contents* of any solid which is similar to a given solid.

State the question as in Proportion, and cube the given sides, diameters, altitudes, or circumferences, and the fourth term of the proportion is the answer required.

548. To find the *side, diameter, altitude,* or *circumference* of any solid, which is similar to a given solid.

State the question as in Proportion, and cube the given sides, diameters, altitudes, or circumferences, and the cube root of the fourth term of the proportion is the answer required.

549. To find the *side* of a cube that shall be equal in solidity to any given solid.

Find the cube root of the contents of the given solid, and that root will be the side of the cube required.

550. To find *a mean proportional* (Art. 333) between any two numbers.

Find the square root of the product of the two numbers, and that root will be the mean proportional required.

551. To find *two mean proportionals* between two given numbers.

Find the cube root of the quotient of the greater of the two numbers divided by the less. The product of the less number by that root will be the least mean proportional; and the quotient of the greater number by the same root will be the other mean proportional.

552. To find any two numbers, whose *sum* and *product* are given.

From the square of half the sum of the two numbers subtract their product, and the square root of the remainder will equal half the difference of the two numbers, which added to half their sum will give the larger, and subtracted from half their sum will give the smaller, of the numbers required.

553. To find any two numbers, when their sum and the difference of their squares are given.

The difference of their squares divided by the sum of the numbers will give their difference; and half of their difference added to half of their sum will give the larger, and half of their difference subtracted from half of their sum will give the smaller, of the numbers required.

EXAMPLES.

1. A certain general has an army of 141376 men. How many must he place in rank and file to form them into a square? Ans. 376.

2. If the area of a circle be 1760 yards, how many feet must the side of a square measure to contain that quantity?
Ans. 125.857+ feet.

3. If a line 144 feet long will reach from the top of a fort to the opposite side of a river 64 feet wide, on whose brink it stands, what is the height of the fort? Ans. 128.99+.

4. A certain room is 20 feet long, 16 feet wide, and 12 feet high; how long must a line be to extend from one of the lower corners to the upper corner farthest from it? Ans. 28.28ft.

5. A certain field is 40 rods square; what must be the length of one of the equal sides of another field that shall contain only one fourth as much area? Ans. 20 rods.

6. The areas of two similar triangular-shaped fields are 60 and 90 acres, and a side of the former is 66 rods. Required the corresponding side of the latter?

7. If a lead pipe $\frac{3}{4}$ of an inch in diameter will fill a cistern in 3 hours, what should be its diameter to fill it in 2 hours?
Ans. .918+ inches.

8. If a pipe $1\frac{1}{2}$ inches in diameter will fill a cistern in 50 minutes, how long would it require a pipe that is 2 inches in diameter to fill the same cistern? Ans. 28m. $7\frac{1}{2}$s.

9. If a pipe 6 inches in diameter will draw off a certain quantity of water in 4 hours, in what time would it take 3 pipes of four inches in diameter to draw off twice the quantity?
Ans. 6 hours.

10. The first term of a proportion is 40, and the fourth term 90. Required a mean proportional between them. Ans. 60.

11. In a pair of scales a body weighed $31\frac{1}{4}$ pounds in one scale, and only 20 pounds in the other scale. Required its true weight. Ans. 25 pounds.

12. I wish to set out an orchard of 2400 mulberry-trees, so that the length shall be to the breadth as 3 to 2, and the distance between any two adjacent trees 7 yards. How many trees must there be in the length, and how many in the

breadth; and on how many square yards of ground will they stand? Ans. 60 in length; 40 in breadth; 112749 sq. yd.

13. The sum of two persons' ages is 50 years, and their product is 600 years. What are their ages?

Ans. Of the one, 20 years; of the other, 30 years.

14. Two ships sail from the same port; one goes due north 128 miles, the other due east 72 miles; how far are the ships from each other? Ans. 146.86+.

15. There are two columns in the ruins of Persepolis left standing upright; one is 70 feet above the plane, and the other 50; in a straight line between these stands a small statue, 5 feet in height, the head of which is 100 feet from the summit of the higher, and 80 feet from the top of the lower column. Required the distance between the tops of the two columns.

16. The sum of two numbers is 44, and the square of their difference is 16. Required the numbers.

Ans. 24 the larger number; 20 the smaller.

17. A tree 80 feet in height stands on a horizontal plane; at what height from the ground must it be broken off, so that the top of it may fall on a point 40 feet from the bottom of the tree, the end where it was broken off resting on the stump?

Ans. 30 feet.

18. The height of a tree, growing in the centre of a circular island, 100 feet in diameter, is 160 feet; and a line extending from the top of it to the farther shore is 400 feet. What is the breadth of the stream, provided the land on each side of the water be level? Ans. 316.6+ feet.

19. A ladder 70 feet long is so planted as to reach a window 40 feet from the ground, on one side of the street, and without moving it at the foot it will reach a window 30 feet high on the other side; what is the breadth of the street?

20. If an iron wire $\frac{1}{10}$ of an inch in diameter will sustain a weight of 450 pounds, what weight might be sustained by a wire an inch in diameter? Ans. 45000lb.

21. A gentleman proposes to plant a vineyard of 10 acres. If he places the vines 6 feet apart, how many more can he plant by setting them in the quincunx order than in the square order, allowing the plat to lie in the form of a square, and no vine to be set nearer its edge than 1 foot in either case?

Ans. 1870 more in the quincunx order.

22. Four men, A, B, C, and D, bought a grindstone, the diameter of which was 40 inches and the place for the shaft 4 inches in diameter. It was agreed that A should grind off his share first, then in turn B, C, and D. Required how many inches each man will grind off from the semidiameter, providing they each paid the same sum.

Ans. A, 2.651in.; B, 3.137in.; C, 4.064in.; and D, 8.148in.

23. I have a board whose surface contains $49\frac{5}{8}$ square feet; the board is $1\frac{1}{2}$ inches thick, and I wish to make a cubical box of it. Required the length of one of its equal sides.

Ans. 36 inches.

24. A carpenter has a plank 1 foot wide, $22\frac{3}{12}$ feet long, and $2\frac{1}{2}$ inches thick; and he wishes to make a box whose width shall be twice its height, and whose length shall be twice its width. Required the contents of the box.

Ans. 5719 cubic inches.

25. If a ball, 3 inches in diameter, weigh 4 pounds, what will be the weight of a ball that is 6 inches in diameter?

Ans. 32lbs.

26. If a globe of gold, one inch in diameter, be worth $120, what is the value of a globe $3\frac{1}{2}$ inches in diameter?

27. If the weight of a well-proportioned man, 5 feet 10 inches in height, be 180 pounds, what must have been the weight of Goliath of Gath, who was 10 feet $4\frac{3}{5}$ inches in height? Ans. 1015.1+lb.

28. If a bell, 4 inches in height, 3 inches in width, and $\frac{1}{4}$ of an inch in thickness, weigh 2 pounds, what should be the dimensions of a similar bell that would weigh 2000 pounds?

Ans. 3ft. 4in. high, 2ft. 6in. wide, and $2\frac{1}{2}$in. thick.

29. What are the two mean proportionals between 56 and 12096? Ans. 336 and 2016.

30. Having a small stack of hay, 5 feet in height, weighing 1cwt., I wish to know the weight of a similar stack that is 20 feet in height. Ans. 64cwt.

31. If a man dig a small square cellar, which will measure 6 feet each way, in one day, how long would it take him to dig a similar one that measured 10 feet each way?

Ans. 4.629+ days.

32. If an ox, whose girth is 6 feet, weighs 600lb., what is the weight of an ox whose girth is 8 feet? Ans. 1422.2+lb.

33. Four women own a ball of yarn, 5 inches in diameter. It is agreed that each shall wind off her share from the ball. How many inches of its diameter shall each wind off?

Ans. First, .45+ inches; second, .57+ inches; third, .82+ inches; fourth, 3.149+ inches.

34. John Jones has a stack of hay in the form of a quadrangular pyramid. It is 16 feet in height, and 12 feet wide at its base. It contains 5 tons of hay, worth $ 17.50 per ton. Mr. Jones has sold this hay to Messrs. Pierce, Row, Wells, and Northend. As the upper part of the stack has been injured, it is agreed that Mr. Pierce, who takes the upper part, shall have 10 per cent. more of the hay than Mr. Rowe; and Mr. Rowe, who takes his share next, shall have 8 per cent. more than Mr. Wells; and Mr. Northend, who has the bottom of the stack, that has been much injured, shall have 10 per cent. more than Mr. Wells. Required the quantity of hay, and how many feet of the height of the stack, beginning at the *top*, each receives.

Ans. Pierce receives $27\frac{54}{273}$cwt. and 10.366+ feet in height; Rowe, $24\frac{198}{273}$cwt. and 2.493 feet; Wells, $22\frac{244}{273}$cwt. and 1.666 feet; Northend, $25\frac{50}{273}$cwt. and 1.474 feet.

PROGRESSION, OR SERIES.

554. A SERIES is a succession of numbers that depend on one another by some fixed law.

The numbers constituting a series are called its *terms;* of which the *first* and *last* are called *extremes*, and the other terms the *means*.

ARITHMETICAL PROGRESSION.

555. ARITHMETICAL PROGRESSION, or PROGRESSION BY DIFFERENCE, is a series that increases or decreases by a constant number, called the *common* DIFFERENCE.

The series is said to be an *ascending* one when each term

after the first exceeds the one before it; and a *descending* one when each term after the first is less than the one before it.

Thus, 1, 5, 9, 13, 17, 21, 25, 29, 33, is an ascending series, in which each term after the first is derived from the one preceding it by the addition of the common difference 4; and 25, 22, 19, 16, 13, 10, 7, 4, 1, is a descending series, in which each term after the first is derived from the one preceding it by the subtraction of the common difference 3.

556. In arithmetical progression, *the first term, the last term, the number of terms, the common difference, and the sum of the terms*, are so related to each other, that, *three* of these being given, the *two* others may be readily determined.

557. To find the *common difference*, the extremes and number of terms being given.

Ex. 1. The extremes of an arithmetical series are 3 and 45, and the number of terms is 22. Required the common difference. Ans. 2.

OPERATION.

$$\frac{45 - 3}{22 - 1} = 2.$$

It is evident that the number of common differences in any series must be 1 less than the number of terms. Therefore, since the number of terms in this series is 22, the number of common differences will be $22 - 1 = 21$, and their sum will be equal to the difference of the extremes; hence the difference of the extremes, $45 - 3 = 42$, divided by the number of common differences, 21, gives 2 as the common difference required.

RULE. — *Divide the difference of the extremes by the number of terms less one, and the quotient will be the common difference.*

EXAMPLES.

2. A certain school consists of 19 teachers and scholars, whose ages form an arithmetical series; the youngest is 3 years old, and the oldest 39. What is the common difference of their ages? Ans. 2 years.

3. A man is to travel from Albany to a certain place in 11 days, and to go but 5 miles the first day, increasing the distance equally each day, so that the last day's journey may be 45 miles. Required the daily increase. Ans. 4 miles.

558. To find the *number of terms*, the extremes and common difference being given.

Ex. 1. If the extremes of an arithmetical series are 3 and 19, and the common difference is 2, what is the number of terms? Ans. 9.

OPERATION.

$$\frac{19 - 3}{2} + 1 = 9.$$

It is evident, that, if the difference of the extremes be divided by the common difference, the result will be the number of common differences; thus $19 - 3 = 16$; $16 \div 2 = 8$. Then, as the number of terms must be 1 more than the number of common differences, $8 + 1 = 9$ is the number of terms in the series.

RULE. — *Divide the difference of the extremes by the common difference, and the quotient increased by* 1 *will be the required number of terms.*

EXAMPLES.

2. A man going a journey travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles. How many days did he travel? Ans. 12.

3. In what time can a debt be discharged, supposing the first week's payment to be \$1, and the payment of every succeeding week to increase by \$2, till the last payment shall be \$103? Ans. 52 weeks.

559. To find the *sum of all the terms*, the extremes and number of terms being given.

Ex. 1. The extremes of an arithmetical series are 3 and 19, and the number of terms 9. Required the sum of the series. Ans. 99.

OPERATION.

$$\frac{3 + 19}{2} \times 9 = 99, \text{ Ans.}$$

Or, $3 + 19 = 22$; $22 \times 4\frac{1}{2} = 99$, Ans.

In an arithmetical series the sum of the extremes is equal to the sum of two terms that are equally distant from them, or to double the middle term, if the number of terms be odd. Thus, in the series, 3, 5, 7, 9, 11, 13, 15, 17, 19, the sum of 3 and 19 is equal to the sum of 5 and 17, or of 7 and 15, and is double the middle term, 11. The reason of this is evident, since 5 and 7 exceed the less extreme by the same quantities by which 17 and 15 are respectively less than the other extreme.

Hence, in this latter series, it is evident that, if each term were made 11, half the sum of the extremes, the *sum* of the whole would remain the same; therefore the sum of the series must equal half the sum of the extremes multiplied by the number of terms, or the sum of the extremes multiplied by half the number of terms.

RULE. — *Multiply half the sum of the extremes by the number of terms.* Or,

Multiply the sum of the extremes by half the number of terms.

EXAMPLES.

2. If the least term of a series of numbers in arithmetical progression be 4, the greatest 100, and the number of terms 17, what is the the sum of the terms? Ans. 884.

3. Suppose a number of stones were laid a rod distant from each other, for thirty miles, the first stone being a rod from a basket. What distance will that man travel who gathers them up singly, returning with them one by one to the basket?

Ans. 288090 miles 2 rods.

560. To find the *sum of the terms*, the extremes and common difference being given.

Ex. 1. If the two extremes are 3 and 19, and the common difference is 2, what is the sum of the series?

OPERATION.

$$\frac{19 - 3}{2} + 1 = 9;$$

$$\frac{19 + 3}{2} \times 9 = 99, \text{ Ans.}$$

It has been shown (Art. 558) that, if the difference of the extremes be divided by the common difference, the quotient will be the number of terms less one. Therefore the number of terms less one will be $\frac{19-3}{2} = 8$; and $8 + 1$ will equal the number of terms. It has also been shown (Art. 559) that, if the number of terms be multiplied by the sum of the extremes, and the product divided by 2, the quotient will be the sum of the series; therefore $\frac{19+3}{2} \times 9 = 99$, the answer required.

RULE. — *Divide the difference of the extremes by the common difference, and to the quotient add* 1; *by this sum multiply half the sum of the extremes, and the product will be the sum required.*

EXAMPLES.

2. If the extremes are 3 and 45, and the common difference 2, what is the sum of the series? Ans. 528.

3. A owes B a certain sum, to be discharged in a year, by

paying 6 cents the first week, 18 cents the second week, and thus to increase every week by 12 cents, till the last payment should be \$ 6.18. What is the debt? Ans. \$ 162.24.

561. To find one of the extremes, when the other extreme and the number and sum of the terms are given.

Ex. 1. If 3 be the first term of a series, 9 the number of terms, and 99 the sum of the series, what is the last term?

OPERATION.

$$\frac{99 \times 2}{9} - 3 = 19, \text{ [Ans.}$$

It has been shown (Art. 559) that, if the sum of the extremes be multiplied by the number of terms, the product will be twice the sum of the series; therefore, if twice the sum of the series be divided by the number of terms, the quotient will be the sum of the extremes. If from this we subtract the given extreme, the remainder must be the other extreme.

RULE. — *Divide twice the sum of the series by the number of terms; from the quotient take the given term, and the remainder will be the term required.*

EXAMPLES.

2. The sum of a series of ten thousand even numbers is 100010000, and the last term of the series is 20000. Required the first term. Ans. 2.

3. A merchant, being indebted to 22 creditors \$ 528, ordered his clerk to pay the first \$ 3, and the rest sums increasing in arithmetical progression. What is the difference of the payments, and the last payment?

Ans. Difference 2; last payment \$ 45.

562. To find any number of arithmetical *means*, the extremes and the number of terms being given.

Ex. 1. If the first term of an arithmetical series is 1 the last term 99, and the number of terms 8, what are the second and seventh terms of the series?

Ans. The second term, 15; the seventh, 85.

OPERATION.

$$\frac{99 - 1}{8 - 1} = 14;$$

$$1 + 14 = 15; \ 99 - 14 = 85.$$

We find the common difference, 14, as in Art. 557, the first term, 1, plus the common difference, 14, gives 15 for the second term, and the last term, 99, minus the common difference, 14, gives the seventh term.

RULE. — *Find the common difference, which, added to the less extreme, or subtracted from the greater, will give one mean. From that mean derive others in the same way, till those required are found.*

EXAMPLES.

2. The extremes of a series are 4 and 49, and the number of terms 6. Required the middle two terms.

Ans. 22 and 31.

3. Insert five arithmetical means between 20 and 30.

Ans. $21\frac{2}{3}$, $23\frac{1}{3}$, 25, $26\frac{2}{3}$, and $28\frac{1}{3}$.

GEOMETRICAL PROGRESSION.

563. Geometrical Progression, or progression by quotients, is a series of numbers that increase or decrease by a constant multiplier or divisor, called the common ratio.

The series is an *ascending* one, when each term after the first increases by a constant ratio; and a descending one, when each term after the first decreases by a constant ratio.

Thus 2, 6, 18, 54, 162, 486 is an ascending geometrical series; and 64, 32, 16, 8, 4, 2 is a descending geometrical series. Of the former, 3 is the common ratio, and of the latter, 2.

564. In geometrical progression the *first term, the last term, the number of terms, the common ratio, and the sum of the terms* are so related to each other, that, any three of these being given, the other two may be readily determined.

565. To find *any proposed term*, one of the extremes, the ratio, and the number of terms being given.

Ex. 1. If the first term of a geometrical series be 3, the ratio 2, and the number of terms 8, what is the last term?

Ans. 384.

OPERATION.

$3 \times 2^7 = 3 \times 128 = 384.$

It is evident that the successive terms are the result of repeated multiplications by the ratio; thus the second term must be the product of the first term by the ratio, the third term the product of the second term by the ratio, and so on. The eighth, or last term, therefore, must be the result of seven

such multiplications, or the product of the first term, 3, by 2^7, or $3 \times 128 = 384$.

If the last term had been given and the first required, the process would evidently have been by division, since every less term is the result of a division of the term next larger by the ratio.

RULE. — *Raise the ratio to the power whose index is one less than the number of terms; by which multiply the least term to find the greatest, or divide the greatest to find the least.*

NOTE 1. — When the ratio requires to be raised to a high power, the process may be abridged, as in Art 516.

NOTE 2. — The rule may be applied in computing compound interest, the principal being the first term, the amount of one dollar for one year the ratio, the time, in years, one less than the number of terms, and the amount the last term.

EXAMPLES.

2. If the first term be 5, and the ratio 3, what is the seventh term? Ans. 3645.

3. If the series be 72, 24, 8, &c., and the number of terms 6, what is the last term? Ans. $\frac{8}{27}$.

4. If the larger extreme be 885735, the ratio 3, and the number of terms 12, what are the tenth and the eleventh terms? Ans. 15 and 45.

5. If the seventh term is 5, and the ratio $\frac{1}{3}$, what is the first term? Ans. 3645.

6. If the first term is 50, the ratio 1.06, and the number of terms 5, what is the last term? Ans. 63.123848.

7. If I were to buy 30 oxen, giving 2 cents for the first ox, 4 cents for the second, 8 cents for the third, &c., what would be the price of the last ox? Ans. $ 10737418.24.

8. What is the amount of $ 160.00 at compound interest for 6 years? Ans. $ 226.96305796096.

9. What is the amount of $ 300.00 at compound interest at 5 per cent. for 8 years? Ans. $ 443.236+.

10. What is the amount of $ 100.00 at compound interest at 6 per cent. for 30 years?

Ans. $ 574.349117291325011626410633231080264584635-7252196069357387776.

566. To find the *sum of a series*, the first term, the ratio, and the number of terms being given.

Ex. 1. If the first term be 1, the ratio 3, and the number of terms 5, required the sum of the terms. Ans. 121.

OPERATION.

$$\frac{(81 \times 3) - 1}{3 - 1} = 121.$$

If we multiply the series 1, 3, 9, 27, 81 by the ratio 3, we shall obtain as a second series 3, 9, 27, 81, 243, whose sum is three times the sum of the first series, and the difference between whose sum and the sum of the first series is evidently twice the sum of the first series. Now it will be observed that the two series have their terms alike, with the exception of the first term in the first series, and the last in the second series. We have then only to subtract the first term in the first series from the last term in the second, and the remainder is twice the sum of the first series; and half this being taken gives the required sum of the series. Therefore the sum of the first series must be $242 \div 2 = 121$.

RULE. — *Find the last term as in Art.* 565, *multiply it by the ratio, and the product less the first term divide by the ratio less* 1; *the result will be the sum of the series.*

NOTE 1. — If the ratio is less than a unit, the product of the last term mulplied by the ratio must be subtracted from the first term, and the remainder divided by unity or 1 decreased by the ratio.

NOTE 2. — When a descending series is continued to infinity, it becomes what is called an *infinite* series, whose last term must always be regarded as 0, and its ratio as a fraction. To find the sum of an infinite series, —

Divide the first term by 1 *decreased by the fraction denoting the ratio, and the quotient will be the sum required.*

This process furnishes an expeditious way of finding the value of *circulating decimals*, since they are composed of numbers in geometrical progression, whose common ratios are $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, &c. according to the number of factors contained in the repetend. Thus, $.\dot{3}33\dot{3}$, &c. represents the geometrical series $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, &c. whose first term is $\frac{3}{10}$ and common ratio $\frac{1}{10}$.

EXAMPLES.

2. The first term of a series is 5, the ratio $\frac{2}{3}$, and the number of terms 6; required the sum of the series.

$$(\tfrac{2}{3})^5 \times 5 = \tfrac{160}{243};\ \tfrac{160}{243} \times \tfrac{2}{3} = \tfrac{320}{729};\ 5 - \tfrac{320}{729} = \tfrac{3325}{729};$$
$$\tfrac{3325}{729} \div (1 - \tfrac{2}{3}) = \tfrac{3325}{243} = 13\tfrac{166}{243},\text{ Ans.}$$

3. Find the value of the circulating decimal $.\dot{2}3\dot{2}32\dot{3}$, &c.

$$\tfrac{23}{100} \div (1 - \tfrac{1}{100}) = \tfrac{23}{99},\text{ Ans.}$$

4. What is the sum of the series 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, &c., continued to an infinite number of terms?

$$4 \div (1 - \tfrac{1}{4}) = 5\tfrac{1}{3},\text{ Ans.}$$

5. If the first term is 50, the ratio 1.06, and the number of terms 4, what is the sum of the series? Ans. 218.7308.

6. A gentleman offered a house for sale on the following terms; that for the first door he should charge 10 cents, for the second 20 cents, for the third 40 cents, and so on in a geometrical ratio. If there were 40 doors, what was the price of the house? Ans. \$ 109951162777.50.

7. If the series $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, &c. were carried to infinity, what would be its sum? Ans. $1\frac{1}{3}$.

8. A gentleman deposited annually \$ 10 in a bank, from the time his son was born until he was 20 years old. Required the amount of the 21 deposits at 6 per cent., compound interest, when his son was 21 years old? Ans. \$ 423.92.

9. Find the value of $.008\dot{4}9713\dot{3}$, &c., continued to infinity. Ans. $\frac{83}{9768}$.

10. If a body be put in motion by a force which moves it 10 miles in the first portion of time, 9 miles in the second equal portion, and so, in the ratio of $\frac{9}{10}$, for ever, how many miles will it pass over? Ans. 100 miles.

567. To find the *ratio*, the extremes and number of terms being given.

Ex. 1. If the extremes of a series are 3 and 192, and the number of terms 7, what is the ratio?

OPERATION.

$192 \div 3 = 64$; $\sqrt[6]{64} = 2$, Ans.

It has been shown (Art. 565) that the last term of a geometrical series is equal to the product of the ratio by the first term raised to a power whose index is one less than the number of terms; hence the ratio must equal the root of the quotient of the last term by the first whose index is one less than the number of terms.

RULE. — *Divide the last extreme by the first, and extract that root of the quotient whose index is one less than the number of terms.*

NOTE. — When the sum of the series and the extremes are given, the ratio may be found by *dividing the sum of all the terms except the first, by the sum of all the terms except the last.*

EXAMPLES.

2. If the last term of a series is 1, the largest term 512, and the number of terms 10; what is the ratio? Ans. 2.

3. If the extremes are 5 and 885735, and the sum of the series 1328600, what is the ratio? Ans. 3.

4. What debt can be discharged in a year by monthly pay-

ments, in geometrical progression, of which the first payment is \$ 1, and the last \$ 2048; and what will be the ratio of the series? Ans. Ratio, 2; debt, \$ 4095.

568. To insert any number of *geometrical means*, or *mean proportionals*, between two given numbers.

Ex. 1. Insert three geometrical means between 4 and 324.
Ans. 12, 36, and 108.

OPERATION.

$$324 \div 4 = 81;\ \sqrt[4]{81} = 3.$$
$$4 \times 3 = 12;\ 12 \times 3 = 36;\ 36 \times 3 = 108.$$

Since the series will include, beside the inserted terms, the two extremes, the number of terms will be 5. Then, having the number of terms and the extremes, we find the ratio, as in the last article, to be 3; and by multiplying the first term by the ratio, we obtain the first of the terms to be inserted. That term multiplied by the ratio gives the next, and that multiplied by the ratio gives the other required mean.

RULE. — *Take the two given numbers as the extremes of a geometrical series, and consider the number of terms in the series greater by two than the required number of means. Then find the ratio, as in Art.* 567, *and the product of the ratio and the first extreme will give one of the means, and the product of this mean and the ratio will give another, and so on.*

NOTE. — When only a single mean is required to be inserted, it may be found as in Art. 550; when only two, as in Art. 551.

EXAMPLES.

2. Insert three geometrical means between $\frac{1}{2}$ and 128.
Ans. 2, 8, and 32.

3. Required five mean proportionals between the numbers 3 and 2187. Ans. 9, 27, 81, 243, and 729.

569. To find the *number of terms*, the extremes and ratio being given.

Ex. 1. If the extremes are 5 and 3645, and the ratio 3, what is the number of terms?

OPERATION.

$$3645 \div 729;\ 3^6 = 729.$$
$$6 + 1 = 7, \text{ Ans.}$$

By Art. 565 it is seen that the ratio raised to the power whose index is one less than the number of terms, and multiplied by the least term, equals the largest term; hence, the largest term divided by the least term will equal a power of the ratio whose index is one less than the number of terms.

RULE. — *Divide the largest term by the least; involve the ratio to a power equal to the quotient; and the index of that power, increased by 1, will be the number of terms.*

EXAMPLES.

2. If the extremes are 5 and 20480, and the ratio 4, what is the number of terms? Ans. 7.

3. In what time will a certain debt be discharged by monthly payments in geometrical progression, if the first and last payments are $ 1 and $ 2048, and the ratio 2?

Ans. In 12 months.

ANNUITIES.

570. ANNUITIES are fixed sums of money payable at the ends of equal periods of time, such as years, or half-years.

Annuities in perpetuity are such as continue for ever.

Annuities certain are such as commence at a fixed time, and continue for a certain number of years.

Annuities contingent are those whose commencement or continuance, or both, depend on some contingent event, as the death of one or more individuals.

Annuities deferred, or *in reversion*, are such as do not commence till after a fixed number of years, or till after some particular event has taken place.

571. An annuity *forborne*, or *in arrears*, is one whose periodical payments, instead of being paid when due, have been allowed to accumulate.

572. The *amount of an annuity* at compound interest, at any time, is the sum to which it will amount, supposing it to have been improved at compound interest during the intervening period.

573. The *present value* of an annuity at compound interest, for any given period, is the sum of the present values of all the payments of that annuity.

TABLE,

SHOWING THE AMOUNT OF AN ANNUITY OF ONE DOLLAR PER ANNUM, IMPROVED AT COMPOUND INTEREST FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY.

Years.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
2	2.030 000	2.035 000	2.040 000	2.050 000	2.060 000	2.070 000
3	3.090 900	3.106 225	3.121 600	3.152 500	3.183 600	3.214 900
4	4.183 627	4.214 943	4.246 464	4.310 125	4.374 616	4.439 943
5	5.309 136	5.362 466	5.416 323	5.525 631	5.637 093	5.750 739
6	6.468 410	6.550 152	6.632 975	6.801 913	6.975 319	7.153 291
7	7.662 462	7.779 408	7.898 294	8.142 008	8.393 838	8.654 021
8	8.892 336	9.051 687	9.214 226	9.549 109	9.897 468	10.259 803
9	10.159 106	10.368 496	10.582 795	11.026 564	11.491 316	11.977 989
10	11.463 879	11.731 393	12.006 107	12.577 893	13.180 795	13.816 448
11	12.807 796	13.141 992	13.486 351	14.206 787	14.971 643	15.783 599
12	14.192 030	14.601 962	15.025 805	15.917 127	16.869 941	17.888 451
13	15.617 790	16.113 030	16.626 838	17.712 983	18.882 138	20.140 643
14	17.086 324	17.676 986	18.291 911	19.598 632	21.015 066	22.550 488
15	18.598 914	19.295 681	20.023 588	21.578 564	23.275 970	25.129 022
16	20.156 881	20.971 030	21.824 531	23.657 492	25.670 528	27.888 054
17	21.761 588	22.705 016	23.697 512	25.840 366	28.212 880	30.840 217
18	23.414 435	24.499 691	25.645 413	28.132 385	30.905 653	33.999 033
19	25.116 868	26.357 180	27.671 229	30.539 004	33.759 992	37.378 965
20	26.870 374	28.279 682	29.778 079	33.065 954	36.785 591	40.995 492
21	28.676 486	30.269 471	31.969 202	35.719 252	39.992 727	44.865 177
22	30.536 780	32.328 902	34.247 970	38.505 214	43.392 290	49.005 739
23	32.452 884	34.460 414	36.617 889	41.430 475	46.995 828	53.436 141
24	34.426 470	36.666 528	39.082 604	44.501 999	50.815 577	58.176 671
25	36.459 264	38.949 857	41.645 908	47.727 099	54.864 512	63.249 030
26	38.553 042	41.313 102	44.311 745	51.113 454	59.156 383	68.676 470
27	40.709 634	42.759 060	47.084 214	54.669 126	63.705 766	74.483 823
28	42.930 923	46.290 627	49.967 583	58.402 583	68.528 112	80.697 691
29	45.218 850	48.910 799	52.966 286	62.322 712	73.639 798	87.346 529
30	47.575 416	51.622 677	56.084 938	66.438 848	79.058 186	94.460 786
31	50.002 678	54.429 471	59.328 335	70.760 790	84.801 677	102.073 041
32	52.502 759	57.334 502	62.701 469	75.298 829	90.889 778	110.218 154
33	55.077 841	60.341 210	66.209 527	80.063 771	97.343 165	118.933 425
34	57.730 177	63.453 152	69.857 909	85.066 959	104.183 755	128.258 765
35	60.462 082	66.674 013	73.652 225	90.320 307	111.434 780	138.236 878
36	63 271 944	70.007 603	77.598 314	95.836 323	119.120 867	148.913 460
37	66.174 223	73.457 869	81.702 246	101.628 139	127.268 119	160.337 400
38	69.159 449	77.028 895	85.970 336	107.709 546	135.904 206	172.561 020
39	72.234 233	80.724 906	90.409 150	114.095 023	145.058 458	185.640 292
40	75.401 260	84.550 278	95.025 516	120.799 774	154.761 966	199.635 112
41	78.663 298	88.509 537	99.826 536	127.839 763	165.047 684	214.609 570
42	82.023 196	92.607 371	104.819 598	135.231 751	175.950 645	230.632 240
43	85.483 892	96.848 629	110.012 382	142.993 339	187.507 577	247.776 496
44	89.048 409	101.238 331	115.412 877	151.143 006	199.758 032	266.120 851
45	92.719 861	105.781 673	121.029 392	159.700 156	212.743 514	285.749 311
46	96.501 457	110.484 031	126.870 568	168.685 164	226.508 125	306.751 763
47	100.396 501	115.350 973	132.945 390	178.119 422	241.098 612	329.224 386
48	104.408 396	120.388 297	139.263 206	188.025 393	256.564 529	353.270 093
49	108.540 648	125.601 846	145.833 734	198.426 663	272.958 401	378.999 000
50	112.796 867	130.999 910	152.667 084	209.347 976	290.335 905	406.528 929

TABLE,

SHOWING THE PRESENT WORTH OF AN ANNUITY OF ONE DOLLAR PER ANNUM, TO CONTINUE FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY.

Years.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	Years.
1	0.970 874	0.966 184	0.961 538	0.952 381	0.943 396	0.934 579	1
2	1.913 470	1.899 694	1.886 095	1.859 410	1.833 393	1.808 017	2
3	2.828 611	2.801 637	2.775 091	2.723 248	2.673 012	2.624 314	3
4	3.717 098	3.673 079	3.629 895	3.545 951	3.465 106	3.387 209	4
5	4.579 707	4.515 052	4.451 822	4.329 477	4.212 364	4.100 195	5
6	5.417 191	5.328 553	5.242 137	5.075 692	4.917 324	4.766 537	6
7	6.230 283	6.114 544	6.002 055	5.786 373	5.582 381	5.389 286	7
8	7.019 692	6.873 956	6.732 745	6.463 213	6.209 744	5.971 295	8
9	7.786 109	7.607 687	7.435 332	7.107 822	6.801 692	6.515 228	9
10	8.530 203	8.316 605	8.110 896	7.721 735	7.360 087	7.023 577	10
11	9.252 624	9.001 551	8.760 477	8.306 414	7.886 875	7.498 669	11
12	9.954 004	9.663 334	9.385 074	8.863 252	8.383 844	7.942 671	12
13	10.634 955	10.302 738	9.985 648	9.393 573	8.852 683	8.357 635	13
14	11.296 073	10.920 520	10.563 123	9.898 641	9.294 984	8.745 452	14
15	11.937 935	11.517 411	11.118 387	10.379 658	9.712 249	9.107 898	15
16	12.561 102	12.094 117	11.652 296	10.837 770	10.105 895	9.446 632	16
17	13.166 118	12.651 321	12.165 669	11.274 066	10.477 260	9.763 206	17
18	13.753 513	13.189 682	12.659 297	11.689 587	10.827 603	10.059 070	18
19	14.323 799	13.709 837	13.133 939	12.085 321	11.158 116	10.335 578	19
20	14.877 475	14.212 403	13.590 326	12.462 210	11.469 421	10.593 997	20
21	15.415 024	14.697 974	14.029 160	12.821 153	11.764 077	10.835 527	21
22	15.936 917	15.167 125	14.451 115	13.163 003	12.041 582	11.061 241	22
23	16.443 608	15.620 410	14.856 842	13.488 574	12.303 379	11.272 187	23
24	16.935 542	16.058 368	15.246 963	13.798 642	12.550 358	11.469 334	24
25	17.413 148	16.481 515	15.622 080	14.093 945	12.783 356	11.653 583	25
26	17.876 842	16.890 352	15.982 769	14.375 185	13.003 166	11.825 779	26
27	18.327 031	17.285 365	16.329 586	14.643 034	13.210 534	11.986 709	27
28	18.764 108	17.667 019	16.663 063	14.898 127	13.406 164	12.137 111	28
29	19.188 455	18.035 767	16.983 715	15.141 074	13.590 721	12.277 674	29
30	19.600 441	18.392 045	17.292 033	15.372 451	13.764 831	12.409 041	30
31	20.000 428	18.736 276	17.588 494	15.592 811	13.929 086	12.531 814	31
32	20.388 766	19.068 865	17.873 552	15.802 677	14.084 043	12.646 555	32
33	20.765 792	19 390 208	18.147 646	16.002 549	14.230 230	12.753 790	33
34	21.131 837	19.700 684	18.411 198	16.192 204	14.368 141	12.854 009	34
35	21.487 220	20.000 661	18.664 613	16.374 194	14.498 246	12.947 672	35
36	21.832 252	20.290 494	18.908 282	16.546 852	14.620 987	13.035 208	36
37	22.167 235	20.570 525	19.142 579	16.711 287	14.736 780	13.117 017	37
38	22.492 462	20.841 087	19.367 864	16.867 893	14.846 019	13.193 473	38
39	22.808 215	21.102 500	19.584 485	17.017 041	14.949 075	13.264 928	39
40	23.114 772	21.355 072	19.792 774	17.159 086	15.046 297	13.331 709	40
41	23.412 400	21.599 104	19.993 052	17.294 368	15.138 016	13.394 120	41
42	23.701 359	21.834 883	20.185 627	17.423 208	15.224 543	13.452 449	42
43	23.981 902	22.062 689	20.370 795	17.545 912	15.306 173	13.506 962	43
44	24.254 274	22.282 791	20.548 841	17.662 773	15.383 182	13.557 908	44
45	24.518 713	22.495 450	20.720 040	17.774 070	15.455 832	13.605 522	45
46	24.775 449	22.700 918	20.884 654	17.880 067	15.524 370	13.650 020	46
47	25.024 708	22.899 438	21.042 936	17.981 016	15.589 028	13.691 608	47
48	25.266 707	23.091 244	21.195 131	18.077 158	15.650 027	13.730 474	48
49	25.501 657	23.276 564	21.341 472	18.168 722	15.707 572	13.766 799	49
50	25.729 764	23.455 618	21.482 185	18.255 925	15.761 861	13.800 746	50

574. To find the *amount* of an annuity, at compound interest, forborne, or in arrears, for any number of years.

Ex. 1. What will an annuity of \$ 60, unpaid, or in arrears, 4 years, amount to, at 6 per cent. compound interest?

Ans. \$ 262.476.

OPERATION.

$$\frac{1.06^4 - 1}{1.06 - 1} \times 60 = 262.476.$$

$$\text{Or, } 4.374616 \times 60 = 262.476.$$

The amounts of the several payments form a geometrical series, of which the annuity is the first term, the amount of \$ 1 for one year the ratio, the years the number of the terms, and the amount required is the sum of the series. Hence,

RULE. — *Find the sum of the series as in geometrical progression.* Or,

Multiply the amount of \$ 1 for the given time, found in the table, by the annuity, and the product will be the required amount.

NOTE. — The amount of an annuity at *simple* interest corresponds to the sum of an arithmetical series, of which the annuity is the first term, the interest on the annuity for one term the common difference, and the time in years the number of terms.

2. What will an annuity of \$ 500 amount to for 5 years, at 6 per cent. compound interest? Ans. \$ 2818.546.

3. What is the amount of an annuity of \$ 80, unpaid, or in arrears, for 9 years, at 5 per cent. compound interest?

Ans. \$ 882.125.

4. What is the amount of an annuity of \$ 1000, forborne for 15 years, at $3\frac{1}{2}$ per cent. compound interest?

Ans. \$ 19295.68.

5. What will an annuity of \$ 30, payable semiannually, amount to, in arrears for 3 years, at 7 per cent. compound interest?

6. Suppose a salary of \$ 600 per year, payable quarterly, to remain unpaid $5\frac{1}{2}$ years; to what sum will it amount, at 6 per cent. compound interest? Ans. \$ 3875.63.

575. To find the *present worth* of an annuity, at compound interest.

Ex. 1. What is the present worth of an annuity of \$ 60, to be continued 4 years, at 6 per cent. compound interest?

Ans. \$ 207.906.

OPERATION.

$\$3.465106 \times 60 = \$207.906+.$

The present worth required evidently may be obtained by finding the amount of the given annuity, by the last articles, and then finding in the usual way the present worth of that amount. A more expeditious method, however, is to find, in the table, the present worth of an annuity of $ 1 for the given time and rate, and take that sum as many times as there are dollars in the given annuity, as in the operation.

RULE. — *Multiply the present worth of an annuity of* $ 1 *for the given time and rate by the number denoting the given annuity.*

2. What is the present worth of an annuity of $100, for 9 years, at 6 per cent.? Ans. $ 680.169.

3. What is the present worth of an annuity of $ 200, for 7 years, at 5 per cent.? Ans. $ 1157.27.

4. Required the present worth of an annuity of $ 500, to continue 40 years, at 7 per cent.

5. A gentleman wishes to purchase an annuity, which shall afford him, at 6 per cent. compound interest, $ 500 a year, for ten years. What sum must he deposit in the annuity office to produce it? Ans. $ 3680.04.

6. If a widow be entitled to $ 160 a year, payable semi-annually, from a fund, for 8 years, what is its value at present, at 6 per cent. compound interest? Ans. $ 1004.88.

576. To find the *present worth* of an annuity in *perpetuity.*

Ex. 1. What is the present worth of a perpetual lease, which yields an income of $ 600, the rate of interest being that of 6 per cent.? Ans. $ 10000.

OPERATION.

$\$600 \div .06 = \$10000.$

The question is evidently the same as one requiring what principal in one year, at 6 per cent. interest, will yield $ 600.

RULE. — *Divide the given annuity by the number denoting the interest of* $ 1 *for one year.*

NOTE. — When the annuity is payable quarterly, semiannually, or in any other periods less than a whole year, the annuity must be increased by the interest which may thus accrue on the parts of the annuity payable before the end of the year, before dividing by the interest of $ 1 for one year.

2. A ground rent in the city of Philadelphia yields an annual income of $ 963, at 6 per cent. interest. What is the value of the estate? Ans. $ 16050.

3. What is the present value of a perpetual lease, yielding an income of $ 6335, interest being at 7 per cent.

Ans. $ 90500.

4. What sum should be paid for a perpetual annuity of $ 1200, payable semiannually, interest being at 5 per cent.?

Ans. $ 24000.

577. To find the *present worth* of an annuity in *reversion.*

Ex. 1. What is the present worth of an annuity of $ 300, to commence in 3 years, and to continue 5 years, allowing compound interest at 6 per cent.? Ans. $ 1061.03.

OPERATION.

$ 6.209744 — $ 2.673012 = $ 3.536732;
$ 3.53673 × 300 = $ 1061.03.

The present worth of an annuity of $ 1, at 6 per cent., commencing at once, and continuing till the termination of the annuity, or for 3 + 5 = 8 years, is $ 6.209744; and the present worth of the same annuity up to the time of the commencement of the reversion is $ 2.673012. The difference of these present worths multiplied by the number of dollars in the given annuity is the present worth of the reversion.

RULE. — *Find the present worth of an annuity of* $ 1, *commencing immediately and continuing till the reversion* COMMENCES, *and also till the reversion* TERMINATES; *and multiply the difference of these present worths by the number of dollars in the given annuity. The result will be the present worth required.*

2. The reversion of a lease of $ 350 per annum, to continue 11 years, which commences 9 years hence, is to be sold. What is its worth, allowing the purchaser 6 per cent. per annum for his ready money? Ans. $ 1633.70.

3. A father presents to his daughter, for 8 years, a rental of $ 70 per annum, payable yearly, and its reversion for the 12 years succeeding to his son. What is the present value of the gift to his son, allowing 4 per cent. compound interest?

Ans. $ 480.03.

4. What is the present worth of the reversion of a perpetuity of $ 240 per annum, payable yearly, but not to come into possession till the expiration of 100 years, compound interest being allowed at 6 per cent.? Ans. $ 11.78.

578. To find the *annuity*, the present worth, time, and rate being given.

Ex. 1. What annuity, continued for 4 years, at 6 per cent. compound interest, is now worth \$ 207.90?

Ans. \$ 60.

OPERATION.

\$ 207.90 ÷ 3.465 = \$ 60.

The present value represented by the debt, divided by the present worth of \$ 1 for the given time and rate, gives the annuity required.

RULE. — *Divide the given present worth by the present worth of an annuity of \$ 1 for the given time and rate, and the result will be the annuity required.*

NOTE. — When the amount of an annuity, the time and rate, are given, the *annuity* may be found *by dividing the given amount by the amount of \$ 1 for the given time and rate.*

2. The present value of an annuity, to be continued 10 years, at 6 per cent. compound interest, payable annually, is \$ 3680.04; required the annuity. Ans. \$ 500.

3. An annuity, remaining unpaid for 9 years, at 5 per cent. compound interest, amounted to \$ 882.125; what was the annuity?

4. A yearly pension which has been forborne for 6 years, at 6 per cent., amounts to \$ 279; what was the pension?

Ans. \$ 40.

PERMUTATIONS AND COMBINATIONS.

579. PERMUTATION is the process of finding the number of changes that can be made in the arrangement of any given number of things.

580. COMBINATION shows how often a less number of things combined can be taken out of a greater, without respect to their order.

581. To find the number of changes that can be made with any given number of things, taken all at once.

Ex. 1. How many changes of order do the first three letters of the alphabet admit of? Ans. 6.

OPERATION.

$1 \times 2 \times 3 = 6.$

By trial we shall find that two are all the possible permutations that can be made of the first two letters of the alphabet; as, *a b* and *b a*. If we take an additional letter, 6 are all the possible permutations; as, *abc*, *acb*, *bca*, *bac*, *cab*, *cba*. Now, the same result may be obtained, in the case of the two letters, by multiplying together the first two digits, and in case of the three letters by multiplying together the first three digits, as in the operation.

RULE. — *Multiply together all the terms of the natural series of numbers, from* 1 *up to the given number, inclusive, and the product will be the number required.*

2. How many changes may be rung on 6 bells?

Ans. 720 changes.

3. For how many days can 10 persons be placed in a different position at dinner?

4. How many changes may be rung on 12 bells, and how long would they be in ringing, supposing 10 changes to be rung in one minute, and the year to consist of 365 days 5 hours and 49 minutes? Ans. 479001600, and 91y. 26d. 22h. 41m.

5. How many changes do the letters of the alphabet admit of? Ans. 403291461126605635584000000.

582. To find how many changes may be made by taking each time any number of different things less than all.

Ex. 1. How many sets of 4 letters each may be formed out of 8 different letters? Ans. 1680.

OPERATION.

$$8 \times (8 - 1) \times (8 - 2) \times (8 - 3)$$
$$= 8 \times 7 \times 6 \times 5 = 1680.$$

It is evident that each one of the 8 letters may be arranged before each of the others; therefore 2 out of the 8 letters admit of 8×7 permutations. By taking 3 out of the 8 letters, the third letter can be arranged as the first, second, and third, in each of the same permutations, giving $8 \times 7 \times 6$ permutations. In like manner for 4 out of 8, we obtain $8 \times 7 \times 6 \times 5$ permutations.

RULE. — *Take a series of numbers, beginning with the number of things given, and decreasing by* 1, *until the number of terms equals the number of things to be taken at a time, and the product of all the terms will be the answer required.*

2. How many changes can be rung with 4 bells out of 6?

Ans. 360.

3. How many words can be made out of the 26 letters of the alphabet, 6 being taken at once? Ans. 165765600.

583. To find the number of combinations that can be formed from a given number of different things, taken a given number at a time.

Ex. 1. How many combinations can be made of 3 letters out of 4, the letters all being different? Ans. 4.

OPERATION.

$$\frac{4 \times \cancel{3} \times \cancel{2}}{1 \times \cancel{2} \times \cancel{3}} = \frac{4}{1} = 4.$$

We find the number of permutations which may be made by taking 3 out of 4 letters, as in Art. 582, and the number of permutations by the three letters all at once, as by Art. 581, and, dividing the first by the latter, obtain the number of combinations required.

RULE. — *Take the series,* 1, 2, 3, 4, 5, *&c., up to the less number of things, and find the product of the terms. Take also a series of numbers beginning with the greater number of things, and decreasing by* 1 *until the number of terms equals the less number of things, and find the product of the terms. The latter result divided by the former will give the number required.*

2. How many combinations can be made of 7 letters out of 10, the letters all being different? Ans. 120.

3. A successful general, being asked what reward would satisfy him for his services, demanded only a cent for every file of 10 men which he could make with a body of 100 men. What would his demand amount to? Ans. $ 173103094564.40.

ANALYSIS BY POSITION.

584. ANALYSIS BY POSITION is the process of solving analytical questions, by assuming or supposing one or more numbers, and reasoning from them, operated upon as if they were the number or numbers required to be found.

585. Questions in which the required number is in any way increased or diminished in any given ratio, or in which it is multiplied or divided by any number, may be solved by means of a *single* assumption.

586. Questions in which the required numbers, or their parts, or their multiples, are increased or diminished by some given number which is no known part or multiple of the required number, or when a power or root of the required number is either directly or indirectly contained in the result given in the question, may be solved by two assumptions.

NOTE. — When the answer is obtained by means of a *single* assumption, the process is called POSITION; and when obtained by means of two assumptions, it is called DOUBLE POSITION.

Analysis by Position affords often a very compendious method of working questions, whose solution otherwise, except by algebra, would be lengthy and difficult. It is even found very useful in shortening some of the processes of algebra.

587. When the answer may be obtained by means of a single assumption or supposition.

Ex. 1. A schoolmaster, being asked how many scholars he had, replied, that if he had as many more as he now has, and half as many more, he should have 200. Of how many scholars did his school consist? Ans. 80 scholars.

OPERATION.

Assumed number,	6 0
As many more,	6 0
Half as many more,	3 0
	1 5 0

1 5 0 : 2 0 0 :: 6 0 : 8 0, Ans.

By having as many more, and half as many more, he would have had $2\frac{1}{2}$ times the original number; therefore, the required number must be as many as 200 contains times $2\frac{1}{2}$, or 80. The same result may be obtained, as in the operation, thus: We assume the number of scholars to be 60; if to 60 as many more, and half as many more, are added, the sum is 150. As this result has the same ratio to the result in the question as the supposed number has to the number required, we find the answer by a proportion.

RULE. — *Assume any convenient number, and proceed with it according to the nature of the question. Then, if the result be either too much or too little, as the result found is to the result given, so will be the number assumed to the number required.*

2. A person, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, had \$ 60 left; what had he at first? Ans. \$ 144.

3. What number is that, which, being increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, equals 125?

4. A's age is double that of B, and B's is triple that of C, and the sum of all their ages is 140. What is each person's age? Ans. A's 84, B's 42, C's 14 years.

5. A person lent a sum of money at 6 per cent., and at the end of 10 years received the amount, $ 560. What was the sum lent? Ans. $ 350.

588. When two or more assumptions or suppositions are required in finding the answer.

Ex. 1. A lady purchased a piece of silk at 80 cents per yard, and lining for it at 30 cents per yard; the silk and lining contained 15 yards, and the price of the whole was $ 7. How many yards were there of each?

Ans. 5 yards of silk; 10 yards of lining.

OPERATION.

Assume 6 yards of silk,	$ 4.80	Assume 4 yards of silk,	$ 3.20
Lining would be 9yd.,	2.70	Lining would be 11yd.,	3.30
Their sum,	$ 7.50	Their sum,	$ 6.50
Sum in the question,	7.00	Sum in the question,	7.00
First error,	+ $ 0.50	Second error,	— $ 0.50

.50 + .50 : 6 — 4 :: .50 : 1; 6 yards — 1 yard = 5 yards;
15 yards — 5 yards = 10 yards.

Since the silk and lining contain 15 yards, cost $ 7.00, the average price per yard is $46\frac{2}{3}$ cents; and 80 cents — $46\frac{2}{3}$ cents = $33\frac{1}{3}$ cents; $46\frac{2}{3}$ cents — 30 cents = $16\frac{2}{3}$ cents; and the quantity of lining will therefore be to that of the silk as $33\frac{1}{3}$ is to $16\frac{2}{3}$, or as to 2 is to 1. Hence, if the given number of yards, 15, be divided into 3 parts, two of those parts, or 10 yards, will be for the lining, and the other part, or 5 yards, will be for the silk.

The same result is obtained as in the operation, thus: We assume the quantity of silk to be 6 yards; then the lining would be 9 yards. We find the cost of each of these at the prices given, and, adding, have as the sum of their costs $ 7.50. This is a result too large by $ 0.50, when compared with the sum in question. By assuming the quantity of the silk to be 4 yards, and proceeding in a like manner, we obtain for the cost of the silk and the lining $ 6.50, a sum too small by $ 0.50. The first error, arising from a result too large, is marked with the sign plus (+), and the second error, arising from a result too small, is marked with the sign minus (—). Then, since one of the results is too large, and the other too small, we then say, as the sum of the errors, or .50 + .50, is to the difference of the assumptions, or 6 — 4, so is .50 the less error, to 1, the correction; which, being added to 4 (yards), the sum, 5, expresses the number of yards of silk; and consequently that of the lining must be 10 yards.

RULE. — *Assume two different numbers, perform on them separately the operations indicated in the question, and note the* ERRORS *of the re-*

sults. Then, as the difference of the errors, if the results be both too great or both too small, or as the sum of the errors, if one result be too great and the other too small, is to the difference of the assumed numbers, so is either error to the correction to be applied to the number which produced that error.

NOTE 1. — The rule usually given fails in an important class of questions; but the rule here given, if not the simplest in the resolution of some questions, has the advantage of being applicable in *every* case.

NOTE 2. — In relation to all questions which in algebra would be resolved by equations of the first degree, the differences between the true and the assumed numbers are proportional to the differences between the result given in the question and the results arising from the assumed numbers. But the principle does not hold exactly in relation to other questions; hence, when applied to them, the above rule, or any other of the kind that can be given, will only produce *approximations* to the true results. In which case the assumed numbers should be taken as nearly true as possible. Then, to approximate more nearly to the required number, assume for a second operation the number found by the first, and that one of the first two assumptions which was nearer the true answer, or any other number that may appear to be still nearer to it. In this way, by repeating the operation as often as may be necessary, the true results may be approximated to any assigned degree of accuracy. This process is sometimes applied with advantage in extracting the higher roots, when approximate results, differing but slightly from entire correctness, will answer.

2. A and B invested equal sums in trade; A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost $ 225; then A's money was double that of B's. What did each invest? Ans. $ 600.

3. A person, being asked the age of each of his sons, replied, that his eldest son was 4 years older than the second, his second 4 years older than the third, his third 4 years older than the fourth, or youngest, and his youngest half the age of the oldest. What was the age of each of his sons?

Ans. 12, 16, 20, and 24 years.

4. A gentleman has two horses, and a saddle worth $ 50. Now if the saddle be put on the first horse, it will make his value double that of the second horse; but if it be put on the second, it will make his value triple that of the first. What was the value of each horse? Ans. The first, $ 30; second, $ 40.

5. A gentleman was asked the time of day, and replied, that $\frac{2}{3}$ of the time past from noon was equal to $\frac{8}{33}$ of the time to midnight. What was the time? Ans. 12 minutes past 3.

6. A and B have the same income. A saves $\frac{1}{12}$ of his, but B, by spending $ 100 per annum more than A, at the end of 10 years finds himself $ 600 in debt. What was their income? Ans. $ 480.

7. A gentleman hired a laborer for 90 days on these conditions: that for every day he wrought he should receive 60 cents, and for every day he was absent he should forfeit 80 cents. At the expiration of the term he received $ 33. How many days did he work, and how many days was he idle?

Ans. He labored 75 days, and was idle 15 days.

8. There is a fish whose head weighs 15 pounds, his tail weighs as much as his head and $\frac{1}{6}$ of his body, and his body weighs as much as his head and tail. What is the weight of the fish? Ans. 72lb.

9. If 12 oxen eat $3\frac{1}{3}$ acres of grass in 4 weeks, and 21 oxen eat 10 acres in 9 weeks, how many oxen would it require to eat 24 acres in 18 weeks, the grass growing uniformly?

Ans. 36 oxen.

10. What number exceeds three times its square root by 11? (Note 2.) Ans. 26.4201648.

SCALES OF NOTATION.

589.° THE *scale* of any system of notation is the law of relation existing between its units of different orders.

590.° The *radix* of any scale is the number of units it takes of one order to make a unit of the next higher. Thus, 10 is the radix of the *decimal* or *denary* system, 2 of the *binary*, 3 of the *ternary*, 4 of the *quaternary*, 5 of the *quinary*, 6 of the *senary*, 7 of the *septenary*, 8 of the *octary*, 9 of the *nonary*, 11 of the *undenary* or *undecimal*, 12 of the *duodenary* or *duodecimal*, 20 of the *vigesimal*, 30 of the *trigesimal*, 60 of the *sexagesimal*, and 100 of the *centesimal.*

591.° In writing any number in a uniform scale, as many distinct characters or symbols are required as there are units in the radix of the given system. Thus, in the decimal or denary scale 10 characters are required, in the binary scale 2 characters, in the duodenary or duodecimal 12 characters, and so on. In the binary scale use is made of the characters 1 and 0, in

the ternary, 1, 2, and 0, &c., the cipher being one of the characters in each scale. In the duodenary scale, eleven characters being required beside the cipher, the first nine may be supplied by the nine digits, the tenth by *t*, the eleventh by *e*, and the twelfth by 0.

592.° To change any number expressed in the decimal scale to any other required scale of notation.

Ex. 1. Express the common number 75432, in the senary and duodenary scales. Ans. 1341120 and 377*e*0.

OPERATION.

6)75432		12)75432	
6)12572	0	12)6286	0
6)2095	2	12)523	10, or *t*
6)349	1	12)43	7
6)58	1	3	7
6)9	4		
1	3		

By dividing the given number by 6, it is distributed into 12572 classes, each containing 6, with 0 remainder. By the second division by 6, these classes are distributed into 2095 classes, each containing 6 times 6, or the second power of 6, with a remainder of 2 of the former class, each containing 6. By the third division the classes last found are distributed into 349 classes, each containing 6 of the latter, which were each the second power of 6, and therefore these are the third power of 6, with a remainder 1 time the second power of 6 In like manner, the next quotient expresses 58 times the fourth power of 6, with a remainder 1 time the third power of 6; the next quotient expresses 9 times the fifth power of 6, with a remainder 4 times the fourth power of 6; and the last quotient expresses 1 time the sixth power of 6, with a remainder 3 times the fifth power of 6. Hence, the given number is found to be equal to $1 \times 6^6 + 3 \times 6^5 + 4 \times 6^4 + 1 \times 6^3 + 1 \times 6^2 + 2 \times 6 + 0$, or according to the senary system of notation 1341120.

By proceeding in like manner, we find the given number to be equal to $3 \times 12^4 + 7 \times 12^3 + 7 \times 12^2 + 10 \times 12 + 0$, or, according to the duodenary scale, 377*e*0.

RULE. — *Divide the given number by the radix of the required scale repeatedly, till the quotient is less than the radix; then the last quotient, with the several remainders in the retrograde order annexed, placing ciphers where there is no remainder, will be the the given number expressed in the required scale.*

2. Change 37 from the decimal to the binary scale.

Ans. 100101.

3. Reduce 1000000 in the decimal scale to the ternary and also to the nonary. Ans. 1212210202001, and 1783661.

4. How will 476897 in the decimal scale be expressed in the duodecimal scale? Ans. 1*tee*95.

593.° To change any number into the decimal scale, when expressed in any other scale of notation.

Ex. 1. Change 377*t*0 from the duodecimal to the decimal scale. Ans. 75432.

OPERATION.

3 7 7 *t* 0
1 2
4 3
1 2
5 2 3
1 2
6 2 8 6
1 2
7 5 4 3 2

We multiply the left-hand figure by the radix, and add to the product the next figure; then we multiply this sum by the radix, and add to the product the next figure, and so proceed till all the figures have been employed; and we thus have, as the values of the several figures collected into one sum, 75432, obtained in a manner similar to the reduction of compound numbers.

RULE. — *Multiply the left-hand figure of the given number by the given radix, and to the product add the next figure; then multiply this sum by the radix, and add to this product the next figure; and so proceed till all the figures of the given number have been added. The result will be the given number in the decimal scale.*

NOTE. — When it is required to change a number from a scale other than decimal to another scale also other than decimal, *first change the number as given into the decimal scale, and then the result into the required scale.*

2. Reduce 234 from the quinary to the decimal scale. Ans. 69.

3. Change 21122 from the ternary to the decimal scale. Ans. 206.

4. Change 100101 in the binary scale to a number in the decimal scale. Ans. 37.

5. Reduce 13579 in the duodecimal scale to the undecimal scale. Ans. 190*t*3.

6. How will 123454321 in the senary scale be expressed in the duodenary scale? Ans. 9873*t*1.

594.° To perform addition, subtraction, multiplication, division, &c. in a scale of notation whose radix is other than 10, we may

Proceed as in the common scale of notation, except that the radix of the given scale must be used in the cases wherein the number 10 would be applied in the decimal system.

Ex. 1. Required the sum and difference of 45324502 and 25405534 in the senary scale, or scale whose radix is 6.

Ans. Sum, 115134440; difference, 15514524.

2. Multiply 2483 by 589 in the undenary scale, or scale whose radix is 11. Ans. 13122*t*5.

3. Divide 1184323 by 589 in the duodenary scale, whose radix is 12. Ans. 2483.

4. Extract the square root of 11122441 in the senary scale. Ans. 2405.

DUODECIMALS.

595. DUODECIMALS are numbers expressed in a scale whose radix is 12, so that 12 units of each lower order make a unit of the next higher.

596. In finding the contents of surfaces and solids, however, it is customary to apply the term *duodecimal* to a mixture of the decimal and duodecimal scales. Thus, in admeasurements in which the foot is the leading unit, though the different orders of units are expressed according to the duodecimal scale, the number of units in each order is usually expressed according to the decimal scale.

597. According to this mixed scale, the foot is divided into 12 equal parts, and each of these parts into 12 other equal parts, and so on indefinitely, giving $\frac{1}{12}$, $\frac{1}{144}$, &c. In writing these fractions without their denominations, to distinguish their orders, or denominations, accents, called indices, are written on the right of the numerators. Thus, inches are called *primes*, and are marked $'$; the next subdivision is called *seconds*, marked $''$; the next is *thirds*, marked $'''$; and so on.

NOTE. — Numbers expressed by the mixed scale of feet, primes, seconds, &c. may be changed to the pure duodecimal scale, and the operations of addition, subtraction, multiplication, division, and so on, then be performed with them, as in Art. 594, observing to place a point between the unit and its lower duodecimal orders, and in the result changing the figures on the left of the point into the decimal scale, and marking those on the right as primes, seconds, &c., according to their places from the order of units.

But the operations of adding, subtracting, &c. are usually performed by other methods, such as are given in the articles that follow.

ADDITION AND SUBTRACTION OF DUODECIMALS.

598. Duodecimals may be added and subtracted in the same manner as compound numbers.

Ex. 1. Add together 121ft. 3′ 9″, 105ft. 11′ 8″, 80ft. 0′ 6″, and 15ft. 10′ 0″ 4‴. Ans. 323ft. 1′ 11″ 4‴.

2. From 462ft. 4′ 9″ take 307ft. 9′ 1″.

3. What is the value of 92ft. 0′ 6″ — 21ft. 9′ 10″ + 19ft. 10′ 3″ 6‴? Ans. 90ft. 0′ 11″ 6‴.

MULTIPLICATION OF DUODECIMALS.

599. *The index of the unit of a product of any two duodecimal orders is equal to the sum of the indices of those factors.* That is, feet multiplied by a number denoting feet produces feet; feet by a number denoting primes produces primes; primes by a number denoting primes produces seconds, &c.

NOTE.—In multiplication of duodecimals, or in other multiplication, the multiplier is always regarded as an abstract number, though the notation of feet, primes, &c. is usually retained, in order the better to note the different orders of units. For the same reason, in division of duodecimals, the divisor usually retains the notation of feet, primes, &c.

600. To multiply one duodecimal by another.

Ex. 1. Required the number of square feet in a platform 6 feet 8 inches long, and 4 feet 5 inches wide.

Ans. 29 sq. ft. 5′ 4″.

FIRST OPERATION.

6ft.	8′	
4ft.	5′	
2	9′	4″
26	8′	
29sq.ft.	5′	4″

We first multiply each of the terms in the multiplicand by the 5′ in the multiplier; thus, 8′×5′=40″ = 3′ and 4″. Writing the 4″ under the multiplier, we reserve the 3′ to add to the next product. Then 6ft. × 5′ = 30′; and 30′ + 3′ = 33′ = 2ft. and 9′, which we write in their order beneath the multiplier. We next multiply by the 4ft., thus: 8′ × 4 feet = 32′ = 2ft. and 8′. We write the 8′ under the primes in the other partial product, and reserve the 2ft. to add to the next product; and 6ft. × 4ft. = 24ft.; 24ft. + 2ft. = 26ft., which we write under the feet in the other partial product. The two being added together, we have 29 sq. ft. 5′ 4″; or (the primes and seconds being changed to a fraction of a foot), $29\frac{66}{144}$sq. ft.

SECOND OPERATION.

$$\begin{array}{r} 6{\cdot}8 \\ 4{\cdot}5 \\ \hline 2\,9\,4 \\ 2\,2\,8 \\ \hline 2\,5{\cdot}5\,4 \end{array} = 29\text{sq. ft. } 5'\ 4''.$$

In the second operation the work is performed as in pure duodecimals (Art. 594). The point (·) separates lower duodecimal orders from those beginning with feet. As to the number of feet in the multiplicand and multiplier, no change is required in that part of either of the given numbers in expressing them according to the duodecimal scale. In performing the multiplication, we make the several reductions required according to the radix 12; and have, after pointing off, 25·54 square feet, expressed according to the duodecimal scale. The units, or feet, at the left of the point, are readily changed to the decimal scale by multiplying the left-hand figure, 2, by 12, the number of units in the radix, and adding the right-hand figure, 5, and giving the figures to the right of the point their proper notation, we have then 29sq. ft. 5′ 4″ for the answer, as before.

RULE. — *Write the multiplier under the multiplicand, so that units of the same orders shall stand in the same column.*

Beginning at the right, multiply each term in the multiplicand by each term of the multiplier, and write the first term of each partial product under its multiplier, observing to carry a unit for every twelve from each lower order to the next higher.

The sum of the partial products will be the product required.

2. How many square feet in a floor 48 feet 6 inches long, and 24 feet 3 inches broad? Ans. 1176 sq. ft. 1′ 6″.

3. The length of a room being 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches, how many square yards of painting are in it, deducting a fireplace of 4 feet by 4 feet 4 inches, and 2 windows, each 6 feet by 3 feet 2 inches?

Ans. $73\frac{2}{27}$ square yards.

4. Required the solid contents of a wall 53 feet 6 inches long, 10 feet 3 inches high, and 2 feet thick.

5. There is a house with four tiers of windows, and four windows in a tier; the height of the first is 6 feet 8 inches; the second, 5 feet 9 inches; the third, 4 feet 6 inches; the fourth, 3 feet 10 inches; and the breadth is 3 feet 5 inches; how many square feet do they contain in the whole?

Ans. 283sq. ft. 7′.

6. How many cords in a pile of wood 97 feet 9 inches long, 4 feet wide, and 3 feet 6 inches high? Ans. $10\frac{177}{256}$ cords.

7. Required the number of cords of wood in a pile 100 feet long, 4 feet wide, and 6 feet 11 inches high. Ans. $21\frac{5}{8}$.

DIVISION OF DUODECIMALS.

601. To divide one duodecimal by another.

Ex. 1. A board in the form of a rectangle, whose area is 27 sq. ft. 8′ 6″, is 1ft. 7in. wide; what is its length?

Ans. 17ft. 6in.

FIRST OPERATION.

```
1ft. 7′ ) 27sq. ft. 8′ 6″ ( 17ft. 6′
          26      11
          ------------
                  9 6
                  9 6
          ------------
```

We find how many times 27 square feet contains the divisor, and obtain 17 feet for the quotient, we multiply the entire divisor by the 17ft., and subtract the product, 26ft. 11′, from the corresponding portion of the dividend, and obtain 9′, to which remainder we bring down the 6″, and, dividing, obtain 6′ for the quotient. Multiplying the entire divisor by 6″, we obtain 0ft. 9′ 6″, which, subtracted as before, leaves no remainder. Therefore 17 feet 6 inches is the length required.

SECOND OPERATION.

```
1·7 ) 23·86 ( 15·6 = 17ft. 6′
      17
      --
       88
       7e
       --
        96
        96
        --
```

In the second operation we reduce the feet of the given multiplicand and multiplier to the duodecimal scale, and thus obtain 23·86 and 1·7. We then conduct the division with reference to the radix 12, as is ordinarily done with respect to 10 (Art. 594). The result obtained is 15·6 in the duodecimal, which, on changing the figures to the left of the point to the decimal scale, and giving the proper notation to the figure on the right of the point, becomes transformed to 17ft. 6′ = 17ft. 6in., the answer, as before.

RULE.—*Find how many times the highest term of the dividend will contain the divisor. By this quotient multiply the entire divisor, and subtract the product from the corresponding terms of the dividend. To the remainder annex the next denomination of the dividend, and divide as before, and so continue till the division is complete.*

2. It required 834 sq. ft. 3′ of board to cover the side of a certain building. The height was 17ft. 9in.; what was the length of the side? Ans. 47 feet.

3. How many feet wide is a plank of uniform width, whose length is 18ft. 9in., thickness 3 inches, and solid contents 84ft. 4′ 6″?

4. An alley has an area of 792ft. 6′ 9″ 2‴. Its width is 12ft. 7′ 8″. Required its length. Ans. 62ft. 8′ 6″.

MISCELLANEOUS EXAMPLES.

1. A merchant engages a clerk at the rate of \$ 20 for the first month, \$ 25 for the second, \$ 30 for the third, &c., thus increasing his salary by \$ 5 per month. How long must the clerk retain his situation, so as to receive on the whole as much as he would have received had his salary been fixed at \$ 52.50 per month? Ans. 14 months.

2. A mason has plastered 3 rooms; the ceiling of each is 20 feet by 16 feet 6 inches, the walls of each are 9 feet 6 inches high, and 90 yards are to be deducted for doors, windows, &c. For how many yards must he be paid? Ans. 251yd. 1ft. 6′.

3. A man of wealth, dying, left his property to his ten sons, and the executor of his will, as follows: to his executor, \$ 1024; to his youngest son, as much and half as much more; and increasing the share of each next elder in the ratio of $1\frac{1}{2}$. What was the share of the eldest?

4. A butcher, wishing to buy some sheep, asked the owner how much he must give him for 20; on hearing his price, he said it was too much; the owner replied, that he should have 10, provided he would give him a cent for each different choice of 10 in 20, to which he agreed. How much did he pay for the 10 sheep, according to the bargain? Ans. \$ 1847.56.

5. If 340 square feet of carpeting are required to cover the floor of a room, how many yards will be required, provided the width of the carpeting is 3 feet 9 inches? Ans. 30yd. 8in.

6. If a clergyman's salary of \$ 700 per annum is 6 years in arrears, how much is due him, allowing compound interest at 6 per cent.? Ans. \$ 4882.72.

7. Suppose a clock to have an hour-hand, a minute-hand, and a second-hand, all turning on the same centre. At 12 o'clock all the hands are together and point at 12.

(1.) How long will it be before the second-hand will be between the other two hands, and at equal distances from each? Ans. $60\frac{780}{1427}$ seconds.

(2.) Also before the minute-hand will be equally distant between the other two hands? Ans. $61\frac{683}{697}$ seconds.

(3.) Also before the hour-hand will be equally distant between the other two hands? Ans. $59\frac{13}{73}$ seconds.

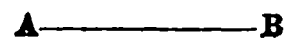
MENSURATION.

DEFINITIONS.

602. A *point* is that which has neither length, breadth, nor thickness, but position only.

603. A *line* is length, without breadth or thickness.

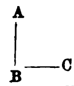
A *straight* line is one which has the same direction in its whole extent; as the line A B.



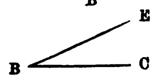
A *curved* line is one which continually changes its direction; as the line C D.

604. An *angle* is the inclination or opening of two lines, which meet in a point.

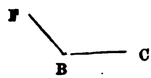
A *right* angle is an angle formed by a straight line and a perpendicular to it; as the angle A B C.



An *acute* angle is one less than a right angle; as the angle E B C.



An *obtuse* angle is one greater than a right angle; as the angle F B C.



605. A *surface* is that which has length and breadth, without thickness.

A *plane* surface, or simply a plane, is that in which, if any two points whatever be taken, the straight line that joins them will lie wholly in it.

Every surface, which is not a plane, or composed of planes, is a *curved* surface.

606. The *area* of a figure is its quantity of surface; and is estimated in the *square* of some unit of measure, as a square inch, a square foot, &c.

607. A *solid*, or *body*, is that which has length, breadth, and thickness.

608. The *solidity*, or *volume* of a solid, is estimated in the cube of some unit of measure; as a cubic inch, a cubic foot, &c.

609. Mensuration is the process of determining the areas of surfaces, and the solidity or volume of solids.

MENSURATION OF SURFACES.

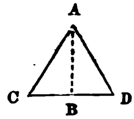
610. A *plane figure* is an enclosed plane surface; if bounded by straight lines only, it is called a *rectilineal figure*, or *polygon.* The perimeter of a figure is its boundary, or contour.

611. Three-sided polygons are called *triangles;* those of four sides, *quadrilaterals;* those of five sides, *pentagons,* and so on.

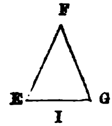
TRIANGLES.

612. An *equilateral triangle* is one whose sides are all equal; as C A D.

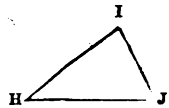
NOTE. — The line A B, drawn from the angle A perpendicular to the base C D, is the altitude of the triangle C A D.



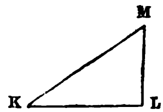
An *isosceles triangle* is one which has two of its sides equal; as E F G.



A *scalene triangle* is one which has its three sides unequal; as H I J.



A *right-angled triangle* is one which has a right angle; as K L M.



613. To find the area of a triangle.

Multiply the base by half the altitude, and the product will be the area. Or,

Add the three sides together, take half that sum, and from this subtract each side separately; then multiply the half of the sum and these remainders together, and the square root of this product will be the area.

Ex. 1. What are the contents of a triangle whose perpendicular height is 12 feet, and whose base is 18 feet? Ans. 108 feet.

2. There is a triangle, the longest side of which is 15.6 feet, the shortest side 9.2 feet, and the other side 10.4 feet. What are the contents? Ans. 46.139+ feet.

3. The triangular gable of a certain building has a base of 40 feet and an altitude of 15 feet; how many square feet of boards will cover it? Ans. 300 sq. ft.

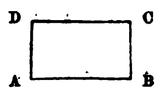
4. The perimeter of a certain field in the form of an equilateral triangle is 336 rods; what is the area of the field?

Ans. 33 acres 152 sq. rd.

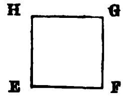
QUADRILATERALS.

614. A *parallelogram* is any quadrilateral whose opposite sides are parallel.

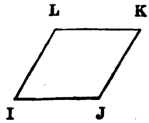
615. A *rectangle* is any right-angled parallelogram; as A B C D.



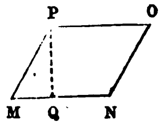
616. A *square* is a parallelogram whose sides are equal, and whose angles are right angles; as E F G H.



617. A *rhombus* is a parallelogram whose sides are equal, and whose angles are not right angles; as I J K L.

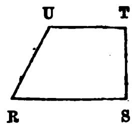


618. A *rhomboid* is a parallelogram whose angles are not right angles; as M N O P.

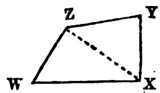


NOTE. — The *altitude* of a parallelogram is the perpendicular distance between any two of its parallel sides taken as bases, as the line P Q, drawn between two sides of the rhomboid M N O P, and perpendicular to the sides M N and O P.

619. A *trapezoid* is a quadrilateral which has only two of its sides parallel; as R S T U.



620. A *trapezium* is a quadrilateral which has no two sides parallel; as W X Y Z.



NOTE. — A *diagonal* of a quadrilateral, or of any polygon of more than four sides, is a straight line which joins the vertices of two opposite angles, or of two angles not adjacent; as the line X Z joining vertices of opposite angles of the trapezium W X Y Z.

621. To find the area of a parallelogram.

Multiply the base by the altitude, and the product will be the area.

Ex. 1. What are the contents of a board 15 feet long and 2 feet wide? Ans. 30 feet.

2. A rectangular state is 128 miles long and 48 miles wide. How many square miles does it contain? Ans. 6144 miles.

3. The base of a rhomboid being 12 feet, and its height 8 feet, required the area. Ans. 96 feet.

4. Required the area of a rhombus of which one of the equal sides is 358 feet, and the perpendicular distance between it and the opposite side is 194 feet. Ans. 69452 sq. ft.

5. The largest of the Egyptian pyramids is square at its base, and measures 693 feet on a side. How much ground does it cover? Ans. 11 acres 4 poles.

6. What is the difference between the area of a floor 40 feet square, and that of two others, each 20 feet square? Ans. 800 feet.

7. There is a square whose area is 3600 yards; what is the side

of a square, and the breadth of a walk along each side and each end of the square, which shall take up just one half of the whole?

Ans. $\begin{cases} 42.42+ \text{ yards, side of the square.} \\ 8.78+ \text{ yards, breadth of the walk.} \end{cases}$

622. To find the area of a trapezoid.

Multiply half of the sum of the parallel sides by the altitude, and the product is the area.

Ex. 1. If the parallel sides of a trapezoid are 75 and 33 feet, and the perpendicular breadth 20 feet, what is the area?

Ans. 1080 sq. ft.

2. Required the area of a meadow in the form of a trapezoid, whose parallel sides are 786 and 473 links, and whose altitude is 986 links. Ans. 6 acres 33 rods 3 yards.

623. To find the area of a trapezium.

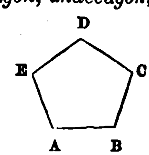
Divide the trapezium into two triangles by a diagonal, and then find the areas of these triangles; their sum will be the area of the trapezium.

Ex. 1. Required the area of a garden in the form of a trapezium, of which the four sides are 328, 456, 572, and 298 feet, and the diagonal, drawn from the angle between the first and second sides, 598 feet. Ans. 3 acres 1 rood 31 rods 29 yards 3.85 feet.

2. Given one of the diagonals of a field, in the form of a trapezium, equal 17 chains 56 links, to compute the area, the perpendiculars to that diagonal from the opposite angles being 8 chains 82 links, and 7 chains 73 links. Ans. 14 acres 2 roods 5 rods.

Pentagons, Hexagons, &c.

624. A *pentagon* is a polygon of five sides; a *hexagon*, one of six sides; a *heptagon*, one of seven sides; an *octagon*, one of eight sides; a *nonagon*, one of nine sides; and so on for a *decagon*, *undecagon*, *dodecagon*, &c.



625. A *regular polygon* is one whose sides and angles are equal; as the pentagon A B C D E.

626. To find the area of a regular polygon.

Multiply the perimeter by half the perpendicular let fall from the centre upon one of the sides. Or,

Multiply the square of one of the sides by the number against the polygon in the following

Table.

Pentagon,	1.720477	Nonagon,	6.181824
Hexagon,	2.598076	Decagon,	7.694209
Heptagon,	3.633913	Undecagon,	9.365641
Octagon,	4.828427	Dodecagon,	11.196152

Ex. 1. What is the area of a regular pentagon, of which the side is 250 feet, and the perpendicular from the centre to one side 172.05 feet? Ans. 107531.25 sq. ft.

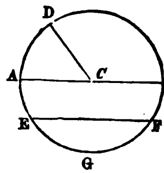
2. What is the area of a regular hexagon whose side is 356 yards, and whose perpendicular is 308.305 yards? Ans. 329269.74yd.

3. The side of a regular octagonal enclosure is 60 yards; how many acres are included? Ans. 3 acres 2 roods 14 rods 19 yards.

4. The side of a field, whose shape is that of a regular decagon, is 243 feet; what is its area?

Ans. 10 acres 1 rood 28 rods 24 yards 6.347 feet.

CIRCLES.



627. A *circle* is a plane figure bounded by a line, every part of which is equally distant from a point within called the *centre;* as A E F G B D.

The *circumference* or periphery of a circle is the line that bounds it.

A *radius* is a line drawn from the centre to the circumference; as C A, or C D.

A *diameter* is a line which passes through the centre, and is terminated by the circumference; as A B.

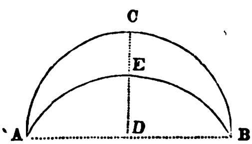
An *arc* is any portion of the circumference; as A D, A E, or E G F.

The *chord* of an arc is the straight line joining its extremities; as E F, which is the chord of the arc E G F.

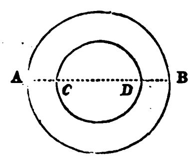
628. The *segment* of a circle is the portion included by an arc and its chord; as the space included by the arc E G F and the chord E F.

629. The *sector* of a circle is the portion included by two radii and the intercepted arc; as the space A C D A.

630. A *zone* is the space between two parallel chords of a circle; as the spaee A E F B A.



631. A *lune*, or *crescent*, is the space included between the intersecting arcs of two eccentric circles; as A C B E A.



632. A *circular ring* is the space included between the circumference of two concentric circles; as the space between the rings A B and C D.

633. To find the *circumference* of a circle, the *diameter* being given.

Multiply the diameter by 3.141592.

Ex. 1. If the diameter of a circle is 144 feet, what is the circumference? Ans. 452.389248 feet.

2. If the diameter of the earth is 7964 miles, what is its circumference? Ans. 25019.638688+ miles.

3. Required the circumference of a circle, whose radius is 512 feet. Ans. 4 furlongs 34 rods 5 yards 1 foot.

634. To find the *diameter* of a circle, the circumference being given.

Multiply the circumference by .318309.

Ex. 1. Required the diameter of a circle, whose circumference is 1043 feet. Ans. 331.997+ feet.

2. If the circumference of a circle is 25000 miles, what is its diameter? Ans. 7957.74+ miles.

3. If the circumference of a round stick of timber is 50 inches, what is its diameter? Ans. 15.91549+ inches.

635. To find the area of a circle, the diameter, or the circumference, or both, being given.

Multiply the square of the diameter by .785398. Or,
Multiply the square of the circumference by .079577. Or,
Multiply half the diameter by half the circumference.

Ex. 1. If the diameter of a circle is 761 feet, what is the area? Ans. 454840.475158 feet.

2. There is a circular island, three miles in diameter; how many acres does it contain? Ans. 4523.89+ acres.

3. Required the area of a circle, of which the circumference is 1284 yards. Ans. 27 acres 17 rods 0.8+ yards.

4. Required the area of a circle, of which the diameter is 169, and the circumference 532 inches. Ans. 17 yards 3 feet 13 inches.

636. To find the *area* of a sector of a circle.

Multiply the length of the arc by half the radius of the circle. Or,
As 360° *are to the degrees in the arc of the sector, so is the area of the circle to the area of the sector.*

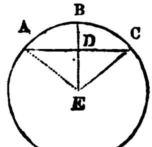
Ex. 1. Required the area of a sector, of which the arc is 79 and the radius of the circle 47 inches. Ans. 1856.5 inches.

2. Required the area of a sector, of which the arc is 26°, and the radius of the circle 25 feet. Ans. 141.8 square feet.

637. To find the area of the segment of a circle.

Find the area of the sector which has the same arc with the segment; and also the area of the triangle formed by the chord and the radii drawn to its extremities. The difference of these areas, when the segment is less, and their sum, when the segment is greater, than the semicircle, will be the area of the segment. Or,

To two thirds of the product of the height of the segment by the chord add the cube of the height, divided by twice the chord.



Ex. 1. Required the area of the segment A B C A, of which the arc A B C is 49.25°, the chord A C 10 feet, and the radii E A, E B, and E C, each 12 feet. Ans. 7.35 sq. ft.

2. Required the area of a segment whose height is 15 rods and whose chord is 24 rods. Ans. 1 acre 3 roods 30 rods 9.4 yards.

638. To find the area of a zone of a circle.

From the area of the whole circle subtract the areas of the segments on the sides of the zone.

Ex. 1. Required the area of a zone whose parallel sides are 23.25 and 20.8 feet, in a circle whose radius is 12 feet. Ans. 206+ sq. ft.

2. Required the area of a zone included between two chords of 16 feet each, the diameter of the circle being 20 feet. Ans. 224.7 sq. ft.

639. To find the area of a lune or crescent.

Find the difference of the areas of the two segments formed by the arcs of the lune and its chord.

Ex. 1. If the chord of two intersecting arcs is 72 feet, and the height of one of the segments is 30, and of the other 20 feet, what is the area of the crescent? Ans. 612 sq. ft.

640. To find the area of a circular ring.

Multiply the sum of the diameters of the two circles by the difference of the diameters, and that product by .7854.

Ex. 1. What is the area of the ring formed by two circles whose diameters are 10 and 20 yards? Ans. 235.62 sq. yd.

2. In the centre of a circular pond there is an island 128 yards in diameter; what is the area of the pond, provided the exact distance from any part of the outer side of the pond to the centre of the island is 78½ yards? Ans. 1 acre 1 rood 14 rods 17 yards 7.4 feet.

641. To find the side of a square that shall equal the area of a circle of a given diameter or circumference.

Multiply the diameter of the circle by .886227. Or,
Multiply the circumference of the circle by .282094.

Ex. 1. I have a round field, 50 rods in diameter; what is the side of a square field that shall contain the same area?
Ans. 44.31135+ rods.

2. I have a circular field 360 rods in circumference; what must be the side of a square field that shall contain the same area?
Ans. 101.55+ rods.

3. John Smith had a farm which was 10,000 rods in circumference, which he sold at $ 71.75 per acre. He purchased another farm con-

taining the same quantity of land in the form of a square; required the length of one of its sides. Ans. 2820.94+ rods.

642. To find the *diameter* of a circle that shall contain the area of a given square.

Multiply the side of the given square by 1.12838.

Ex. 1. The side of a square is 44.31135 rods; required the diameter of a circular field containing the same area.

643. To find the side of the largest equilateral triangle that can be inscribed in a circle of a given diameter or circumference.

Multiply the given diameter by .866025. Or,
Multiply the given circumference by .275664.

Ex. 1. There is a certain piece of round timber 30 inches in diameter; required the side of an equilateral triangular beam that may be hewn from it. Ans. 25.98+ inches.

2. How large an equilateral triangle may be inscribed in a circle whose circumference is 5000 feet? Ans. 1378.320 feet.

3. Required the side of an equilateral triangular beam, that may be hewn from a round piece of timber 80 inches in circumference. Ans. 22.05+ inches.

644. To find the side of the largest square that can be inscribed in a circle of a given diameter or circumference.

Multiply the given diameter by .707106. Or,
Multiply the given circumference by .225079.

NOTE. — To find the circumference of a circle required to exactly admit a square of a given side, *divide the given side by* .225079.

Ex. 1. I have a piece of timber 30 inches in diameter; how large a square stick can be hewn from it? Ans. 21.21+ in. square.

2. Required the side of a square that may be inscribed in a circle 80 feet in diameter. Ans. 56.56848+ feet.

3. I have a circular field whose circumference is 5000 rods; what is the side of the largest square field that can be made in it? Ans. 1125.395+ rods.

4. How large a square stick may be hewn from a piece of round timber 100 inches in circumference? Ans. 22.5+ inches square.

5. What must be the circumference of a tree that, when hewn, shall be 18 inches square? Ans. 79.97+ inches.

6. I have a garden which is 20 rods square; required, in feet, the circumference of a circle that will enclose this garden. Ans. 1466.15+ feet.

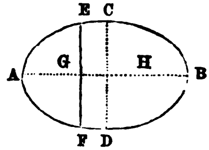
645. To find the diameter of the three largest equal circles that can be inscribed in a circle of a given diameter.

Divide the given diameter by 2.155.

Ex. 1. Required the diameter of each of the largest three circles that can be inscribed in a circle 86.2 inches in diameter. Ans. 40 inches.

Ellipse.

646. An *ellipse* is a plane figure bounded by a curve, from any point of which the sum of the distances to two fixed points is equal to a given distance. The two fixed points are called the *foci*, as G,H in the ellipse A E C B D F A. The *major* or *transverse axis* of an ellipse is its longest diameter, as A B. The *minor* or *conjugate axis* of an ellipse is its shortest diameter, as C D. The *segment* of an ellipse is a portion cut off from the ellipse, as F A E F.



647. To find the area of an ellipse, the two diameters being given.

Multiply the two diameters together, and that product by .785398.

Ex. 1. What is the area of an ellipse, whose two diameters are 24 and 18 inches? Ans. 339.2919 inches.

2. What is the area of an elliptical pond, whose longest diameter is 33 feet 5 inches, and whose shortest diameter 20 feet 3 inches? Ans. 59 sq. yd. 67 sq. in.

MENSURATION OF SOLIDS.

Prisms and Cylinders.

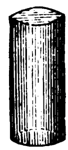
648. A *prism* is a figure whose ends or bases are any plane figures which are equal and similar, and parallel to each other, and whose sides are parallelograms.

A *triangular prism* is one whose base is a triangle; as the figure A B.

A *square prism* is one whose base is a square; a *pentagonal prism*, one whose base is a pentagon; and so on, according to the figure of the ends or bases.

A *parallelopiped* is a prism whose ends or bases, as well as its sides, are parallelograms.

649. A *cylinder* is a round body of uniform diameter, and which has circular bases parallel to each other; as the figure C D.



The *perimeter* of a prism or cylinder is the line that bounds its end or base; and the altitude or height is the distance between the ends or bases.

The *convex* surface of a prism or cylinder is the entire surface, exclusive of the two ends or bases.

650. To find the surface of a prism, or of a cylinder.

Multiply the perimeter of the given prism or cylinder by the height, and to the product add the area of the two ends.

Ex. 1. Required the surface of a triangular prism, of which the dis-

tance between the ends is 13 feet, and the sides of the base 23, 34, and 19 inches. Ans. 85.22+ square feet.

2. Required the surface of a pentagonal prism, whose length is 14 feet, and each side of whose base is 33 inches. Ans. 218.52 sq. ft.

3. Required the surface of a cylinder 13 feet long, the circumference of whose base is 57 inches. Ans. 65.34 square feet.

4. How often must a cylinder, 5 feet 3 inches long, whose diameter is 21 inches, revolve, to roll an acre? Ans. 1509.18 times.

5. Required the wall-surface of a square room, whose sides are each 16 feet long and 10 feet high. Ans. $71\frac{1}{9}$ square yards.

651. To find the contents or volume of a prism or cylinder.

Multiply the area of the base of the given prism or cylinder by the height.

Ex. 1. What are the contents of a triangular prism, whose length is 12 feet, and each side of whose base is $2\frac{1}{2}$ feet? Ans. 32.47+ cu. ft.

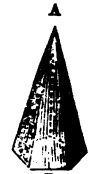
2. Required the volume of a triangular prism, whose length is 10 feet, and the three sides of whose triangular end or base are 5, 4, and 3 feet. Ans. 60 cu. ft.

3. How many cubic feet in a block of marble, whose length is 3 feet 2 inches, breadth 2 feet 8 inches, and depth 2 feet 6 inches? Ans. $21\frac{1}{9}$ cu. ft.

4. What is the volume of a cylinder, whose length is 9 feet, and the circumference of whose base is 6 feet? Ans. 25.78+ cu. ft.

Pyramids and Cones.

A



B

652. A *pyramid* is a solid having for its base some rectilineal figure, and for its sides triangles meeting in a common point called the vertex; as the figure A B.

The *slant height* of a pyramid is a line drawn from the vertex to the middle of one of the sides of the base.

653. A *cone* is a solid having a circle for its base, and tapering uniformly to a point called the *vertex.*

The *slant height* of a cone is a line drawn from the vertex to the circumference of the base.

654. The *altitude* or *height* of a pyramid or of a cone is a line drawn from the vertex perpendicular to the plane of the base.

D

C

655. The *frustum of a solid* is the part that remains after cutting off the top by a plane parallel to the base; as the frustum of a cone C D.

656. To find the surface of a pyramid or of a cone.

Multiply the perimeter or the circumference of the base by half of the slant height, and to the product add the area of the base.

Ex. 1. Required the area of the surface of a square pyramid, whose base is 2 feet 8 inches square, and whose slant height is 3 feet 9 inches.

2. What is the convex surface of a cone, whose slant height is 20 feet, and the circumference of whose base is 9 feet? Ans. 90 feet.

657. To find the volume of a pyramid, or of a cone.

Multiply the area of the base by one third of the altitude.

Ex. 1. What is the solidity of a cone, whose height is $12\frac{1}{2}$ feet, and the diameter of whose base is $2\frac{1}{2}$ feet? Ans. 20.45+ feet.

2. What are the contents of a triangular pyramid, whose height is 14 feet 6 inches, and the sides of whose base are 5, 6, and 7 feet? Ans. 71.035+ feet.

658. To find the surface of a frustum of a pyramid, or of a cone.

Multiply the sum of the perimeters or of the circumferences of the two ends by half of the slant height; and to the product add the areas of the two ends.

Ex. 1. Required the surface of a frustum of a pentagonal pyramid, whose slant height is 10 inches, and the sides of whose base are 3 and 5 inches.

2. What is the surface of the frustum of a cone, the diameters of the bases being 43 and 23 inches, and the slant height 9 feet? Ans. 90.72+ sq. ft.

659. To find the volume of a frustum of a pyramid, or of a cone.

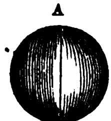
Multiply the areas of the two ends together, and extract the square root of the product. To this root add the two areas, and multiply their sum by one third of the altitude.

Ex. 1. If the length of a frustum of a square pyramid be 18 feet 8 inches, the side of its greater base 27 inches, and that of its less 16 inches, what is the volume? Ans. 61.228+ cu. ft.

2. What are the contents of a stick of timber, whose length is 40 feet, the diameter of the larger end being 24 inches, and of the smaller end 12 inches? Ans. $73\frac{1}{3}$ cu. ft., nearly.

SPHERES, SPHEROIDS, &c.

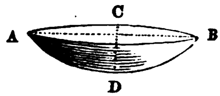
660. A *sphere* is a solid, bounded by a curved surface, every part of which is equally distant from a point within, called the centre.



The *axis* or *diameter* of a sphere is a line passing through the centre, and terminated by the surface; as the line A B.

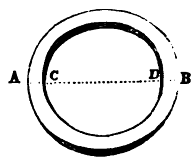
The *radius* of a sphere is a line drawn from the centre to any part of the surface.

661. A *segment* of a sphere is a part of it cut off by any plane; as the figure A C B D. The plane is the *base* of the segment; the perpendicular distance from the centre of the base to the convex surface is the *height* of the segment; as C D.



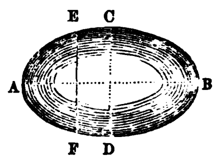
662. A *spherical zone* is a part of the surface of a sphere included between two parallel planes, which form its *bases;* and the *height* of a spherical zone is the perpendicular distance between the planes forming its bases.

663. A *cylindrical ring* is a figure formed by bending a cylinder uniformly till the two ends meet; as A C D B.



664. A *spheroid* is a figure resembling a sphere, and which may be formed by the revolution of an ellipse about one of its axes; as A E C B D F A.

If the ellipse revolves about its longer or transverse axis or diameter, the spheroid is *prolate*, or oblong; if about its shorter or conjugate diameter, the spheroid is *oblate*, or flattened.



665. A *segment* of a spheroid is a part cut off by any plane, as F A E F.

666. To find the surface of a sphere.

Multiply the diameter by the circumference.

Ex. 1. Required the convex surface of a globe, whose diameter is 24 inches. Ans. 1809.55+ inches.

2. Required the surface of the earth, its diameter being 7957$\frac{3}{4}$ miles, and its circumference 25,000 miles. Ans. 198943750 square miles.

667. To find the solidity of a sphere.

Multiply the cube of the diameter by .523598.

Ex. 1. What is the solidity of a sphere, whose diameter is 12 inches? Ans. 904.78+ inches.

2. Required the solidity of the earth, supposing its circumference to be 25,000 miles. Ans. 263858149120.06886875 miles.

668. To find the convex surface of a segment or of a zone of a sphere.

Multiply the height of the segment or zone by the circumference of the sphere of which it is a part.

Ex. 1. If the diameter of a sphere is $12\frac{1}{2}$ feet, what is the convex surface of a segment cut off from it, whose height is 2 feet?

Ans. 78.54 sq. ft.

2. If the diameter of the earth, considered as a perfect sphere, is 7970 miles, and the height of each temperate zone is taken at 2143.623553 miles, what is the surface of each temperate zone?

Ans. 53673093.12+ sq. m.

669. To find the solidity of a segment of a sphere.

Multiply the square of the height plus three times the square of the radius of the base, by the height, and this product by .5236.

Ex. 1. Required the solidity of a spherical segment, whose height is 3 feet, and the radius of whose base is $4\frac{1}{2}$ feet. Ans. 109.56 cu. ft.

2. Required the solidity of the segment of a sphere, whose height is 9 feet, and the diameter of whose base is 20 feet.

Ans. 1795.42 cu. ft.

670. To find the surface of a cylindrical ring.

Multiply the sum of the thickness and the inner diameter by the thickness, and that product by 9.8696.

Ex. 1. Required the surface of a cylindrical ring, whose inner diameter is 21 inches, and whose thickness is 4 inches.

Ans. 986.96 sq. in.

671. To find the solidity of a cylindrical ring.

Multiply the sum of the thickness and the inner diameter by the square of the thickness, and that product by 2.4674.

Ex. 1. Required the solidity of a cylindrical ring, whose inner diameter is 25 inches, and whose thickness is 5 inches.

Ans. 1850.55 cu. in.

672. To find the solidity of a spheroid.

Multiply the square of the revolving axis by the fixed axis, and that product by .523598.

Ex. 1. If the fixed axis of a spheroid is 32 inches, and the revolving axis 20 inches, what is the solidity? Ans. 6702.08 cu. in.

2. Required the contents of a balloon in the form of a prolate spheroid, having its longer diameter 48 feet, and its shorter 38 feet.

Ans. 36291.76 cu. ft.

MENSURATION OF LUMBER.

673. Boards are usually measured by the square foot.

Planks, joists, beams, &c. are usually measured by board measure, the board being considered to be 1 inch in thickness.

Round timber is sometimes measured by the ton, and sometimes by board measure.

674. To find the number of square feet in a board.

Multiply the length of the board, taken in feet, by its width, taken in inches; and the product divided by 12 *will give the contents in square feet.* Or,

Take both the length and width in feet, and their product will be the contents in feet.

NOTE. — If the board is tapering, take half the sum of the width of its ends for the width.

Ex. 1. What are the contents of a board 24 feet long, and 8 inches wide? Ans. 16 feet.

2. What are the contents of a board 30 feet long, and 16 inches wide? Ans. 40 feet.

3. What are the contents of a tapering board, 30 feet long, whose ends are, the one 26 inches, and the other 14 inches wide?

675. To find the number of feet, board measure, in a plank, joist, beam, &c.

Multiply the width taken in inches by the thickness in inches, and this product by the length, in feet; and the last product divided by 12 *will give the contents in feet, board measure.*

NOTE. — If the plank, joist, &c. is tapering in width, take half the sum of the width of the ends for the width; and if the taper be both of the width and the thickness, the common rule of obtaining the contents in cubic feet is, to *multiply half the sum of the areas of the two ends by the length, and divide the product by* 144.

Ex. 1. How many feet are there in 3 joists, which are 15 feet long, 5 inches wide, and 3 inches thick? Ans. $56\frac{1}{4}$ feet.

2. How many feet in 20 joists, 10 feet long, 6 inches wide, and 2 inches thick? Ans. 200 feet.

3. How many feet in a beam 20 feet long, 10 inches thick, whose width tapers from 18 to 16 inches? Ans. $283\frac{1}{3}$ feet.

676. To find the contents of round timber.

Multiply the length, taken in feet, by the square of one fourth of the mean girth, taken in inches; and this product divided by 144 *will give the contents in cubic feet.*

NOTE. — The girth of tapering timber is usually taken about $\frac{1}{3}$ the distance from the larger to the smaller end.

The rule is that in common use, though very far from giving the actual number of cubic feet in round lumber measured by it. 40 cubic feet, as given by the rule, are in fact equal to $50\frac{92}{100}$ true cubic feet. The following rule gives results more nearly accurate, requiring to be diminished by only one foot in 190, to give exact contents. *Multiply the square of one fifth of the mean girth, taken in inches, by twice the length, in feet; and divide by* 144.

Ex. 1. How many cubic feet in a stick of timber which is 30 feet long, and whose girth is 40 inches? Ans. $20\frac{5}{6}$ feet.

2. If a stick of timber is 50 feet long, and its girth is 56 inches, what number of cubic feet does it contain? Ans. $68\frac{1}{18}$ feet.

3. What are the contents of a log 90 feet long, and whose circumference is 120 inches? Ans. $562\frac{1}{2}$ feet.

GAUGING OF CASKS.

677. Gauging is the process of finding the capacities of casks or other vessels.

Casks are generally considered to be of four varieties: 1. Having the staves nearly straight; 2. Having the staves very little curved; 3. Having the staves of a medium curve; 4. Having the staves considerably curved.

NOTE. — Casks of the first variety approach very nearly the form of a cylinder; those of the third variety are of the shape of a molasses hogshead; those of the second variety have a curvature of stave between that of the first and third; and the fourth have a greater curvature than that of the third.

678. In gauging casks, it is necessary first to find the *mean diameter*. This is found by taking the end and middle diameters, and the length in inches; and then adding to the end diameter the product of the difference between the end and middle diameters by .55, .60, .65, or .70, as the cask may be of the first, second, third, or fourth variety.

679. To find the capacity of a cask in gallons

Multiply the square of the mean diameter, in inches, by the length, in inches; and the product multiplied by 0034 *will give the capacity in liquid or wine gallons.*

NOTE 1. — If the capacity is required in ale or beer gallons, use for a multiplier .0028 instead of .0034. If imperial gallons are required, multiply the liquid or wine gallon, as found by the rule, by .833.

NOTE 2. — The contents of any vessel being known in cubic inches, its capacity in liquid gallons may be found by dividing by 231; in ale or beer gallons, by dividing by 282; and in bushels, by dividing by 2150.42.

Ex. 1. Required the capacity in gallons of a cask of the fourth variety, whose middle diameter is 35 inches, head diameter 27 inches, and length 45 inches. Ans. 162.6.

2. What is the capacity in gallons of a cask of the third variety, whose middle diameter is 38 inches, head diameter 30 inches, and length 42 inches?

3. What are the contents in liquid measure of a tub 40 inches in diameter at the top, 30 inches at the bottom, and whose height is 50 inches? Ans. 209.66gal.

4. How many wine gallons will a cubical box contain, that is 10 feet long, 5 feet wide, and 4 feet high? Ans. $1496\frac{8}{77}$gal.

5. How many ale gallons will a trough contain, that is 12 feet long, 6 feet wide, and 2 feet high? Ans. $882\frac{18}{47}$gal.

6. How many bushels of grain will a box contain that is 15 feet long, 5 feet wide, and 7 feet high? Ans. 421.8bu.

TONNAGE OF VESSELS.

680. The tonnage of a ship is the number of tons burden it will carry, with safety, under the ordinary circumstances of navigation.

The *light-loaded water-line* of a vessel is the line made by the water upon the outside of the hull as it floats without load; and the *deep-loaded water-line* is that made in like manner when it is fully laden.

The number of cubic feet of the hull between these two water-lines, divided by 35, the number of cubic feet of sea-water which must be taken to weigh a ton, represents the weight of water displaced in sinking the vessel from the light to the deep-loaded water-line, and therefore its true tonnage.

681. Government has by law established a rule by which the custom-house officers are to be guided in collecting tonnage duties. But as it does not always give the actual tonnage, builders, and others, usually make their estimates by some other rule.

Government Rule.

For Single-decked Vessels. *Take the length on deck from the forward side of the main stem to the after side of the stern post, and the breadth at the broadest part above the main wales; take the depth from the under side of the deck plank to the ceiling of the hold; and deduct from the length three fifths of the breadth; multiply the remainder by the breadth, and the product by the depth; and divide the last product by* 95.

For Double-decked Vessels. *Proceed as with single-decked vessels, except for the depth take half the breadth.*

Note. — The government rule is differently construed. The length is usually taken in a line with the deck; the depth at the main hatch. But with regard to the breadth, there is a great want of uniformity among measurers; most take the breadth about 45 inches below the plank-sheer at the broadest part; some consider the upper wales, and others the lower, at the main wales, thus making a considerable difference in their results.

The government rule for single-decked vessels operates very well, but the rule for double-decked vessels, which is also intended to include all vessels of more than one deck, often fails to give the true tonnage. A more accurate method would, for double-decked vessels, *take the breadth* 5 *feet below the upper deck, at the broadest part, and for* three-decked vessels 7 *feet below the upper deck; and in each case for depth of hold three fifths of the breadth.*

Ex. 1. A. & G. T. Sampson, of East Boston, have contracted to build a clipper ship $191\frac{6}{12}$ feet long, $36\frac{5}{12}$ feet wide, $22\frac{2}{12}$ feet deep; what is the government tonnage of the ship? Ans. $1184\frac{427}{8200}$ tons.

2. What is the government tonnage of the ship Meridian, whose length is $184\frac{6}{12}$, width $38\frac{11}{12}$, and depth 28 feet?
Ans. $1284\frac{296047}{547200}$ tons.

3. The ship Mattakeeset is $195\frac{2}{12}$ feet long, $39\frac{4}{12}$ wide, and $27\frac{6}{12}$ deep; what is the government tonnage of the same?
Ans. $1397\frac{182}{12825}$ tons.

4. Required the tonnage of a single-decked vessel, whose length is 78 feet, width 21 feet, and depth 9 feet. Ans. $130\frac{53}{475}$ tons.

5. What is the government tonnage of a double-decked vessel, whose length is 159 feet, and width 30 feet? Ans. $667\frac{17}{19}$ tons.

6. What is the government tonnage of Noah's ark, admitting its length to have been 479 feet, its breadth 80 feet, and its depth 48 feet? Ans. $14517\frac{17}{19}$ tons.

MISCELLANEOUS EXAMPLES.

1. What number is that to which, if $\frac{2}{7}$ of $\frac{5}{9}$ be added, the sum will be 1? Ans. $\frac{53}{63}$.

2. A certain gentleman, at the time of his marriage, agreed to give his wife $\frac{2}{3}$ of his estate, if, at the time of his death, he left only a daughter, and if he left only a son, she should have $\frac{1}{3}$ of his property; but as it happened, he left a son and a daughter, in consequence of which the widow received in equity \$ 2400 less than she would have received if there had been only a daughter. What would have been his wife's dowry if he had left only a son? Ans. \$ 2100.

3. A gentleman being asked what o'clock it was, said that it was between 5 and 6; but, to be more particular, he said that the minute-hand had passed as far beyond the 6 as the hour-hand wanted of having reached the 6; that is, that the hour and minute-hands made equal acute angles with a line passing from the 12 through the 6. Required the time of day. Ans. 32m. $18\frac{6}{13}$s. past 5.

4. Divide 97deg. 55m. 7fur. 35rd. 4ft. 6in. by 6.

5. A, B, and C are to share \$ 100,000 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, respectively; but C's part being lost by his death, it is required to divide the whole sum properly between the other two.

Ans. A's part is \$ $57,142\frac{6}{7}$, and B's \$ $42,857\frac{1}{7}$.

6. A father devised $\frac{7}{18}$ of his estate to one of his sons, and $\frac{7}{18}$ of the residue to the other, and the remainder to his wife. The difference of his sons' legacies was found to be 257£. 3s. 4d. What money did he leave for his widow? Ans. 635£. 0s. $10\frac{80}{49}$d.

7. In the walls of Balbec, in Turkey, the ancient Heliopolis, there are three stones laid end to end, now in sight, that measure 61 yards in length, one of which is 63 feet long, 12 feet thick, and 12 feet broad; what is its weight, supposing its specific gravity to be 3 times that of water? Ans. $850\frac{1}{2}$ tons.

8. A burden of 200lb., suspended on a pole 4ft. in length, the point of suspension being 6in. from the middle, is carried by two men, the ends of the pole resting on their shoulders; how much of this load is borne by each man? Ans. 125lb. and 75lb.

9. The court-house in Boston has eight pillars of granite, each 25ft. 4in. in length, 4ft. 5in. in diameter at one end, and 3ft. 5in. in diameter at the other end. How many cubic feet do they contain, and what is their weight, allowing a cubic foot to weigh 3000 ounces? Ans. 2455.03 cub. ft.; 230.15+ tons.

10. A father, dying, left his son a legacy, $\frac{1}{4}$ of which he spent in 8 months; $\frac{3}{7}$ of the remainder lasted him 12 months longer, after which he had only \$ 410 left. What amount did his father bequeath him? Ans. \$ $956.66\frac{2}{3}$.

11. A merchant sold goods to a certain amount, on a commission of 4 per cent., and, having remitted the net proceeds to the owner, received $\frac{1}{4}$ per cent. for prompt payment, which amounted to \$ 15.60. What was the amount of his commission? Ans. \$ 260.

12. A, of Boston, remits to B, of New York, a bill of exchange on London, the avails of which he wishes to be invested in goods on his account. B, having disposed of the bill at $7\frac{1}{2}$ per cent. advance, received \$ 9675; and having reserved for himself $\frac{1}{4}$ per cent. on the sale of the bill, and 2 per cent. for commission, he invests the remainder. What is the amount invested, and for how much was the bill drawn? Ans. Investment, \$ $9461.58\frac{8}{34}$; the bill was £ 2025.

13. Bunker Hill Monument is 30ft. square at its base, and 15ft. square at its top; its height is 220 feet. From the bottom to the top, through its centre, is a conical avenue 15ft. in diameter at the bottom, and 11ft. at the top. How many cubic feet are there in the monument? Ans. 86,068.518+ ft.

14. A hired a house for one year for \$ 300; at the end of 4 months he takes in M as a partner, and at the end of 8 months he takes in P. At the end of the year, what rent must each pay?

Ans. A pays \$ $183\frac{1}{3}$; M pays \$ $83\frac{1}{3}$; P pays \$ $33\frac{1}{3}$.

15. A merchant receives on commission three kinds of flour; from A he receives 20 barrels, from B 25 barrels, and from C 40 barrels. He finds that A's flour is 10 per cent better than B's, and that B's is 20 per cent. better than C's. He sells the whole at \$ 6 per barrel. What in justice should each man receive?

Ans. A receives \$ $139\frac{161}{241}$; B, \$ $158\frac{172}{241}$; C, $211\frac{149}{241}$.

16. Bought 100 barrels of flour, at \$ 5 per barrel, and immediately sold it on a credit of six months. The note which I received for pay I got discounted at the Suffolk Bank, and, on examining my money, I found that I had gained 20 per cent. on my purchase. What did I receive per barrel for the flour? Ans. \$ $6.18\frac{1698}{1939}$.

17. Required the greatest possible number of hills of corn that can be planted on a square acre, the hills to occupy only a mathematical point, and no two hills to be nearer than three and a half feet. Ans. 4165.

18. Lent a friend \$ 700, which he kept 20 months. Some years after I borrowed of him \$ 300; how long should I keep it to balance the favor? Ans. $46\frac{2}{3}$ months.

19. John Lee gave $\frac{1}{2}$ of his estate to his wife, $\frac{1}{5}$ of the remainder to his oldest son, and $\frac{1}{4}$ of the residue to his oldest daughter, and $\frac{1}{2}$ of what then remained, which was \$ 1500, was to be equally distributed among his other children, who received \$ 150 each; required the number of his children, and the value of his estate.

20. A and B set out to travel round a certain island, which is 80 miles in circumference. A travels 5 miles a day, and B 7 miles a day. How far must B travel to overtake A? Ans. 280 miles.

21. If 24.4 cubic inches of lead weigh 16 pounds, required the number of feet of lead pipe that can be made from 80 pounds of lead, the caliber of the pipe to be 1 inch, and the thickness of it $\frac{1}{4}$ of an inch. Ans. 10.35+ feet.

22. How long a tube can be made from a cylinder of lead 8 inches long and 2 inches in diameter, and through the centre of which is a hole $\frac{3}{4}$ of an inch in diameter; the tube or pipe to be $\frac{3}{4}$ of an inch in caliber, and $\frac{1}{8}$ of an inch in thickness? Ans. 16.29+ in.

23. Four men, A, B, C, and D, bought a stack of hay, containing

8 tons, for \$ 100. A is to have 12 per cent. more of the hay than B, B is to have 10 per cent. more than C, and C is to have 8 per cent. more than D. Each man is to pay in proportion to the quantity he receives. The stack is 20 feet high, and 12 feet square at its base, it being an exact pyramid; and it is agreed that A shall take his share first from the top of the stack, B is to take his share the next, and then C and D. How many feet of the perpendicular height of the stack shall each take, and what sum shall each pay?

Ans. A takes 13.22+ft., and pays \$ $28.93\frac{12287}{28741}$; B takes 3.14+ft., and pays \$ $25.83\frac{11997}{28741}$; C takes 2.06+ft., and pays \$ $23.48\frac{16182}{28741}$; D takes 1.58+ft., and pays \$ $21.74\frac{17068}{28741}$.

24. A, B, and C bought a grindstone, for which they paid \$ 10.60. B paid 20 per cent. more than A, and 10 per cent. less than C. The diameter of the stone was 65 inches, and the diameter of the place for the shaft 3 inches. What sum did each pay, and how much must each grind off from the semidiameter to obtain his proper share of the stone?

Ans. A paid \$ 3, B \$ 3.60, and C \$ 4. A grinds off 5 inches; B $7\frac{1}{2}$ inches, and C $18\frac{1}{2}$ inches.

25. A servant draws off a gallon on each day, for 20 days, from a cask containing 10 gallons of wine, each time supplying the deficiency by the addition of a gallon of water; and then, to escape detection, he again draws off 20 gallons, supplying the deficiency each time by a gallon of wine. How much water still remains in the cask?

Ans. 1.0679577 gallons, or more than a gallon and half a pint.

26. The dimensions of a bushel measure are $18\frac{1}{2}$ inches wide, and 8 inches deep; what should be the dimensions of a similar measure that would contain 8 bushels? Ans. 37in. wide, 16in. deep.

27. What is the weight of a hollow spherical iron shell 5 inches in diameter, the thickness of the metal being 1 inch, and a cubic inch of iron weighing $\frac{129}{500}$ of a pound? Ans. 13.2387lb.

28. At a certain time between 2 and 3 o'clock, the minute-hand was between 3 and 4. Within an hour after, the hour-hand and minute-hand had exactly changed places with each other. What was the precise time when the hands were in the first position?

Ans. 2h. 15m. $56\frac{92}{143}$s.

29. Required the contents of the largest cube that can be inscribed in a sphere 20 inches in diameter. Ans. 1539.58+ cu. in.

30. If in a pair of scales a body weigh 90 pounds in one scale, and only 40 pounds in the other, required the true weight, and the proportion of the lengths of the two arms of the balance-beam on each side of the point of suspension.

Ans. Weight 60lb., and the proportions 3 to 2.

31. In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer wheel made two turns, while the inner wheel made but one; the wheels were each 4 feet high; and supposing them fixed at the distance of 5 feet asunder on the axletree, what was the circumference of the track described by the outer wheel? Ans. 62.83+ feet.

32. The ball on the top of St. Paul's Church is 6 feet in diameter. What did the gilding of it cost, at $3\frac{1}{2}$d. per square inch?

33. There is a conical glass, 6 inches high, 5 inches wide at the top, and which is $\frac{1}{8}$ part filled with water. What must be the diameter of a ball, let fall into the water, that shall be immersed by it?

Ans. 2.445+ inches.

34. A certain lady, the mother of three daughters, had a farm of 500 acres, in a circular form, with her dwelling-house in the centre. Being desirous of having her daughters settled near her, she gave to them three equal parcels, as large as could be made in three equal circles included within the periphery of her farm, one to each, with a dwelling-house in the centre of each; that is, there were to be three equal circles, as large as could be made within a circle that contained 500 acres. How many acres did the farm of each daughter contain, how many acres did the mother retain, how far apart were the dwelling-houses of the daughters, and how far was the dwelling-house of each daughter from that of the mother?

Ans. Each daughter's farm contained 107 acres 2 roods 31.22+ rods. The mother retained 176 acres 3 roods 26.34+ rods. The distance from one daughter's house to the other was 148.119817+ rods. The mother's dwelling-house was distant from her daughters' 85.51+ rods.

35. James Page has a circular garden, 10 rods in diameter; how many trees can be set in it, so that no two shall be within ten feet of each other, and no tree within two and a half feet of the fence enclosing the garden? Ans. 241.

36. A and B engaged to reap a field for 90 shillings; and as A could reap it in 9 days, they promised to complete it in 5 days. They found, however, that they were obliged to call in C, an inferior workman, to assist them for the last two days, in consequence of which B received 3s. 9d. less than he otherwise would have done. In what time could B and C each reap the field?

Ans. B could reap it in 15 days, and C in 18 days.

37. A merchant tailor bought 40 yards of broadcloth, $2\frac{1}{4}$ yards wide; but on sponging it, it shrunk in length upon every 4 yards half a quarter, and in width, one nail and a half upon every $1\frac{1}{4}$ yards. To line this cloth, he bought flannel 5 quarters wide, which, being wet, shrunk the whole width on every 20 yards in length, and in width it shrunk half a nail. Required the number of yards of flannel used in lining the cloth. Ans. $71\frac{7}{18}$ yards.

38. I have a garden in the form of an equilateral triangle, whose sides are 200 feet. At each corner stands a tower; the height of the first is 30 feet, the second 40, and the third 50. At what distance from the base of each tower must a ladder be placed, that it may just reach the top of each? And what is the length of the ladder, the garden being a horizontal plane?

Ans. The foot of the ladder from the base of the first tower 118.811+ feet; second tower, 115.827+ feet; third tower, 111.875+ feet. Length of the ladder, 122.535+ feet.

THE END.